## Introduction

Consider this invariant and postcondition:


!B \&\& P implies R

What condition along with the invariant tells us that the postcondition is true? Without mentioning indices, we can say that the query segment must be empty. When is it empty? The formula Follower minus First tells us when j -h is 0 , or when $\mathrm{h}=\mathrm{j}$. So, the loop should stop when this is not the case. Thus, we write:

$$
\text { while }(\mathrm{h}!=\mathrm{j})\{\ldots\} \quad \text { or } \quad \text { while }(\mathrm{h}<\mathrm{j})\{\ldots\}
$$

Wasn't that easy? In general, with array diagrams, the loop will terminate with some segment empty, and we just have to write down the condition for it being empty.

The condition for a segment to be empty is: First $=$ Follower or First $>=$ Follower
Some people will write
while (j ! = h) $\{\ldots\}$
or
while $(\mathrm{j}>\mathrm{h})\{\ldots\}$

While these are correct, we don't like them because $h$ is visually before $j$ in the invariant, so we like it that way in formulas as well. Anything we can do to make relationships easy to see helps.

Here's a similar example, but $j$ is before rather than after the second vertical line:

R: b


When we see such a case, we don't need to deal with the Follower minus First formula. We recognize the pattern, and we know that segment $\mathrm{b}[\mathrm{h} . \mathrm{j}]$ has a value in it if $\mathrm{h} \leq \mathrm{j}$. (Note that if $\mathrm{h}=\mathrm{j}, \mathrm{b}[\mathrm{h} . . \mathrm{j}]$, or $\mathrm{b}[\mathrm{h} . . \mathrm{h}]$, contains one value.) So we immediately write the loop condition

$$
\text { while }(\mathrm{h}<=\mathrm{j})\{\ldots\}
$$

Below, we give you two exercises. For each, determine the loop condition B, given the invariant and postcondition. The answers are given at the end of the pdf script for this video.

## Exercises

1. (From a binary search algorithm)


Practice with the second loopy question and array diagrams
2. (From the Dutch National Flag problem)


## Answers

1. The loop condition is $\mathrm{h}+1!=\mathrm{t}$ or $\mathrm{h}+1<\mathrm{t}$.
2. The loop condition is $\mathrm{h}<=\mathrm{k}$.
