Prelim 2 Solution

CS 2110, November 19, 2015, 7:30 PM

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<th>Question</th>
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<tr>
<td>Max Score</td>
<td>20</td>
<td>15</td>
<td>13</td>
<td>14</td>
<td>17</td>
<td>21</td>
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The exam is closed book and closed notes. Do not begin until instructed.

You have 90 minutes. Good luck!

Write your name and Cornell **NetID** at the top of EACH page! There are 6 questions on 8 numbered pages, front and back. Check that you have all the pages. When you hand in your exam, make sure your booklet is still stapled together. If not, please use our stapler to reattach all your pages!

We have scrap paper available. If you do a lot of crossing out and rewriting, you might want to write code on scrap paper first and then copy it to the exam, so that we can make sense of what you handed in.

Write your answers in the space provided. Ambiguous answers will be considered incorrect. You should be able to fit your answers easily into the space provided.

In some places, we have abbreviated or condensed code to reduce the number of pages that must be printed for the exam. In others, code has been obfuscated to make the problem more difficult. This does not mean that its good style.
1. **True / False** (20 points)

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</table>
If the value of function `equals(Object)` in a class depends only on fields \( b \) and \( c \), function `hashCode()` in that class has to depend also on only those two fields.

One can’t use a “for-each” statement to process the entries of a `HashMap` because the keys of a `HashMap` are not ordered in any way.

The purpose of interface `Iterable` is to make it possible to use a for-each statement.

2. **Short Answer** (15 points)

2.a **Hashing** (9 points)

Consider implementing a set using hashing with linear probing. Assume an array of 6 elements of class `Integer`, as shown below. We define the hash function \( f(i) = (3 * i) \mod 6 \), where 6 is the table size. For example, hashing 4 gives 12 mod 6, which is 0.

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<th>2</th>
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</thead>
<tbody>
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<td>null</td>
<td>5</td>
<td>null</td>
<td>1</td>
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</tbody>
</table>

(i) **5 points** Insert the values 5, 3, 5, 4, and 1 into the set, in that order —write the values in the appropriate element in the table above, crossing off the value currently in that element. Do not re-size the array, even though this is the standard way to implement a hash table.

(ii) **2 points** Now remove the value 1 from the set. Do you simply set the array element to `null`? Explain why or why not.

No. We set the value of `isInSet` to `false`. If we simply set the array element to `null`, then the linear probing algorithm might stop searching for an element before it should.

(iii) **2 points** Consider the insertion of values in point (i) above. Which insertion (if any) caused the load factor to surpass 0.4?

Insertion of the “4” changes the load factor to .5.

2.b **Exception Handling** 6 points

(i) **2 points** Consider the statement below, appearing in a method `m`, where `b` is an int array. Does its execution result in a `RuntimeException` being thrown out to the call of `m`? Write your answer and an explanation for it to the right of the statement.

```java
try {
    throw new RuntimeException();
}
```

```java
catch (Exception e) {
    int x= b[-1];
}
```

3 of 8
Yes. The line `int x = b[-1];` in the catch block throws an `ArrayIndexOutOfBoundsException`, which is a subclass of `RuntimeException`, that is not caught in `m`.

(ii) 4 points  To the right below, write down what is printed by the `println` statements during execution of the call `mm(1)`, where method `mm()` is defined as follows:

```java
public static void mm(int x) {
    try {
        System.out.println("11");
        int b = 5/(x-1);
        System.out.println("12");
        return;
    } catch (RuntimeException e) {
        System.out.println("13");
        int c = 5/(x-2);
        System.out.println("14");
    }
    System.out.println("15");
    int d = 5/(x-1);
    System.out.println("16");
    return;
}
```

11
13
14
15

3. Complexity (13 points)

(a) 4 points  For each of the functions `f` below, state the function `g(n)` such that `f(n)` is `O(g(n))`. `g(n)` should be as small as possible. (e.g. one could say that `f(n) = 2n^2` is `O(n^3)`, but the best answer, the one we want, is `O(n^2)`.)

(i) \( f(n) = n \log(n) + n + n^2 \).  \( g(n) = n^2 \)
(ii) \( f(n) = 2 + \frac{1500}{n} + 42n^3 \).  \( g(n) = n^3 \)
(iii) \( f(n) = 2^{n+4} + 300n^2 \).  \( g(n) = 2^n \)
(iv) \( f(n) = 56 \).  \( g(n) = 1 \)
(b) 3 points State the tightest (smallest) asymptotic time complexity (in terms of \( n \)) of the following statement sequence:

\[
\text{int } s = 0; \\
\text{for (int } k = 0; k < 7; k = k+1) \{ \\
\text{for (int } j = k-n; j < k; j = j+1) \{ \\
\text{ s = s + j*k; } \\
\text{ } \}
\}
\]

\( O(n) \). The outer loop has 7 iterations only; that does not depend on \( n \).

