Assignment A7 available
Due 2 days after prelim 2.

Implement Dijkstra's shortest-path algorithm.
Start with our abstract algorithm, implement it in a specific setting. Our method: 36 lines, including extensive comments.
We will make our solution to A6 available after the deadline for late submissions.

Last semester: median: 4.0, mean: 3.84. But our abstract algorithm is much closer to the planned implementation than last fall, and we expect a much lower median and mean.

Dijkstra's algorithm using Nodes.

An object of class Node for each node of the graph.
Nodes have an identification, (S, A, E, etc).
Nodes contain shortest distance from Start node (red).

Execution times for ArrayList methods, etc.

Several questions on the Piazza about how fast various methods are in ArrayList, HashMap, etc.
Please please look at the Java API documentation for these classes! All the information is there! For example, I will demo googling
ArrayList B java
and show you, in class.
Also, look in the FAQs note for an assignment before asking a question about that assignment!

Backpointers

Shortest path requires not only the distance from start to a node but the shortest path itself. How to do that?
In the graph, red numbers are shortest distance from S.

Need shortest path from S to every node. Storing that info in node S wouldn’t make sense.
Backpointers

Shortest path requires not only the distance from start to a node but the shortest path itself. How to do that?

In the graph, red numbers are shortest distance from S.

In each node, store (a pointer to) previous node on the shortest path from S to that node. **Backpointer**

Each iteration of Dijkstra's algorithm

dist: shortest-path length calculated so far

\[
f= \text{node in Frontier with min spl; Remove } f \text{ from Frontier; for each neighbor } w \text{ of } f:\n\]

if \( w \) in far-off set
then \[ w.\text{spl} = \text{f.dist} + \text{weight(f, w)}; \]
Put \( w \) in the Frontier;
\[ w.\text{backPointer} = f; \]
else if \( f.\text{dist} + \text{weight(f, w)} < w.\text{spl} \)
then \[ w.\text{dist} = f.\text{dist} + \text{weight(f, w)} \]
\[ w.\text{backPointer} = f; \]

Undirected trees

• An undirected graph is a **tree** if there is exactly one simple path between any pair of vertices

Root of tree? It doesn’t matter. Choose any vertex for the root

Facts about trees

Consider a graph with these properties:

1. \(|E| = |V| - 1\)
2. **connected**
3. **no cycles**

Any two of these properties imply the third—and imply that the graph is a tree

A spanning tree of a connected undirected graph \((V, E)\) is a subgraph \((V, E')\) that is a tree

- Same set of vertices \(V\)
- \(E' \subseteq E\)
- \((V, E')\) is a tree

- Same set of vertices \(V\)
- Maximal set of edges that contains no cycle

- Same set of vertices \(V\)
- Minimal set of edges that connect all vertices

Three equivalent definitions
Spanning trees: examples

Finding a spanning tree

A subtractive method

- Start with the whole graph – it is connected
- If there is a cycle, pick an edge on the cycle, throw it out – the graph is still connected (why?)
- Repeat until no more cycles

Finding a spanning tree: Additive method

- Start with no edges
- While the graph is not connected:
  Choose an edge that connects 2 connected components and add it – the graph still has no cycle (why?)

Tree edges will be red.
Dashed lines show original edges.
Left tree consists of 5 connected components, each a node

Minimum spanning trees

- Suppose edges are weighted (> 0), and we want a spanning tree of minimum cost (sum of edge weights)
- Some graphs have exactly one minimum spanning tree. Others have several trees with the same cost, any of which is a minimum spanning tree

Use:
Maximal set of edges that contains no cycle

Use:
Minimal set of edges that connects all vertices

Nondeterministic algorithm
Minimum spanning trees

- Suppose edges are weighted (> 0), and we want a spanning tree of minimum cost (sum of edge weights)
- Useful in network routing & other applications
- For example, to stream a video

Greedy algorithm

A greedy algorithm: follow the heuristic of making a locally optimal choice at each stage, with the hope of finding a global optimum

Example. Make change using the fewest number of coins. Make change for n cents, n < 100 (i.e. < 51)
Greedy: At each step, choose the largest possible coin
If n ≥ 50 choose a half dollar and reduce n by 50;
If n ≥ 25 choose a quarter and reduce n by 25;
As long as n ≥ 10, choose a dime and reduce n by 10;
If n ≥ 5, choose a nickel and reduce n by 5;
Choose n pennies.

