TREES
Announcements

- Prelim 1 on Tuesday!
  - Review session on Sunday 1-3PM
  - Get review handout from website
  - Unless you hear otherwise, take it at your time 5:30 or 7:30 in the Kennedy Auditorium

- A4 will be posted today

- Mid-semester TA evaluations are coming up; please participate! Your feedback will help our staff improve their teaching ---this semester.
Tree Overview

Tree: data structure with nodes, similar to linked list
- Each node may have zero or more successors (children)
- Each node has exactly one predecessor (parent) except the root, which has none
- All nodes are reachable from root

Binary tree: tree in which each node can have at most two children: a left child and a right child
Binary trees were in A1!

You have seen a binary tree in A1.

A PhD object phd has one or two advisors. 
Here is an intellectual ancestral tree!

```
  phd
 /    \
 ad1   ad2
 /      /
 ad1   ad2
 /  \
 ad1 ad1
```
Tree terminology

\(M\): root of this tree
\(G\): root of the left subtree of \(M\)
\(B, H, J, N, S\): leaves (their set of children is empty)
\(N\): left child of \(P\); \(S\): right child of \(P\)
\(P\): parent of \(N\)
\(M\) and \(G\): ancestors of \(D\)
\(P, N, S\): descendants of \(W\)
\(J\) is at depth 2 (i.e. length of path from root = no. of edges)
\(W\) is at height 2 (i.e. length of longest path to a leaf)
A collection of several trees is called a ...?
class TreeNode<T> {
    private T datum;
    private TreeNode<T> left, right;

    /** Constructor: one node tree with datum x */
    public TreeNode (T d) { datum= d; left= null; right= null; }

    /** Constr: Tree with root value x, left tree l, right tree r */
    public TreeNode (T d, TreeNode<T> l, TreeNode<T> r) { 
        datum= d; left= l; right= r; 
    }
}

more methods: getDatum, setDatum, getLeft, setLeft, etc.
Binary versus general tree

In a binary tree, each node has up to two pointers: to the left subtree and to the right subtree:

- One or both could be `null`, meaning the subtree is empty (remember, a tree is a set of nodes)

In a general tree, a node can have any number of child nodes (and they need not be ordered)

- Very useful in some situations ...
- ... one of which may be in an assignment!
Class for general tree nodes

class GTreeNode<T> {
    private T datum;
    private List<GTreeNode<T>> children;
    //appropriate constructors, getters, setters, etc.
}

Parent contains a list of its children
Applications of Tree: Syntax Trees

- Most languages (natural and computer) have a recursive, hierarchical structure

- This structure is *implicit* in ordinary textual representation

- Recursive structure can be made *explicit* by representing sentences in the language as trees: Abstract Syntax Trees (ASTs)

- ASTs are easier to optimize, generate code from, etc. than textual representation

- A *parser* converts textual representations to AST
Applications of Tree: Syntax Trees

In textual representation:
Parentheses show hierarchical structure

In tree representation:
Hierarchy is explicit in the structure of the tree

We’ll talk more about expression and trees on Thursday
Recursion on trees

Trees are defined recursively, so recursive methods can be written to process trees in an obvious way.

Base case
- empty tree (null)
- leaf

Recursive case
- solve problem on each subtree
- put solutions together to get solution for full tree
Searching in a Binary Tree

/** Return true iff x is the datum in a node of tree t*/
public static boolean treeSearch(T x, TreeNode<T> t) {
    if (t == null) return false;
    if (Objects.equals(t.datum, x) return true;
    return treeSearch(x, t.left) || treeSearch(x, t.right);
}

• Analog of linear search in lists:
  given tree and an object, find out if object is stored in tree
• Easy to write recursively, harder to write iteratively
Binary Search Tree (BST)

If the tree data is ordered and has no duplicate values:
in every subtree,
   All left descendants of a node come before the node
   All right descendants of a node come after the node
Search can be made MUCH faster

```java
/** Return true iff x if the datum in a node of tree t. */
boolean treeSearch (int x, TreeNode<Integer> t) {
    if (t==null) return false;
    if (x < t.datum.intValue()) return treeSearch(x, t.left);
    else if (x == t.datum.intValue()) return true;
    else return treeSearch(x, t.right);
}
```
To insert a new item
- Pretend to look for the item
- Put the new node in the place where you fall off the tree
- This can be done using either recursion or iteration

Example
- Tree uses alphabetical order
- Months appear for insertion in calendar order
What can go wrong?

A BST makes searches very fast, *unless...*

- Nodes are inserted in increasing order
- In this case, we’re basically building a linked list (with some extra wasted space for the *left* fields, which aren’t being used)

BST works great if data arrives in random order
Because of ordering rules for a BST, it’s easy to print the items in alphabetical order:

- Recursively print left subtree
- Print the node
- Recursively print right subtree

```java
/** Print BST t in alpha order */
private static void print(TreeNode<T> t) {
    if (t == null) return;
    print(t.left);
    print(t.right);
    System.out.print(t.datum);
}
```
Tree traversals

“Walking” over the whole tree is a tree traversal

- Done often enough that there are standard names

Previous example: in-order traversal
- Process left subtree
- Process root
- Process right subtree

Note: Can do other processing besides printing

Other standard kinds of traversals

- preorder traversal
  - Process root
  - Process left subtree
  - Process right subtree

- postorder traversal
  - Process left subtree
  - Process right subtree
  - Process root

- level-order traversal
  - Not recursive uses a queue. We discuss later
Some useful methods

/** Return true iff node t is a leaf */
public static boolean isLeaf(TreeNode<T> t) {
    return t != null && t.left == null && t.right == null;
}

/** Return height of node t (postorder traversal) */
public static int height(TreeNode<T> t) {
    if (t == null) return -1; //empty tree
    return 1 + Math.max(height(t.left), height(t.right));
}

/** Return number of nodes in t (postorder traversal) */
public static int numNodes(TreeNode<T> t) {
    if (t == null) return 0;
    return 1 + numNodes(t.left) + numNodes(t.right);
}
Useful facts about binary trees

Max # of nodes at depth d: \(2^d\)

If height of tree is h
- min # of nodes: \(h + 1\)
- max # of nodes in tree:
  \[2^0 + \ldots + 2^h = 2^{h+1} - 1\]

Complete binary tree
- All levels of tree down to a certain depth are completely filled
Things to think about

What if we want to delete data from a BST?

A BST works great as long as it’s balanced

How can we keep it balanced? This turns out to be hard enough to motivate us to create other kinds of trees
A tree is a recursive data structure:

- Each node has 0 or more successors (children)
- Each node except the root has exactly one predecessor (parent)
- All nodes are reachable from the root
- A node with no children (or empty children) is called a leaf

Special case: binary tree

- Binary tree nodes have a left and a right child
- Either or both children can be empty (null)

Trees are useful in many situations, including exposing the recursive structure of natural language and computer programs.