

## Prelim 2

5:30 PM, 25 April 2017

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Question	Name	Short answer	Search/sort	Collections stuff	Trees	Graphs	
Max	1	26	18	15	20	20	100
Score							
Grader							

The exam is closed book and closed notes. Do not begin until instructed.

You have **90 minutes**. Good luck!

Write your name and Cornell **NetID**, legibly, at the top of **every** page! There are 5 questions on 7 numbered pages, front and back. Check that you have all the pages. When you hand in your exam, make sure your pages are still stapled together. If not, please use our stapler to reattach all your pages!

We have scrap paper available. If you do a lot of crossing out and rewriting, you might want to write code on scrap paper first and then copy it to the exam so that we can make sense of what you handed in.

Write your answers in the space provided. Ambiguous answers will be considered incorrect. You should be able to fit your answers easily into the space provided.

In some places, we have abbreviated or condensed code to reduce the number of pages that must be printed for the exam. In others, code has been obfuscated to make the problem more difficult. This does not mean that it's good style.

**Academic Integrity Statement:** I pledge that I have neither given nor received any unauthorized aid on this exam. I will not talk about the exam with anyone in this course who has not yet taken prelim 2.

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(signature)

### 1. Name (1 point)

Write your name and NetID at the top of **every** page of this exam.

## 2. Short Answer (26 points.)

(a) **Asymptotic complexity. 8 points.** Be sure to answer both (1) and (2) of this question.

(1) Two correct algorithms  $A_f$  and  $A_g$  have running times given by the functions  $f(n)$  and  $g(n)$  respectively. Assume  $f(n)$  is  $O(g(n))$  and  $g(n)$  is  $O(f(n))$ . Could there be a reason to choose one algorithm over another in a practical setting? Answer YES or NO and give a short reason for your answer.

(2) Give the tightest asymptotic complexity in terms of  $n$  (e.g.  $O(n^3)$ ) of the following algorithm. Explain your answer.

```
int sum= 0;
for (int h= 1; h <= n; h= h + 1) {
    for (int k= Math.min(10000, h); 0 < k; k= k - 1) {
        sum= sum + 10;
    }
}
sum= sum + 9999;
```

(b) **Hashing. 5 points.** We are implementing a set in an array  $b$  using linear probing. The set currently has size  $n$ . Give the tightest worst-case complexity formula for enumerating all elements of the set. Do not use the load factor, which for the purpose of this question is unknown.

Hint: think about what has to be done to find all the elements in the set.

(c) **Hashing. 7 points.** An instance of class CMSGroup below maintains information about two students who grouped for an assignment on the CMS. We include only information that is necessary for this question. Class Student, among other things, has its own functions *equals* and *hashCode*.

We expect some program written by the instructors to use a hash set of objects of CMSGroup to maintain information about grouping, so class CMSGroup needs functions *equals* and *hashCode*. Complete these functions below. Notes: (1) The order of the students shouldn't matter. (2) Choose a reasonable but simple *hashCode*; it could be based on the *hashCodes* of the students.

```
/* An instance maintains info about a grouped pair for a CMS assignment */
public class CMSGroup {
    Student s1; // s1 and s2 are different Students
    Student s2; // according to function equals in class Student.

    /** Return true iff g is a CMSGroup whose students are equal to
        those in this object. */
    public @Override boolean equals(Object g) {

    }

    public @Override int hashCode() {

    }
}
```

(d) **6 points** Write the steps involved in executing a procedure call  $m(p, q)$  on static procedure  $m$ .

### 3. Sorting (18 points.)

(a) **8 points** A stable sorting algorithm maintains the relative order of equal values. For example if  $b[i]$  and  $b[j]$  are equal,  $i < j$ , and  $b[i]$  and  $b[j]$  are moved to  $b[i1]$  and  $b[j1]$  by the sorting algorithm, then  $i1 < j1$ . Suppose a 3-element array contains [3, 2, 3]. A sorting algorithm that changes it to [2, 3, 3] is not stable because it switched the order of the two 3's.

For each of the following sorting algorithms, tell us whether (1) it is stable, (2) it is unstable, or (3) it depends on the implementation.

