Graphs - II

CS 2110, Fall 2016

Announcements

- A6 will be available tonight or tomorrow.
- Gries lunch canceled today
- Gries office hours today from 1 to 2 only

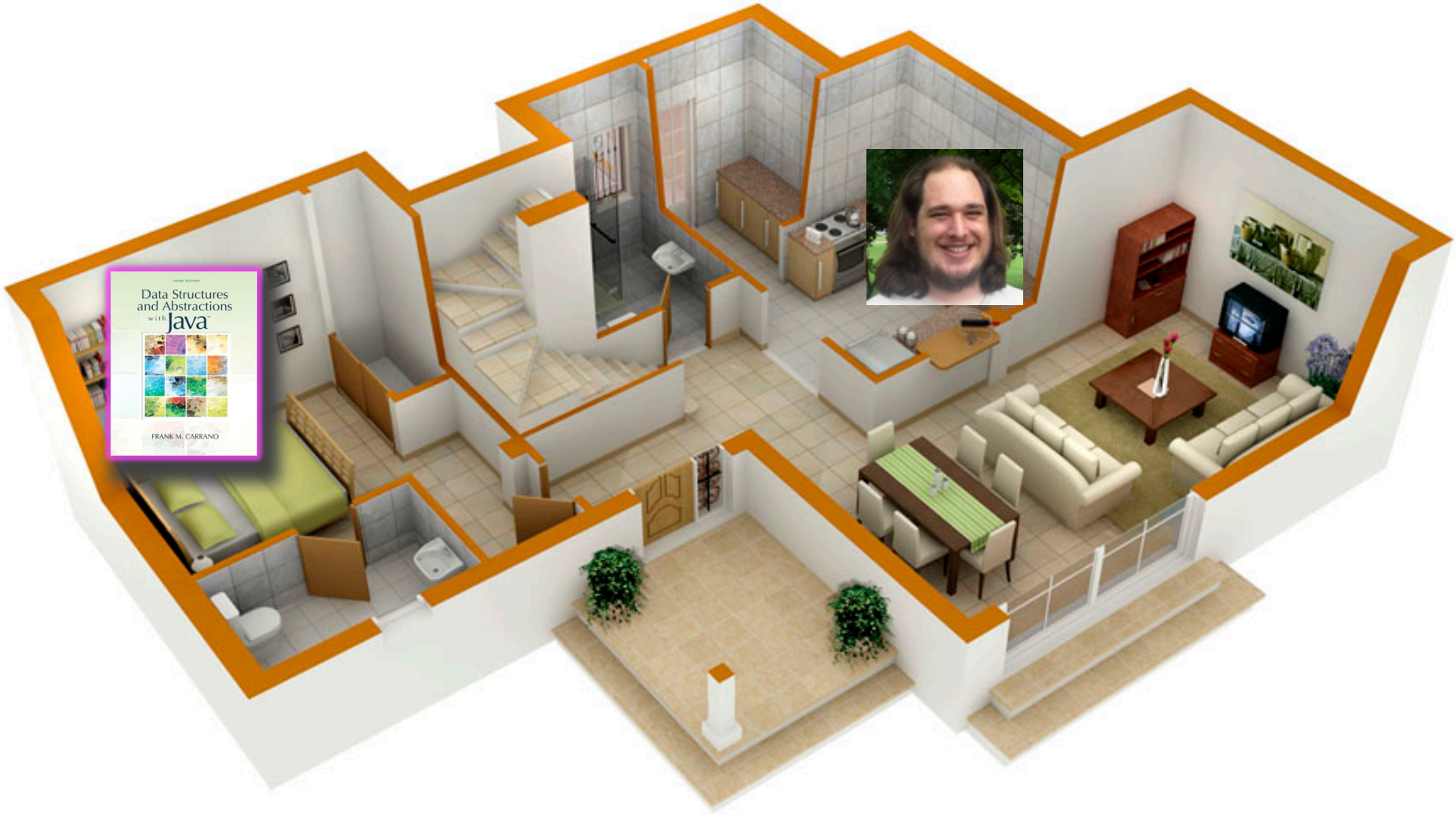
Q. Why do programmers confuse Halloween and Christmas?

Answer. Because oct 31 = dec 25!

Decimal	Octal
00	00
01	01
02	02
03	03
04	04
05	05
06	06
07	07
08	10
09	11
10	12
11	13
12	14

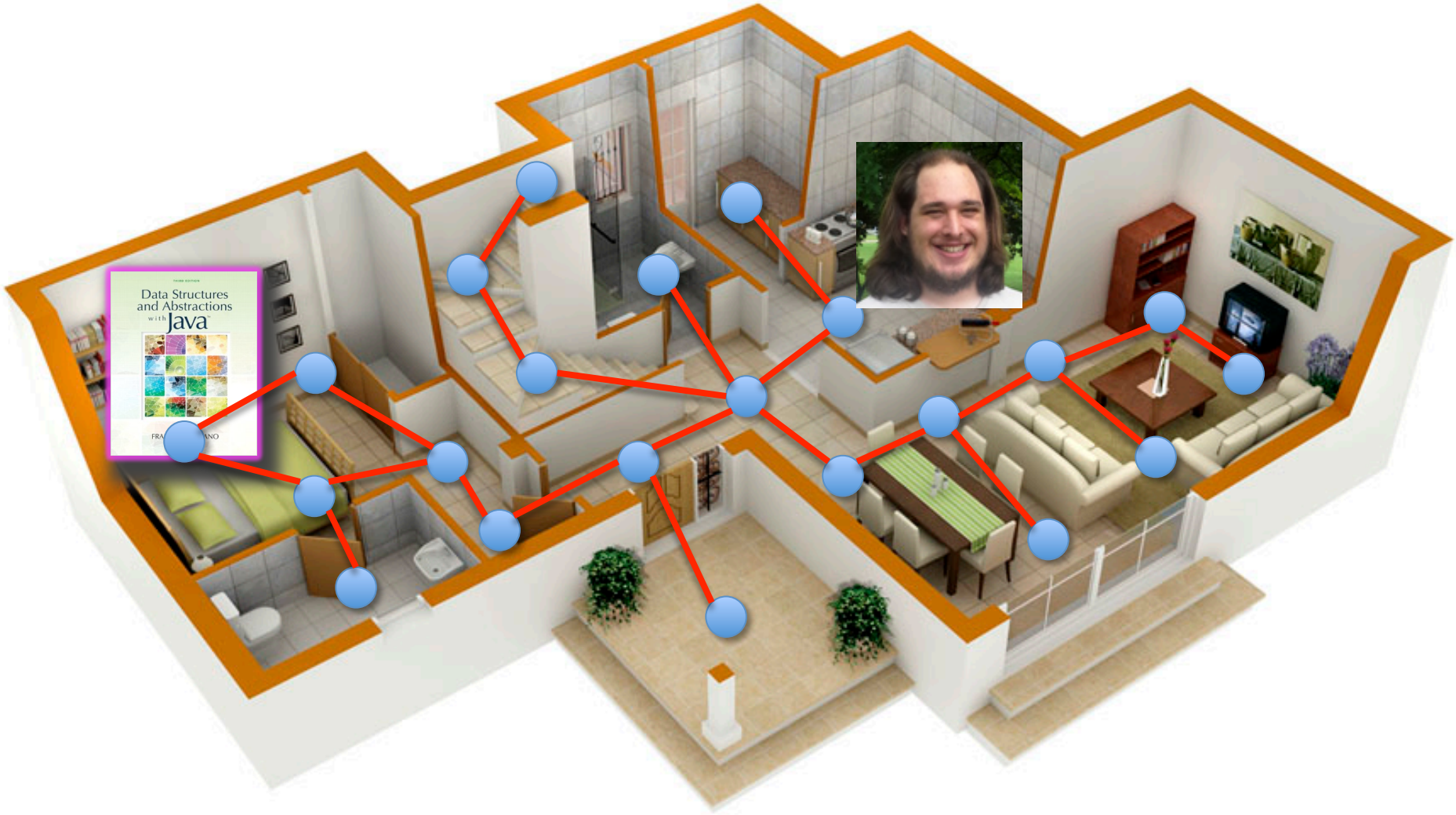
Decimal	Octal
13	15
14	16
15	17
16	20
17	21
18	22
19	23
20	24
21	25
22	26
23	27
24	30
25	31

Where did I leave that book?



http://www.geahvet.com

Where did I leave that book?