Greedy algorithm

A greedy algorithm: follow the heuristic of making a locally optimal choice at each stage, with the hope of finding a global optimum.

Doesn’t always work

Example. Make change using the fewest number of coins. Coins have these values: 7, 5, 1
Greedy: At each step, choose the largest possible coin
Consider making change for 10. The greedy choice would choose: 7, 1, 1, 1.
But 5, 5 is only 2 coins.

Greediness doesn’t work here

You’re standing at point x, and your goal is to climb the highest mountain.
Two possible steps: down the hill or up the hill. The greedy step is to walk up hill. But that is a local optimum choice, not a global one. Greediness fails in this case.

Construct minimum spanning tree (greedy)

As long as there is a cycle:
Find a black max-weight edge – if it is on a cycle, throw it out otherwise keep it (make it red)

We mark a node red to indicate that we have looked at it and determined it can’t be removed because removing it would unconnect the graph (the node is not on a cycle)
Construct minimum spanning tree (greedy)

As long as there is a cycle:
Find a black max-weight edge – if it is on a cycle, throw it out otherwise keep it (make it red)

Throw out edge with weight 6
Can’t throw out 5: make it red

Throw out one 4

No more cycles: done

Maximal set of edges that contain no cycle

Non-deterministic algorithm

Two greedy algorithms for constructing a minimum spanning tree

- **Kruskal**
- **Prim**

Both use this definition of a spanning tree and in a greedy fashion:

Minimal set of edges that connect all vertices

Both are non-deterministic, in that at a point they may choose one of several nodes with equal weight

Kruskal’s algorithm: greedy

At each step, add an edge (that does not form a cycle) with minimum weight

edge with weight 2
edge with weight 3

one of the 4’s
the 5

Dashed edges: original graph
Red edges: the constructed spanning tree

Prim’s algorithm: greedy

Have start node.

Edge with weight 3

Invariant: The added edges (and their nodes) are connected

Edge with weight 5

One of the 4’s

The 2

Tree greedy spanning tree algorithms

1. Algorithm that uses this property of a spanning tree: Maximal set of edges that contains no cycle

2. Algorithms that use this property of a spanning tree: Minimal set of edges that connect all vertices
   (a) Kruskal   (b) Prim

When edge weights are all distinct, or if there is exactly one minimum spanning tree, all 3 algorithms construct the same tree.
Prim’s algorithm (n nodes, m edges)

```c
prim(s) {
    D[s]= 0; // start vertex
    D[i]= ∞ for all i ≠ s;
    while (a vertex is unmarked) {
        v= unmarked vertex
            with smallest D;
        mark v;
        for (each w adj to v)
            D[w]= min(D[w], c(v,w));
    }
}
```

- $O(m + n \log n)$ for adj list
- Use a priority queue PQ
- Regular PQ produces time $O(n + m \log m)$
- Can improve to $O(m + n \log n)$ using a fancier heap

### Application of MST

Maze generation using Prim’s algorithm

More complicated maze generation

![Maze generation using Prim's algorithm](http://en.wikipedia.org/wiki/File:MAZE_30x20_Prim.ogv)

Greedy algorithms

- These are Greedy Algorithms
- Greedy Strategy: is an algorithm design technique like Divide & Conquer
- Greedy algorithms are used to solve optimization problems
- Goal: find the best solution
- Works when the problem has the greedy-choice property:
  A global optimum can be reached by making locally optimum choices

Example: Making change

Given an amount of money, find smallest number of coins to make that amount

Solution: Use Greedy Algorithm:

Use as many large coins as you can.

Produces optimum number of coins for US coin system

May fail for old UK system

Similar code structures

```c
while (a vertex is unmarked) {
    v= best unmarked vertex
    mark v;
    for (each w adj to v)
        update D[w];
    c(v,w) is the v→w edge weight
}
```

Traveling salesman problem

Given a list of cities and the distances between each pair, what is the shortest route that visits each city exactly once and returns to the origin city?

- The true TSP is very hard (called NP complete)… for this we want the perfect answer in all cases.
- Most TSP algorithms start with a spanning tree, then "evolve" it into a TSP solution. Wikipedia has a lot of information about packages you can download…