1. insertion sort
2. selection sort
3. quicksort
4. mergesort

**(b) 6 points** The code for merge sort is given below. It contains errors. Fix the errors. Assume that method `merge` has the specification shown after the method.

```
/** Sort b[h..k]. */
public static void mergeSort(Comparable[] b, int h, int k) {

    if (h < k) return;

    int e= h / 2;

    merge(b, h, e, k);

    mergeSort(b, h, e);

    mergeSort(b, e, k);

}

/** b[h..j] and b[j+1..k] are sorted.
 * Merge them together so that b[h..k] is sorted. */
public static void merge(Comparable[] b, int h, int j, int k) {...}
```

**(4) 4 points** Give the following information about the time- and space-complexity of Quicksort on an array of size  $n$ .

1. Quicksort, worst-case time:
2. Quicksort, expected time:
3. Our initial Quicksort algorithm has worst-case space:
4. The initial Quicksort algorithm can be modified to get worst-case space down to:

#### 4. Collections classes/interfaces (15 points.)

This question concerns interface `Set<E>`, which, among others, has these methods:

- `boolean contains(E ob)`: Return true iff this set contains `ob`.
- `Iterator<E> iterator()`: Return an iterator over the elements in this set.
- `int size()`: Return the number of elements in this set.
- `E[] toArray()`: Return an array containing all the elements in this set.

For this question, consider a deck of  $0 \leq n \leq 52$  cards, which are objects of class `Card`.

**(a) 5 points.** Interface `Set<E>` lacks methods to extract elements from the set, but a `Set<E>` is iterable. Write the body of function `pickAnyCard` below —it will probably use a for-each loop.

```
/** Return any element of s. Precondition: s is not empty. */
public static Card pickAnyCard(Set<Card> s) {

}
}
```

**(b) 5 points.** Is the element returned by (a) guaranteed to be a randomly selected element? BRIEFLY justify your answer. Hint: Remember that `Set<E>` is an interface.

**(c) 5 points.** Assume that a variable `r` of type `Random` is available and that `r` was initialized correctly. Variable `r` has an instance method `nextInt(int max)`, and each call `r.nextInt(m)` returns a new random integer in the range `0..m`.

Write a one-line implementation of `pickAnyCard` given below. Hint: look at the methods available in interface `Set<E>`. You don't have to use a for-each loop (but you can)! We give partial credit for solutions that require several lines of code, but for full credit, your solution must (1) be a single return statement, (2) be correct, and (3) if called often enough, would return every card in the deck at least once. Your solution need not be time-efficient.

```
/** Return a random element of s. Precondition: s is not empty. */
public static Card pickAnyCard(Set<Card> s) {

}
}
```

**5. Trees (20 points.)**

(a) **10 points** Consider the following class Node. Complete function isBST given after it.

```
/** An instance is a node of a tree (or a subtree rooted at that node). */
public static class Node {
    public int val; // the value in this node
    public Node left; // left subtree (or null if none)
    public Node rite; // right subtree (or null if none)
    ...
}

/** Return true if n is the root of a binary search tree (BST)
    whose values are in the range h..k.
    Precondition: n is not null and is indeed the root of a binary tree. */
public static boolean isBST(Node n, int h, int k) {

}
}
```

(b) **5 points** A heap of integers is maintained in an int array b. Suppose the heap has size 10 and b[0..9] contains these 10 integers:

[1 4 6 5 6 9 8 8 7 9]

Below, show what b[0..8] is like after one call to poll(). We cannot give partial credit if you don't show your work.

(c) **5 points** It has been said that a binary tree with no duplicate values in its nodes can be reconstructed from its preorder and inorder traversals. Write down the first two steps in doing this —how do you tell (1) what the root of the tree is and (2) what is in its left and right subtrees? Do not give us an example. Just tell us in English what these two steps are.

## 6. Graphs (20 points.)

(a) **8 points** Complete the body of the following procedure. Keep things abstract: to visit a node  $p$  write simply “visit  $p$ ” and to ask whether  $p$  has been visited write “if ( $p$  has been visited)” or “if ( $p$  has not been visited)”. Similarly, you can write about the neighbors of a node in an abstract way, as we have done in lecture.

```
/** Visit all nodes reachable from node n along paths of unvisited nodes. */  
public void DFS(Node n) {
```

```
}
```

(b) **4 points** State the theorem that is proved in our development of Dijkstra’s shortest path algorithm. It talks about a node that can be moved to the Settled set.

(c) **4 points** Topological sort numbers the nodes of a DAG (Directed Acyclic Graph). It is used, for example, to determine an order in which tasks can be performed on an assembly line.

Write down the property that is used to determine which node is chosen first in a topological sort.