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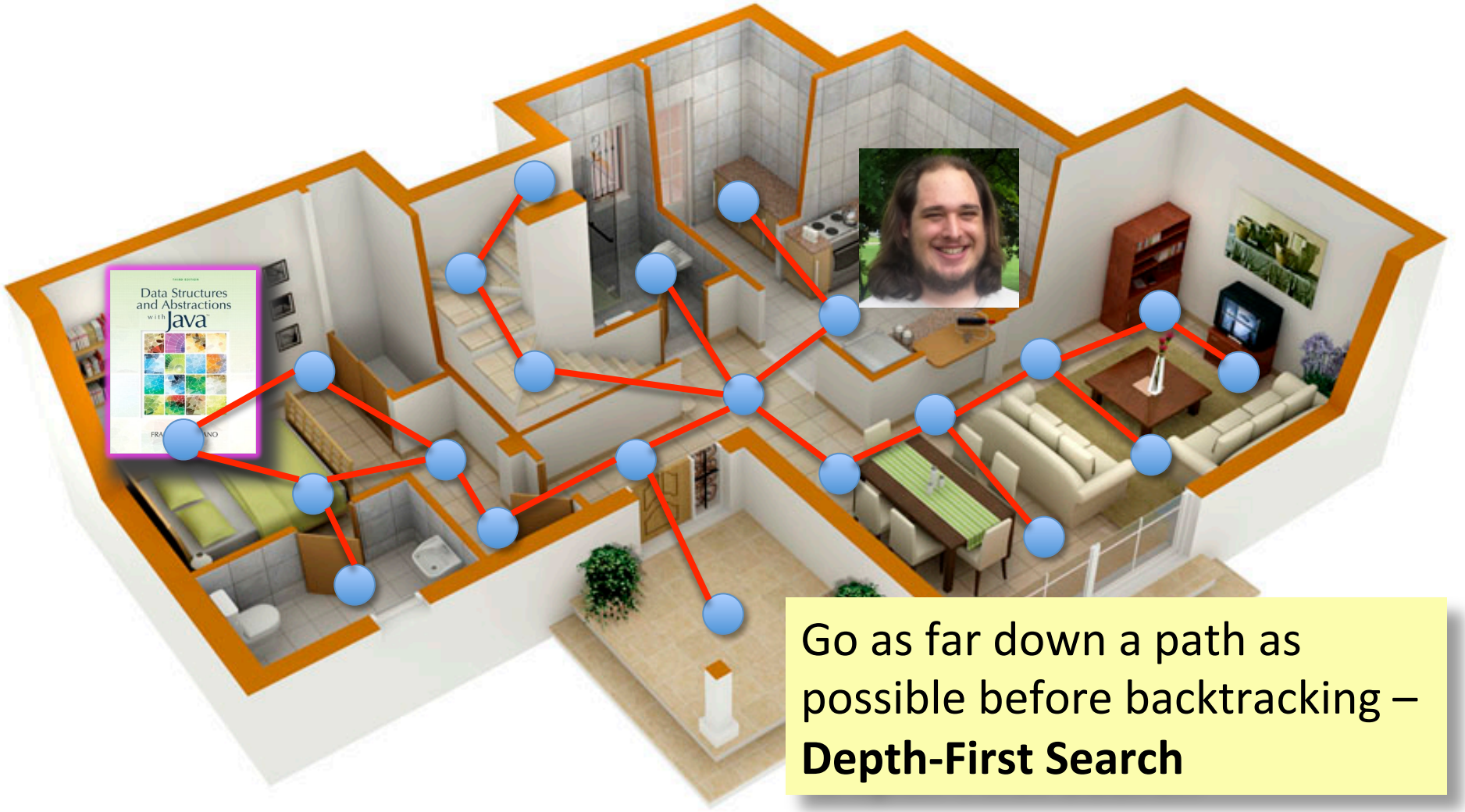
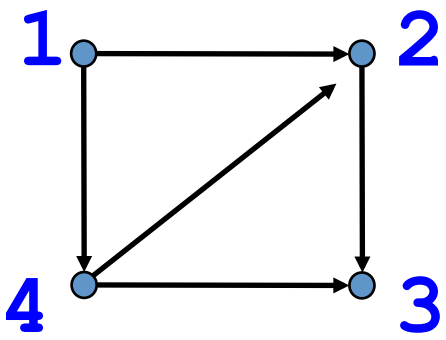


Go as far down a path as possible before backtracking – **Depth-First Search**

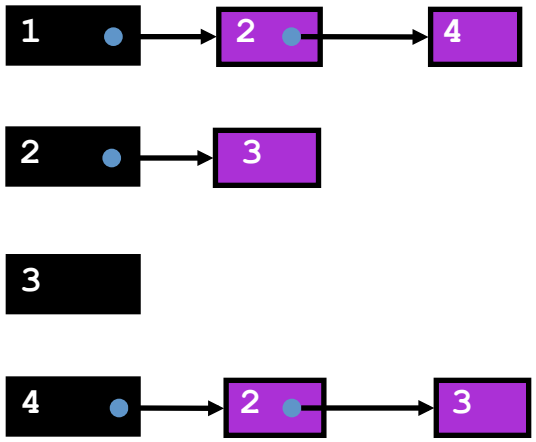
Graph Algorithms

- Search
  - Depth-first search
  - Breadth-first search
- Shortest paths
  - Dijkstra's algorithm
- Minimum spanning trees
  - Prim's algorithm
  - Kruskal's algorithm

Representations of Graphs



Adjacency List



Adjacency Matrix

	1	2	3	4
1	0	1	0	1
2	0	0	1	0
3	0	0	0	0
4	0	1	1	0

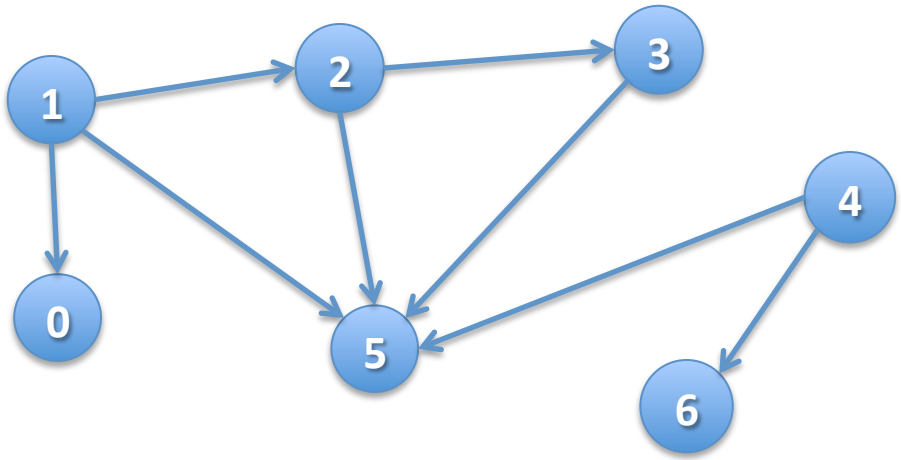
Adjacency Matrix or Adjacency List?