(c) 6 points Give a formal proof that \( f(n) = 30n + 2n^2 \) is \( O(n^2) \).

\[
f(n) = \begin{align*}
\text{<definition of } f > & \\
30n + 2n^2
\end{align*}
\]

\[
\leq \begin{align*}
\text{<for n } &\geq 1 > \\
30n^2 + 2n^2
\end{align*}
\]

\[
\begin{align*}
\text{= <arithmetic> } \\
32n^2
\end{align*}
\]

So, use \( c = 32, N = 1 \)

4. Searching, Sorting, and Invariants (14 points)

(a) 6 points Assume that this procedure has already been written:

```java
/** Partition b[e..f] by a random pivot in b[e..f] 
 * Return the index j of the pivot, so that 
 * b[e..j-1] <= b[j] <= b[j+1..f] */
public static int partition(int[] b, int e, int f) {...}
```

Below, write the body of procedure QS, completely in Java. Solution

```java
/** Sort b[e..f], using the quicksort algorithm. */
public static void QS(int[] b, int e, int f) {
    if (f <= e) return;
    int p = partition(b, e, f);
    QS(b, e, p - 1);
    QS(b, p + 1, f);
}
```
(b) 8 points  Below is the precondition and postcondition of an algorithm to swap the positive values in \( b[e..f] \) into the end of \( b[e..f] \). Note that \( e \) is not necessarily 0 and \( f \) is not necessarily \( b.length-1 \). Do not change \( e \) and \( f \). You may assume the following procedure has already been written:

```java
/** Swap b[i] and b[j]. */
public static void swap(int[] b, int i, int j) {...}
```

Complete procedure \( p \) below using a loop that uses the given loop invariant to accomplish this task.

```
	/** Modify b[e..f] as defined above. */
	public static void p(int[] b, int e, int f) {
		int h = f + 1; // 2 pts for init making inv true
		int k = f; // 2 pts for inv && !cond imply result
		while (e != h) { // 2 pts for progress toward termination
			if (b[h - 1] <= 0) { // 2 pts for maintaining inv
				h= h - 1; // pts may be deducted for doing what
			} else { // shouldn’t be done (e.g. change e or f)
				swap(b, h - 1, k);
				h= h - 1;
				k= k - 1;
			}
		}
	}
```

5. Trees (17 points)

5.a Binary Search Trees (9 points)

Assume that each node of a binary search tree (BST), of class Node, has these fields:
- Node left: the left subtree (null if empty)
- Node right: the right subtree (null if empty)
- int data: the data of the node

Write the body of the following method, which appears in class Node:

Solution
/** Return true if v is in this tree and false otherwise.
 * Takes O(d) time, where d is the maximum depth of this tree
 * Precondition this is a BST */
public boolean contains(int v) {
    if (v == data) return true;
    if (v < data) return left != null && left.contains(v);
    return right != null && right.contains(v);
}

5.b Heaps (8 points)
Consider writing heapsort to sort an int array b into descending order. Complete the implementation of step (2) in the method below. You do not have to concern yourself with the implementation of step (1). In implementing step (2):
• Remember that b[0] is the root of the heap and is the smallest value (it is a min-heap).
• Assume that function int poll(int[] b, int k) has already been written and can be used. It assumes that b[0..k − 1] is a heap, removes the root, does what is necessary to make b[0..k − 2] back into a heap, and returns the removed value.

Solution
/** Sort array b using heapsort */
public static void heapsort(int[] b) {
    // (1) Make b[0..b.length-1] into a min-heap
    heapify(b);

    // (2) Poll values from the heap and put them into their
    // sorted (in descending order) position in b
    for (int i = b.length - 1; i >= 0; i--) {
        b[i] = poll(b, i + 1);
    }
}

6. Graphs (21 points)
(a) 3 points There are two basic ways to implement a graph: (1) an adjacency matrix and (2) an adjacency list. Let a graph have n nodes and e edges. Below, for each point, state to the right which representation of a graph has that property:

(i) Takes time $O(n)$ to iterate over the edges that leave given node $n$: Adjacency matrix
(ii) Takes time $O(n)$ to determine whether there is an edge from node $n_1$ to node $n_2$: Adjacency list
(iii) Uses space $O(n^2)$: Adjacency matrix

(b) 5 points For the graph below, give a list of the nodes that are visited by the recursive depth-first search algorithm starting at node A, in the order visited. Whenever there is a choice of processing nodes in any way, process them in alphabetical order by their names. Example:
to do something with nodes G, A, D, first do A, then D, and then G.

A, B, C, E, D, F, G

(c) 3 points  Is the following graph a planar graph? Write yes or no to its right.

Yes

(d) 3 points  State the difference between Prim’s algorithm and Kruskal’s algorithm for constructing a spanning tree of an undirected graph.

Kruskal’s algorithm selects the smallest edge that does not introduce a cycle and adds it to the tree each iteration. Prim’s algorithm starts with a specific node and selects the smallest edge leaving the connected component being built each iteration.

(e) 7 points  Complete recursive algorithm dfs, given below. Do not be concerned about how visiting occurs. You may simply say “visit m” and “if m is unvisited”.

Solution

```java
/** Visit all nodes that are reachable along unvisited paths from m. 
 * Precondition: m is unvisited. */
public static void dfs(Node m) {
    visit m;
    for (Node v : m.neighbors) { // Could have said this a bit differently.
        if (v is unvisited) { // We were not particular as long as it
            dfs(v); // was understandable and correct
        }
    }
}
```