A picture of a singly linked list:

Node object

pointer

int value

Today: trees!

A tree
Not a tree

Also not a tree
List-like tree

Binary Trees

A binary tree is a particularly important kind of tree where every node has at most two children.

In a binary tree, the two children are called the left and right children.

Not a binary tree
(a general tree)

Binary tree

Binary trees were in A1!

You have seen a binary tree in A1.

A PhD object has one or two advisors. (Confusingly, my advisors are my “children.”)

Adrian Sampson
Luis Ceze
Dan Grossman
Josep Torellas
Greg Morrisett
Tree Terminology

the root of the tree (no parents)

left child of M

right child of M

the leaves of the tree (no children)

Tree Terminology

ancestors of B

descendants of W

Tree Terminology

A node’s depth is the length of the path to the root.
A tree’s (or subtree’s) height is he length of the longest path from
the root to a leaf.

Depth 1, height 2.

Depth 3, height 0.

Tree Terminology

Multiple trees: a forest.

Class for general tree nodes

class GTreeNode<T> {
    private T value;
    private List<GTreeNode<T>> children;
    //appropriate constructors, getters, //setters, etc.
}

Parent contains a list of
its children

General tree
Class for general tree nodes

```java
class GTreeNode<T> {
    private T value;
    private List<GTreeNode<T>> children;
    //appropriate constructors, getters, setters, etc.
}
```

Java.util.List is an interface!
It defines the methods that all implementation must implement.
Whoever writes this class gets to decide what implementation to use — ArrayList? LinkedList? Etc.?

Class for binary tree node

```java
class TreeNode<T> {
    private T value;
    private TreeNode<T> left, right;
    /** Constructor: one-node tree with datum x */
    public TreeNode (T d) {
        datum = d; left= null; right= null;
    }
    /** Constr: Tree with root value x, left tree l, right tree r */
    public TreeNode (T d, TreeNode<T> l, TreeNode<T> r) {
        datum = d; left = l; right = r;
    }
    // more methods: getValue, setValue, getLeft, setLeft, etc.
}
```

Either might be null if the subtree is empty.

Binary versus general tree

In a binary tree, each node has up to two pointers: to the left subtree and to the right subtree:
- One or both could be null, meaning the subtree is empty
  (remember, a tree is a set of nodes)

In a general tree, a node can have any number of child nodes (and they need not be ordered)
- Very useful in some situations ...
- ... one of which may be in an assignment!

An Application: Syntax Trees

```
(1 + (9 - 2)) * 7
```

A Java expression as a string.

```
(1 + (9 - 2)) * 7
```

An expression as a tree.

Applications of Tree: Syntax Trees

- Most languages (natural and computer) have a recursive, hierarchical structure
- This structure is implicit in ordinary textual representation
- Recursive structure can be made explicit by representing sentences in the language as trees: Abstract Syntax Trees (ASTs)
- ASTs are easier to optimize, generate code from, etc. than textual representation
- A parser converts textual representations to AST

Applications of Tree: Syntax Trees

```
In textual representation:
Parentheses show hierarchical structure
Text Tree Representation
-(2+3) -(2 + 3)
```

In tree representation:
Hierarchy is explicit in the structure of the tree
We’ll talk more about expressions and trees in next lecture

```
((2+3) + (5+7))
```

```
(1 + (9 - 2)) * 7
```

```
(1 + (9 - 2)) * 7
```

```
(1 + (9 - 2)) * 7
```

```
(1 + (9 - 2)) * 7
```
A Tree is a Recursive Thing

A binary tree is either null or an object consisting of a value, a left binary tree, and a right binary tree.

Looking at trees recursively

A Recipe for Recursive Functions

Base case:
If the input is “easy,” just solve the problem directly.

Recursive case:
Get a smaller part of the input (or several parts).
Call the function on the smaller value(s).
Use the recursive result to build a solution for the full input.
Recursive Functions on Binary Trees

Base case:
empty tree (null)
or, possibly, a leaf

Recursive case:
Call the function on each subtree.
Use the recursive result to build a solution for the full input.

Searching in a Binary Tree

/** Return true iff x is the datum in a node of tree t */
public static boolean treeSearch(T x, TreeNode<T> t) {
    if (t == null) return false;
    if (x.equals(t.datum)) return true;
    return treeSearch(x, t.left) || treeSearch(x, t.right);
}

Some useful methods – what do they do?