- Definitions:
  - n = number of vertices
  - m = number of edges
  - $d(u)$ = degree of u = number of edges leaving u
- Adjacency Matrix
  - Uses space $O(n^2)$
  - Can iterate over all edges in time $O(n^2)$
  - Can answer "Is there an edge from u to v?" in $O(1)$ time
  - Better for dense graphs (lots of edges)
- Adjacency List
  - Uses space $O(m + n)$
  - Can iterate over all edges in time $O(m + n)$
  - Can answer "Is there an edge from u to v?" in $O(d(u))$ time
  - Better for sparse graphs (fewer edges)

Depth-First Search

- Given a graph and one of its nodes u

  (say node 1 below)



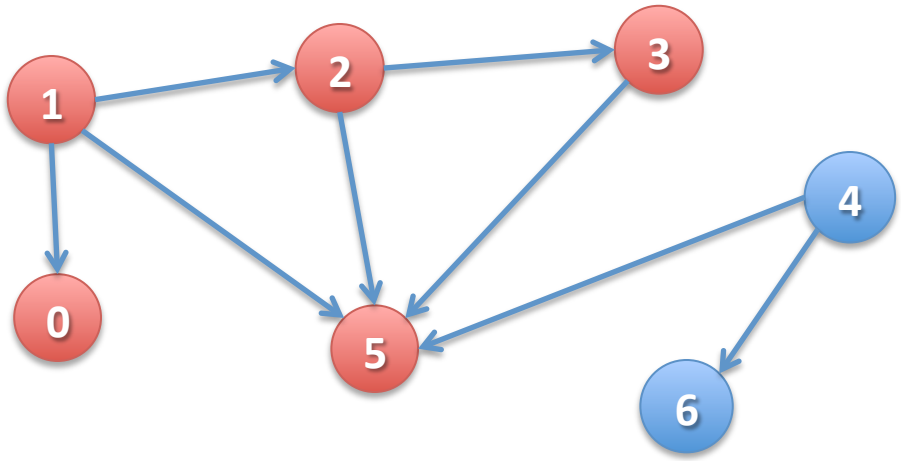
Depth-First Search

- Given a graph and one of its nodes u

  (say node 1 below)

- We want to "visit" each node reachable from u

  (nodes 1, 0, 2, 3, 5)



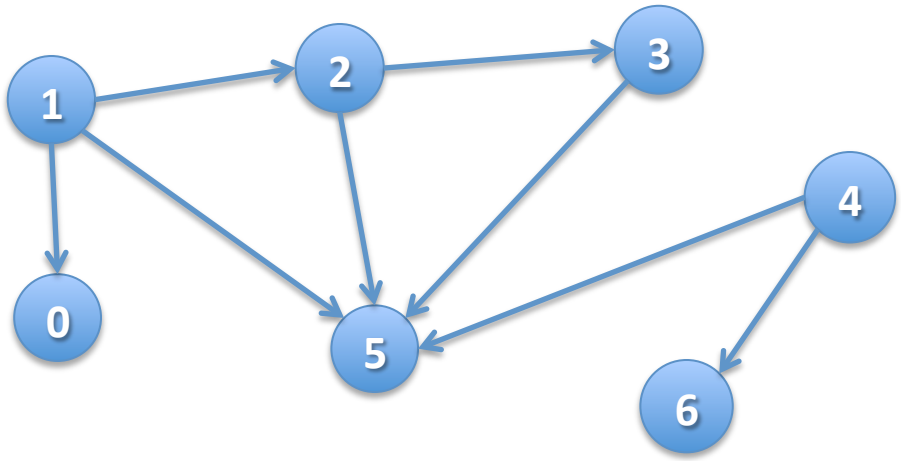
There are many paths to some nodes.

How do we visit all nodes efficiently, without doing extra work?

Depth-First Search

boolean[ ] visited;

- Node u is visited means: visited[u] is true
- To visit u means to: set visited[u] to true
- Node v is REACHABLE from node u if there is a path (u, …, v) in which all nodes of the path are unvisited.



Suppose all nodes are unvisited.

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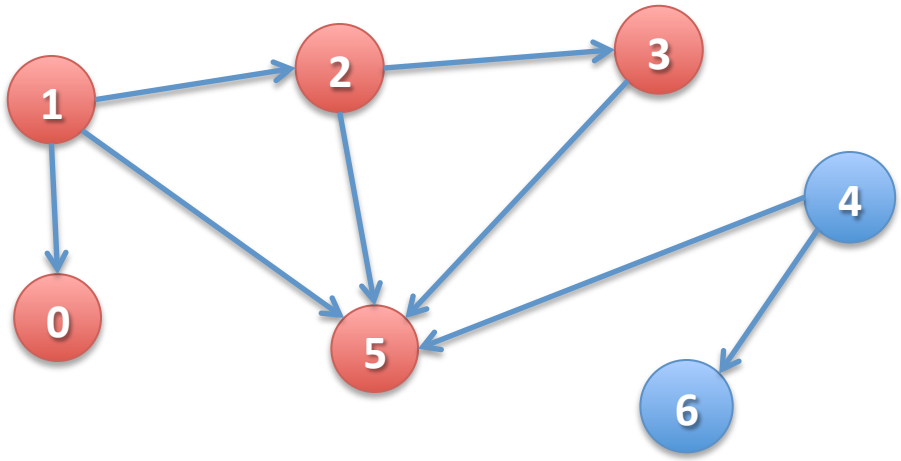

Suppose all nodes are unvisited.

Nodes REACHABLE from node **1**:
{1, 0, 2, 3, 5}

Depth-First Search

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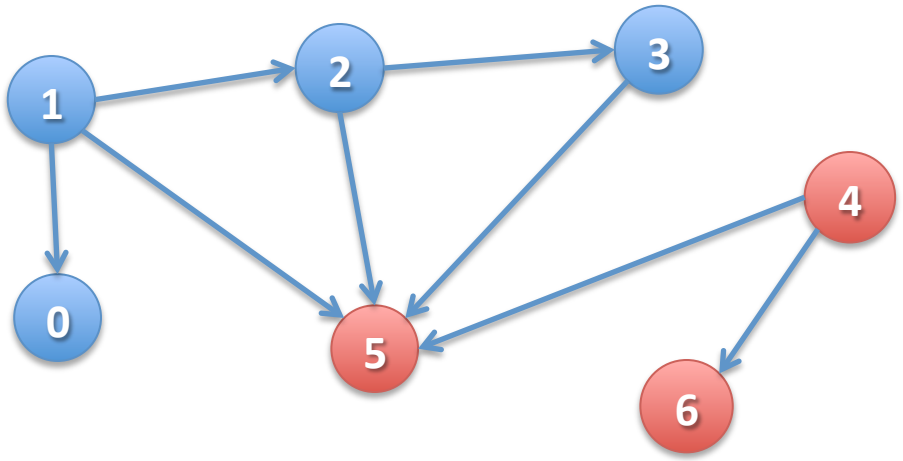

Suppose all nodes are unvisited.

Nodes REACHABLE from node **1**:
{1, 0, 2, 3, 5}

Nodes REACHABLE from **4**: {4, 5, 6}

Depth-First Search

boolean[ ] visited;

- Node u is visited means: visited[u] is true
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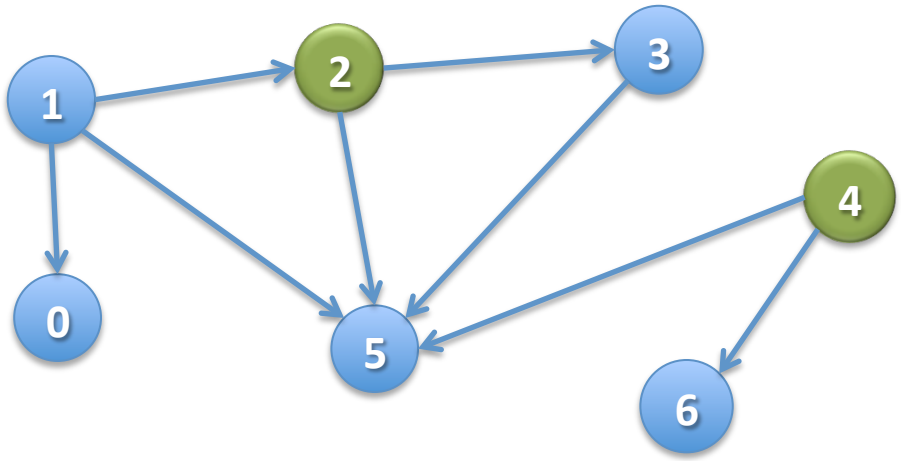
- Node v is REACHABLE from node u if there is a path (u, …, v) in which all nodes of the path are unvisited.



Green: visited
Blue: unvisited

Depth-First Search

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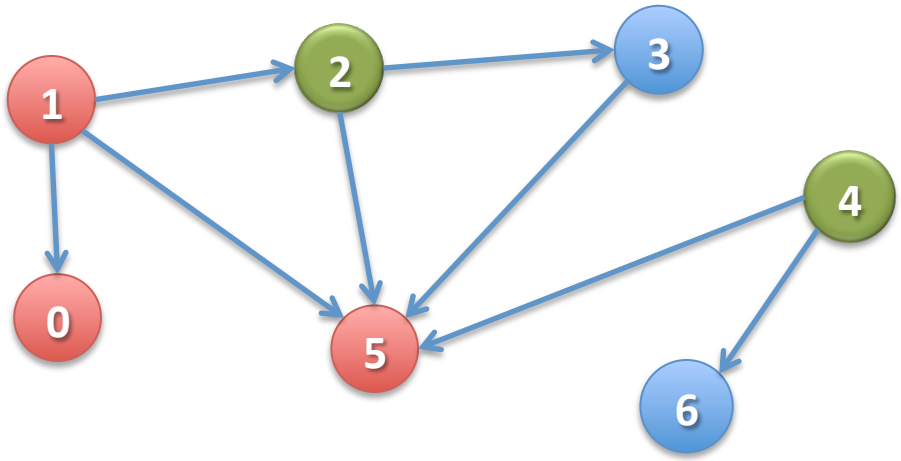

Green: visited
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Nodes REACHABLE from node **1**:
{1, 0, 5}

Depth-First Search

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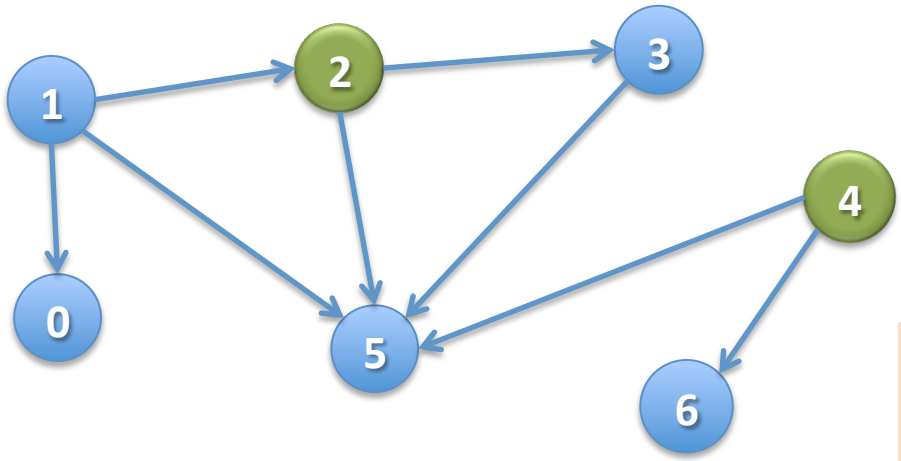


Green: visited
Blue: unvisited

Nodes REACHABLE from node **1**:
{1, 0, 5}

Nodes REACHABLE from **4**: none

Not even **4** itself, because it's already been visited!

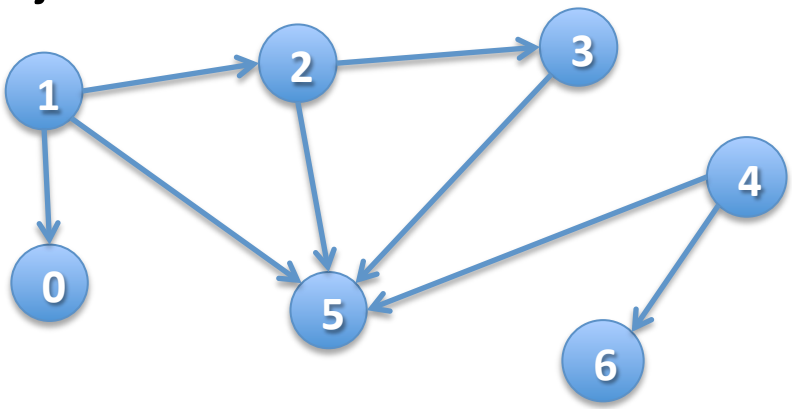
Depth-First Search

```
/** Visit all nodes that are REACHABLE from u.
    Precondition: u is not visited*/
public static void dfs(int u) {
}
```

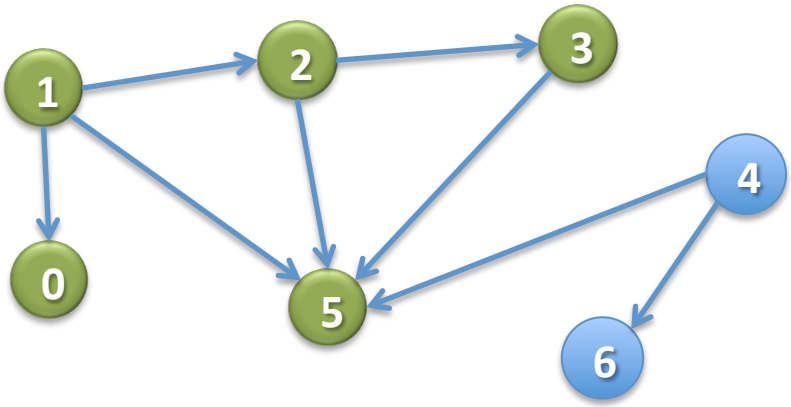
Let u be **1**

The nodes REACHABLE from 1 are 1, 0, 2, 3, 5

Start



End



Depth-First Search

```
/** Node u is unvisited. Visit all nodes
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public static void dfs(int u) {



}
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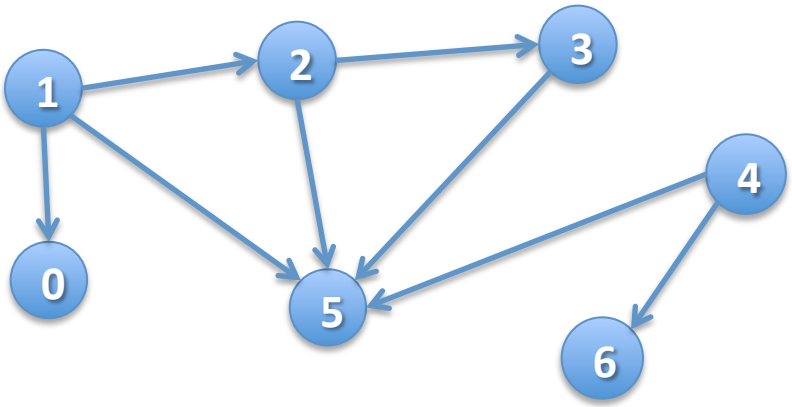

Depth-First Search

```
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    visited[u] = true;


}
```

Let u be **1**

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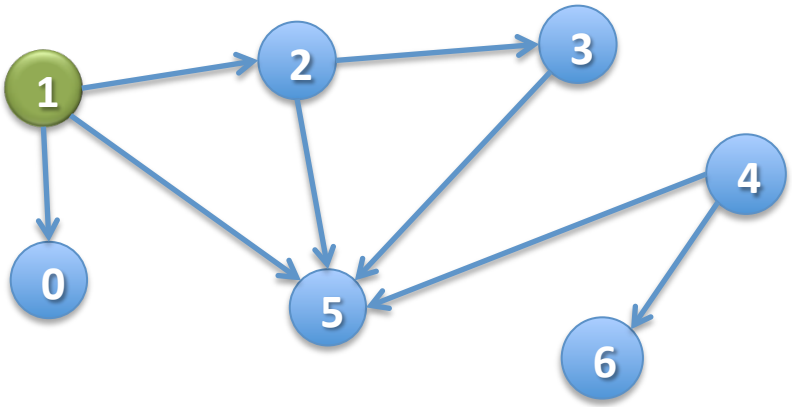

Depth-First Search

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/** Node u is unvisited. Visit all nodes
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public static void dfs(int u) {
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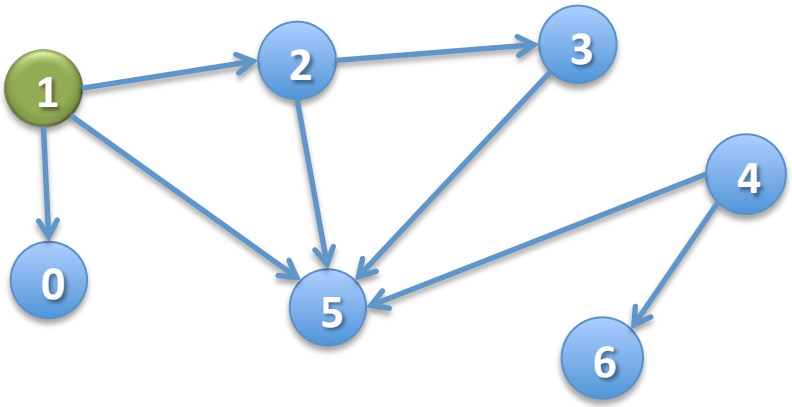
```