/** Method A ??? */
public static boolean A(Node n) {
    return n != null && n.left == null && n.right == null;
}

/** Method B ??? */
public static int B(Node n) {
    if (n == null) return -1;
    return 1 + Math.max(B(n.left), B(n.right));
}

/** Method C ??? */
public static int C(Node n) {
    if (n == null) return 0;
    return 1 + C(n.left) + C(n.right);
}

Searching in a Binary Tree

// An analog of linear search in lists:
given tree and an object, find out if object is stored in tree
Easy to write recursively, harder to write iteratively

Binary Search Tree (BST)

A binary search tree is a binary tree that is ordered and has no duplicate values. In other words, for every node:
- All nodes in the left subtree have values that are less than the value in that node, and
- All values in the right subtree are greater.

A BST is the key to making search way faster.
Binary Search Tree (BST)

Compare binary tree to binary search tree:

```java
boolean searchBST(n, v):
   if n==null, return false
   if n.v == v, return true
   if v < n.v
      return searchBST(n.left, v)
   else
      return searchBST(n.right, v)
```

Building a BST

- To insert a new item:
  - Pretend to look for the item
  - Put the new node in the place where you fall off the tree

```plaintext
<table>
<thead>
<tr>
<th>BUILDING A BST</th>
</tr>
</thead>
<tbody>
<tr>
<td>january</td>
</tr>
<tr>
<td>january</td>
</tr>
<tr>
<td>january</td>
</tr>
</tbody>
</table>
```

```plaintext
<table>
<thead>
<tr>
<th>BUILDING A BST</th>
</tr>
</thead>
<tbody>
<tr>
<td>january</td>
</tr>
<tr>
<td>february</td>
</tr>
</tbody>
</table>
```

```plaintext
<table>
<thead>
<tr>
<th>BUILDING A BST</th>
</tr>
</thead>
<tbody>
<tr>
<td>january</td>
</tr>
<tr>
<td>february</td>
</tr>
</tbody>
</table>
```
Building a BST

- january
- february
- march
- april

Building a BST

- january
- february
- march
- april
- june
- may
- august
- july
- october
- september
- december
- november

Inserting in Alphabetical Order

- april

Inserting in Alphabetical Order

- april

Inserting in Alphabetical Order

- april
- august
Inserting in Alphabetical Order

- April
- August
- December

Inserting in Alphabetical Order

- April
- August
- December
- February
- January

Insertion Order Matters

- A balanced binary tree is one where the two subtrees of any node are about the same size.
- Searching a binary search tree takes $O(h)$ time, where $h$ is the height of the tree.
- In a balanced binary search tree, this is $O(\log n)$.
- But if you insert data in sorted order, the tree becomes imbalanced, so searching is $O(n)$.

Printing contents of BST

Because of ordering rules for a BST, it's easy to print the items in alphabetical order:
- Recursively print left subtree
- Print the node
- Recursively print right subtree

/** Print BST t in alpha order */
private static void print(TreeNode<T> t) {
  if (t == null) return;
  print(t.left);
  System.out.print(t.value);
  print(t.right);
}

Tree traversals

- "Walking" over the whole tree is a tree traversal
  - Done often enough that there are standard names
  - Previous example: in-order traversal
    - Process left subtree
    - Process root
    - Process right subtree
  - Note: Can do other processing besides printing

Other standard kinds of traversals
- preorder traversal
- Process root
- Process left subtree
- Process right subtree
- postorder traversal
- Process left subtree
- Process right subtree
- Process root
- level-order traversal
- Not recursive: uses a queue (we’ll cover this later)

Useful facts about binary trees

Max # of nodes at depth d: $2^d$

If height of tree is $h$
- min # of nodes: $h + 1$
- max # of nodes in tree: $2^0 + \ldots + 2^h = 2^{h+1} - 1$

Complete binary tree:
- All levels of tree down to a certain depth are completely filled
Things to think about

What if we want to delete data from a BST?

A BST works great as long as it’s balanced.
There are kinds of trees that can automatically keep themselves balanced as you insert things!

Tree Summary

- A tree is a recursive data structure
  - Each node has 0 or more successors (children)
  - Each node except the root has exactly one predecessor (parent)
  - All node are reachable from the root
  - A node with no children (or empty children) is called a leaf
- Special case: binary tree
  - Binary tree nodes have a left and a right child
  - Either or both children can be empty (null)
- Trees are useful in many situations, including exposing the recursive structure of natural language and computer programs