Let u be **1** (visited)

The nodes to be visited are 0, 2, 3, 5



Depth-First Search

```
/** Node u is unvisited. Visit all nodes
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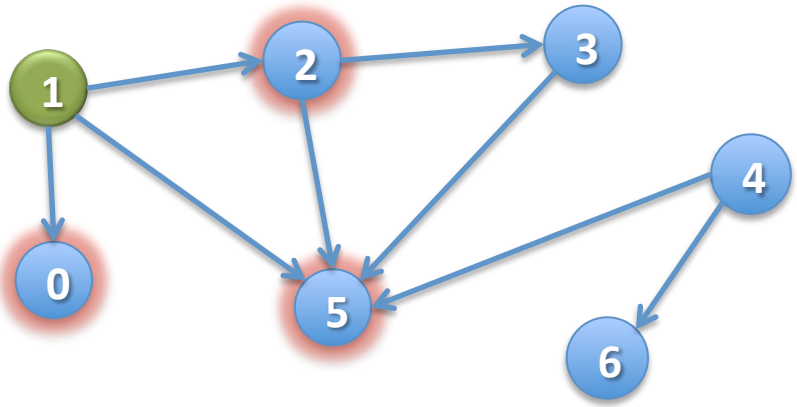
public static void dfs(int u) {
    visited[u] = true;
    for all edges (u, v) leaving u:
        if v is unvisited then dfs(v);
}
```



Let u be **1** (visited)

The nodes to be visited are 0, 2, 3, 5

Have to do DFS on all unvisited neighbors of u!

Depth-First Search

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/** Node u is unvisited. Visit all nodes
that are REACHABLE from u. */
public static void dfs(int u) {
    visited[u] = true;
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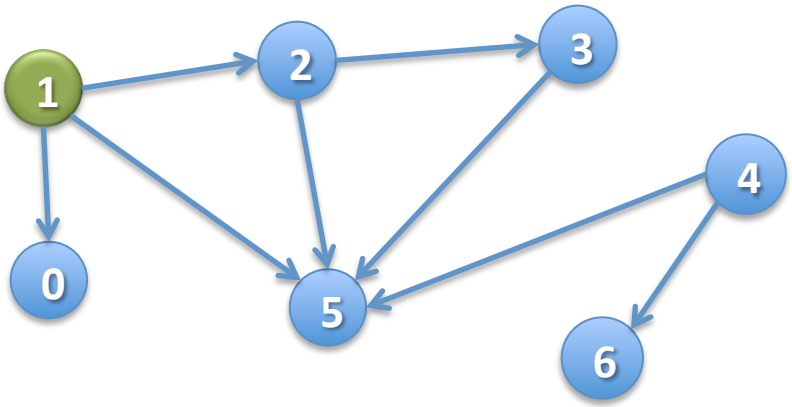
}
```

Suppose the **for** loop visits neighbors in numerical order. Then **dfs(1)** visits the nodes in this order: 1 …



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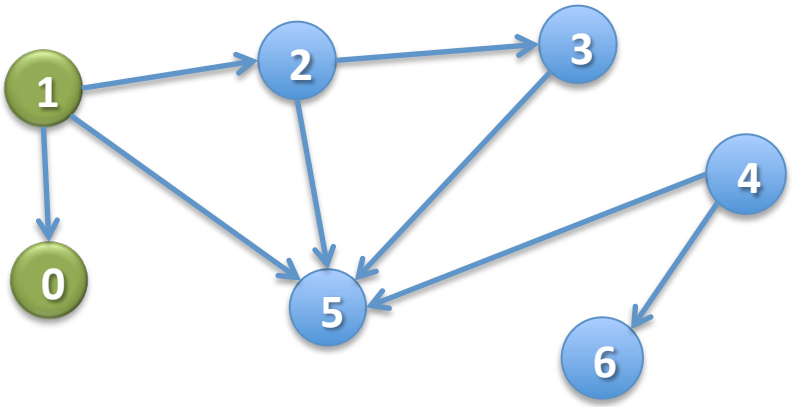

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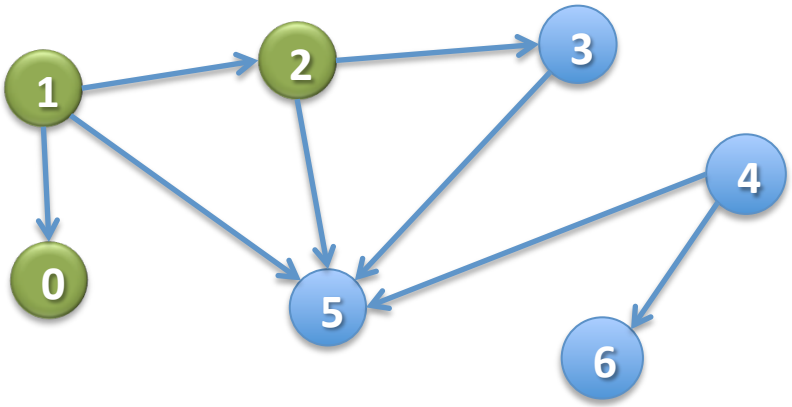

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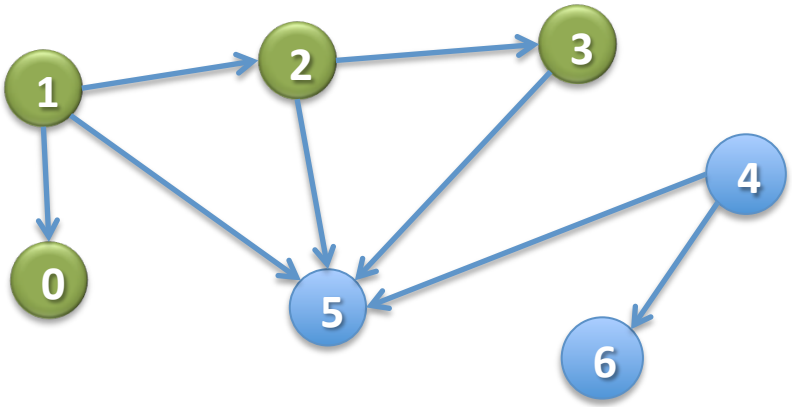
Suppose the **for** loop visits neighbors in numerical order. Then **dfs(1)** visits the nodes in this order: 1, 0, 2, 3 …



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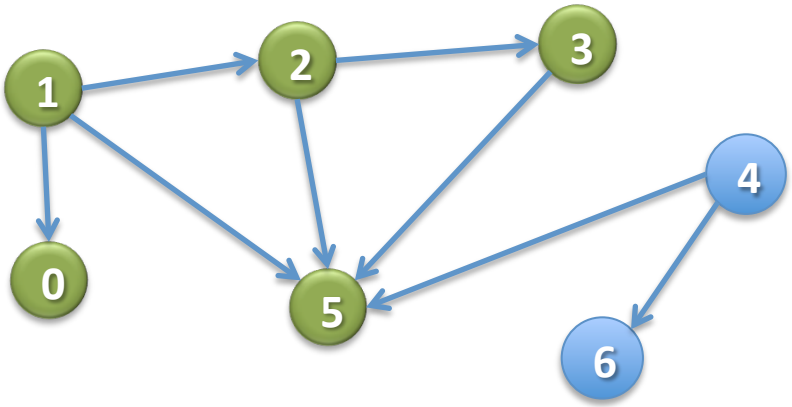

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}
```

Suppose n nodes are REACHABLE along e edges (in total). What is

- Worst-case execution?
- Worst-case space?

Depth-First Search

```
/** Node u is unvisited. Visit all nodes
that are REACHABLE from u. */
public static void dfs(int u) {
    visited[u] = true;
    for all edges (u, v) leaving u:
        if v is unvisited then dfs(v);
}
```

That's all there is to basic DFS. You may have to change it to fit a particular situation.

If you don't have this spec and you do something different, it's probably wrong.

Example: Use different way (other than array visited) to know whether a node has been visited

Example: We really haven't said what data structures are used to implement the graph

Depth-First Search in OO fashion

```
public class Node {
    boolean visited;
    List<Node> neighbors;

    /** This node is unvisited. Visit all nodes
        REACHABLE from this node */
    public void dfs() {
        visited= true;
        for (Node n: neighbors) {
            if (!n.visited) n.dfs();
        }
    }
}
```

Each node of the graph is an object of type Node

No need for a parameter. The object is the node.

Depth-First Search written iteratively

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Stack s= (u);    // Not Java!
    // inv: all nodes that have to be visited are
    //       REACHABLE from some node in s
    while ( s is not empty ) {
        u= s.pop();    // Remove top stack node, put in u
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

Depth-First Search written iteratively

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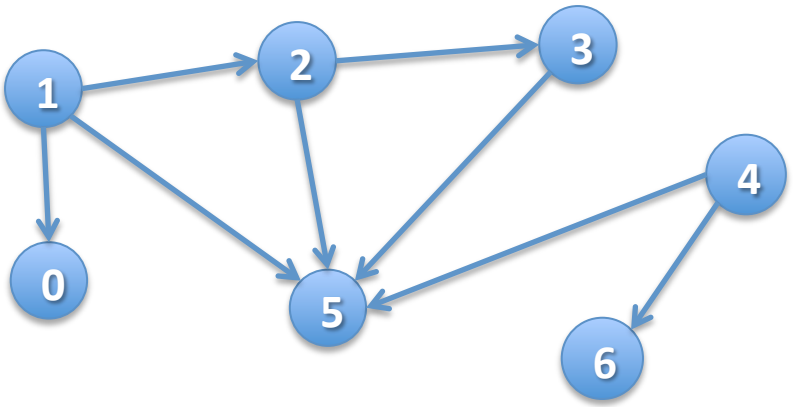
Call dfs(1)



1

Stack s

Depth-First Search written iteratively

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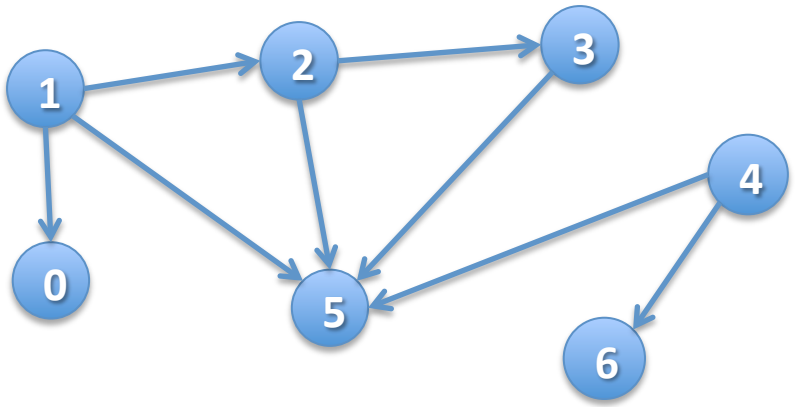
Iteration 0



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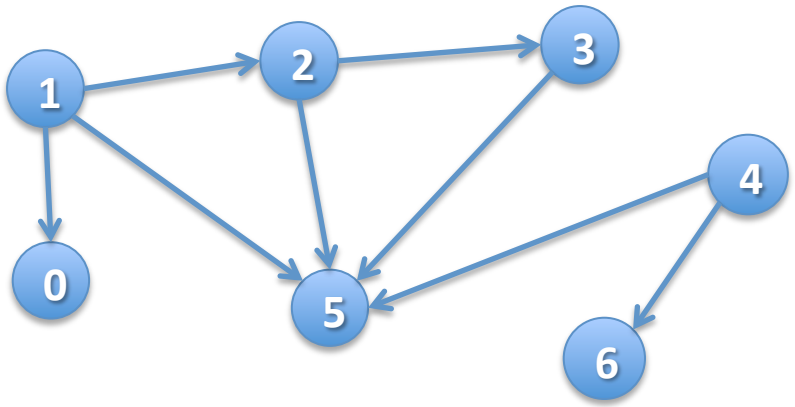

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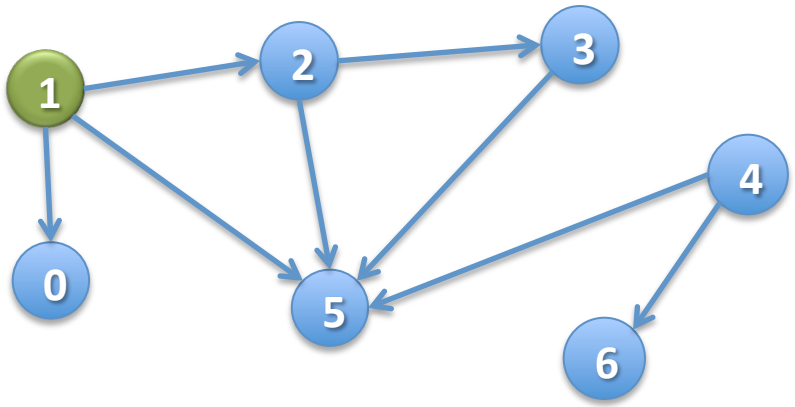
Iteration 0



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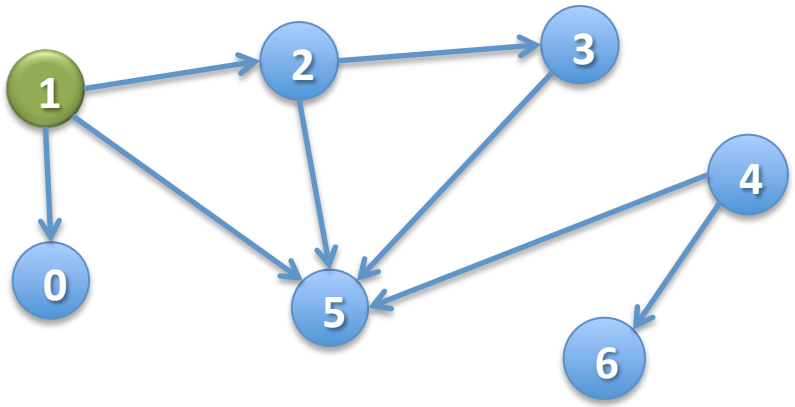
Call dfs(1)

Iteration 0



0
2
5

Stack s

Depth-First Search written iteratively

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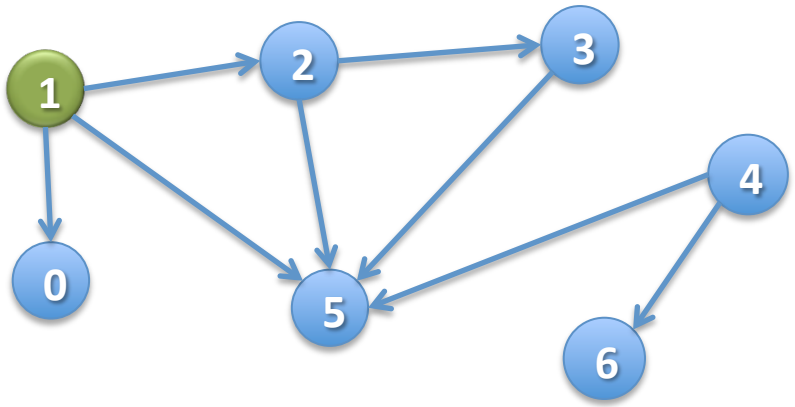
Call dfs(1)

Iteration 1



0
2
5

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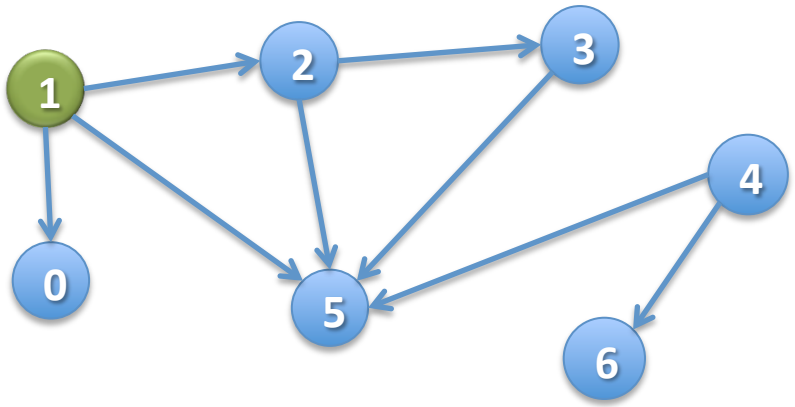

2

5

Stack s

Depth-First Search written iteratively

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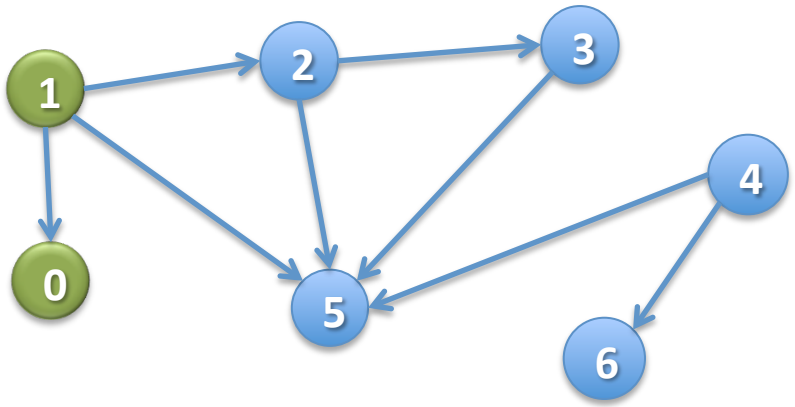
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
      Stack s= (u);
      while (s is not empty) {
            u= s.pop();
            if (u has not been visited) {
                  visit u;
                  for each edge (u, v) leaving u:
                        s.push(v);
            }
      }
}
```

Call dfs(1)

Iteration 1



2

5

Stack s

Depth-First Search written iteratively

```
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      Stack s= (u);
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                  visit u;
                  for each edge (u, v) leaving u:
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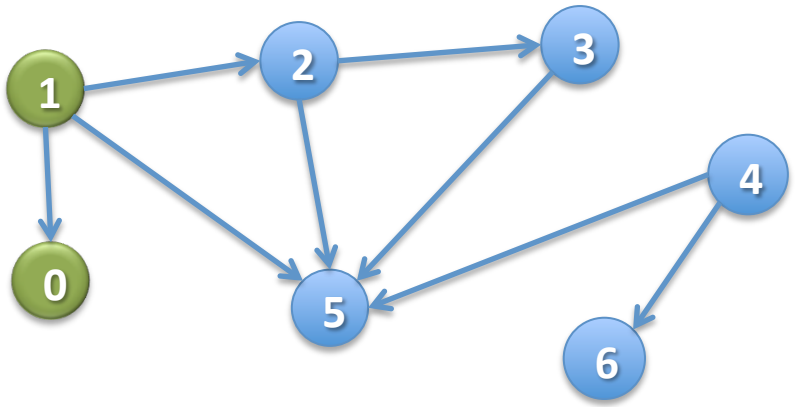
            }
      }
}
```

Call dfs(1)

Iteration 2



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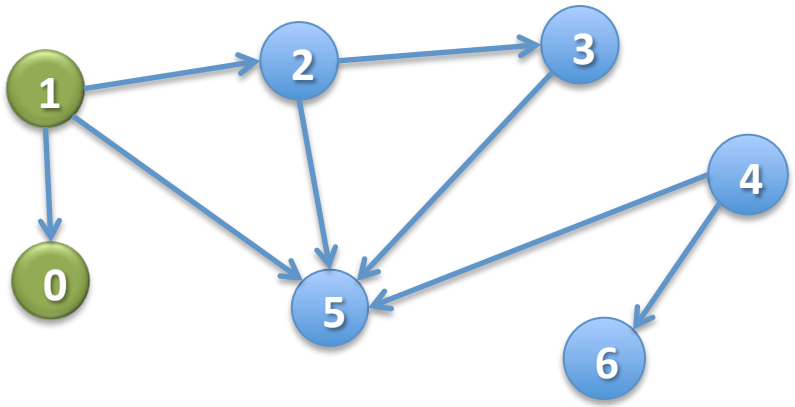
}
```

Call dfs(1)

Iteration 2



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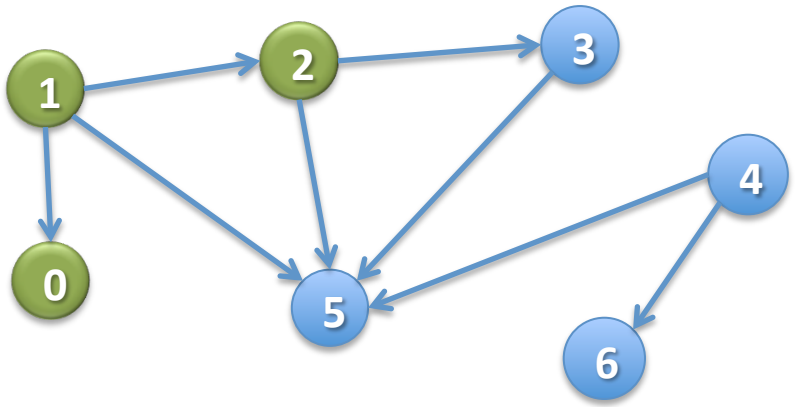
            }
      }
}
```

Call dfs(1)

Iteration 2



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Depth-First Search written iteratively

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                  visit u;
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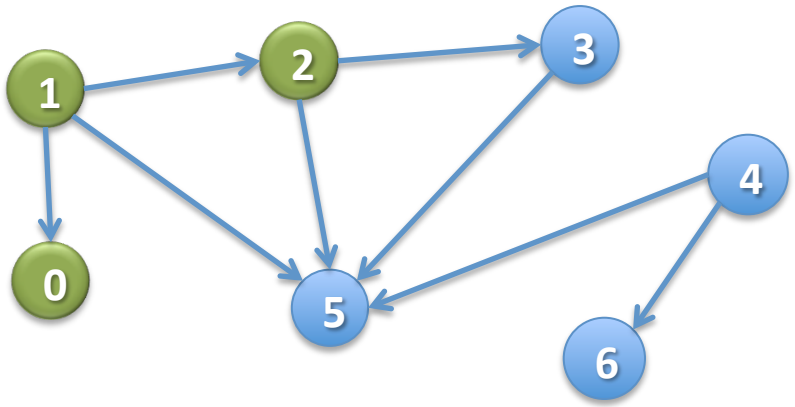
            }
      }
}
```

Call dfs(1)

Iteration 2

Yes, 5 is put on the stack twice, once for each edge to it. It will be visited only once.



3
5
5

Stack s

Breadth-First Search

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
    Queue q= (u);    // Not Java!
    // inv: all nodes that have to be visited are
    //       REACHABLE from some node in s
    while ( q is not empty ) {
        u= q.popFirst();    // Remove first node in queue, put in u
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);    // Add to end of queue
        }
    }
}
```

Breadth-First Search

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
      Queue q= (u);
      while q is not empty) {
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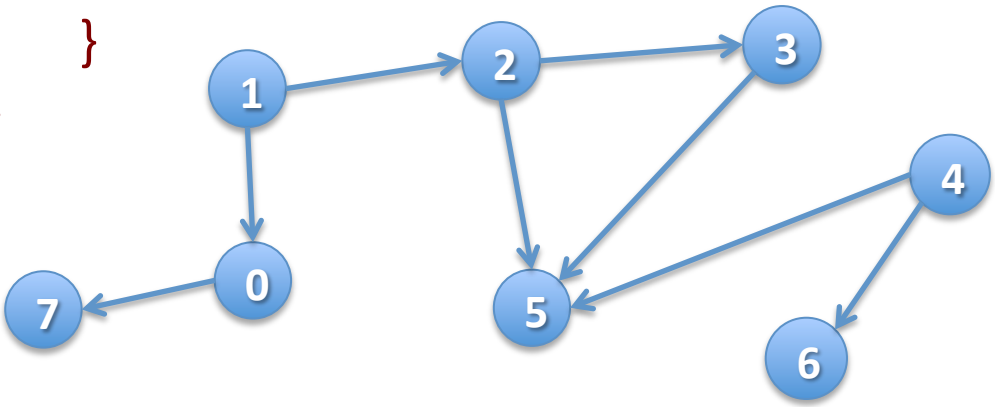
      }
}
```

Call bfs(1)



1

Queue q

Breadth-First Search

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
      Queue q= (u);
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                  visit u;
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                        q.append(v);
            }
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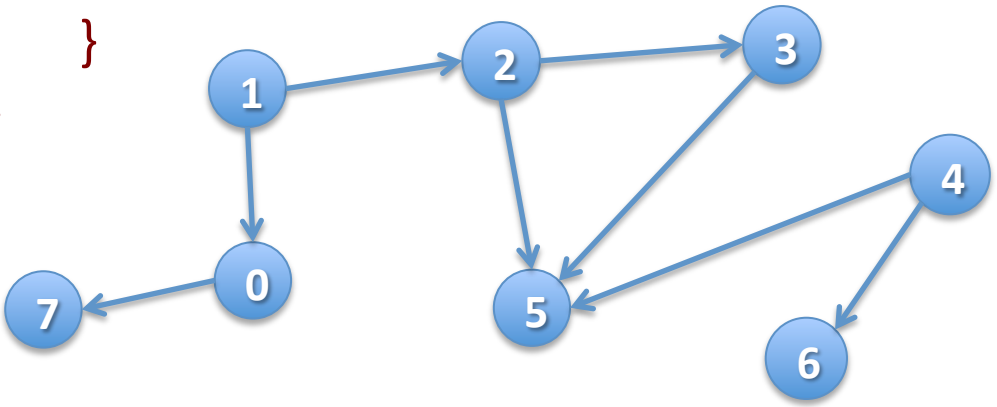
}
```

Call bfs(1)

Iteration 0



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Breadth-First Search

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            }
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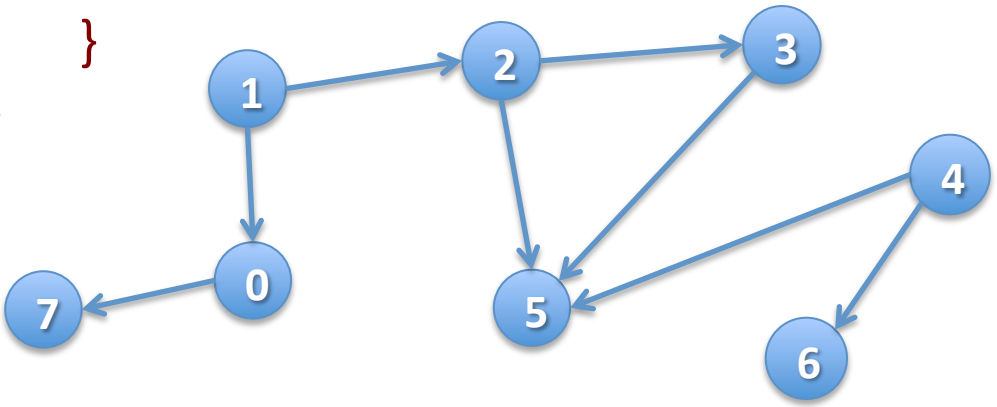
```

Call bfs(1)

Iteration 0



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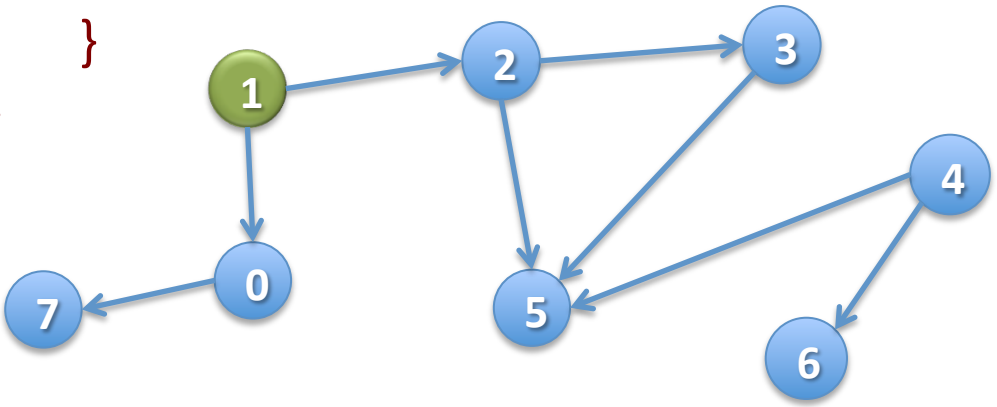

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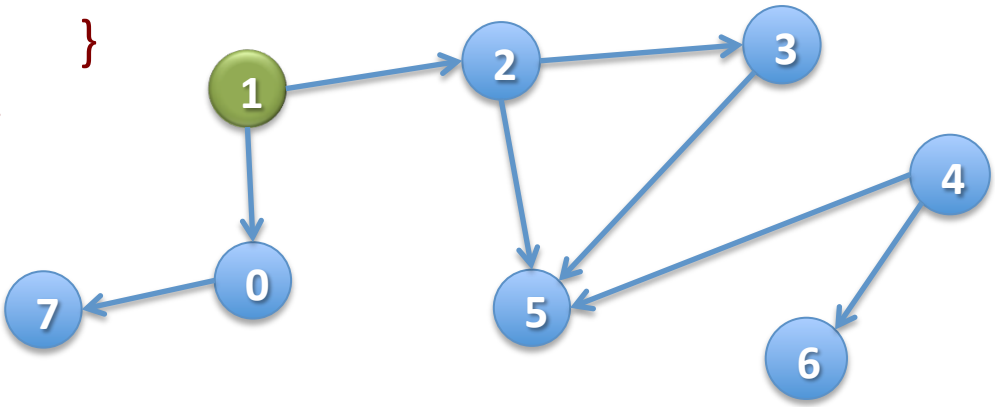
Call bfs(1)

Iteration 0



0 2

Queue q

Breadth-First Search

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
      Queue q= (u);
      while q is not empty) {
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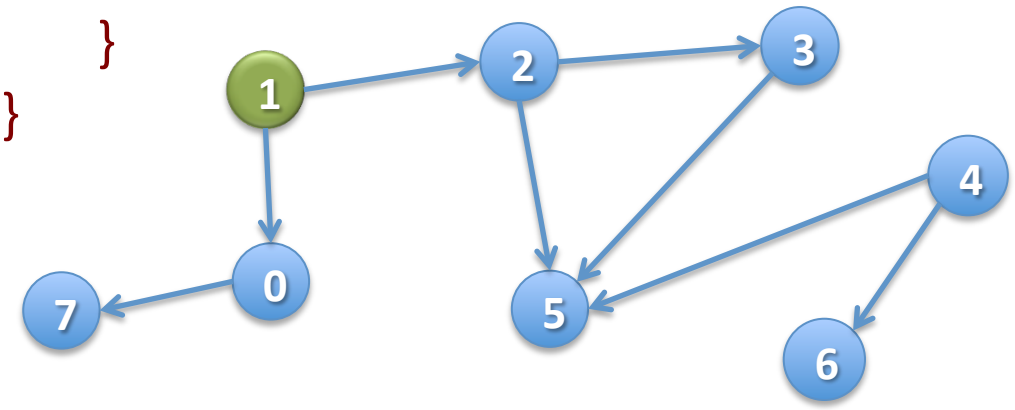
                        q.append(v);
            }
      }
}
```

Call bfs(1)

Iteration 1



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Queue q

Breadth-First Search

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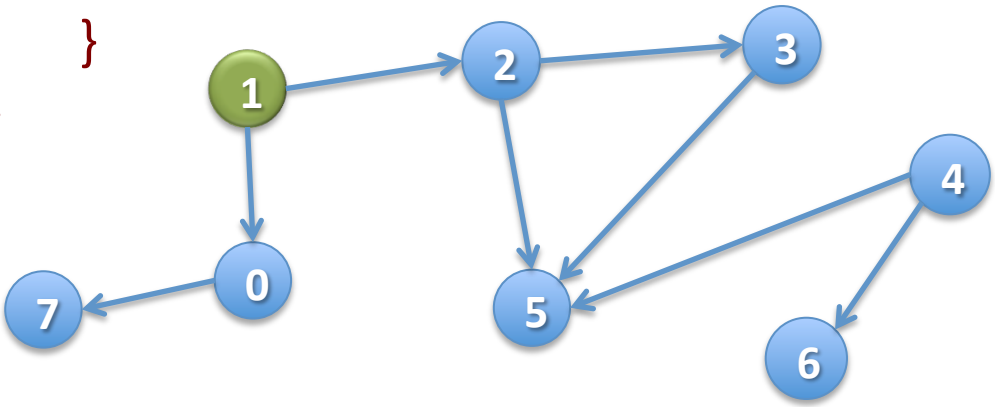
Iteration 1



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Breadth-First Search

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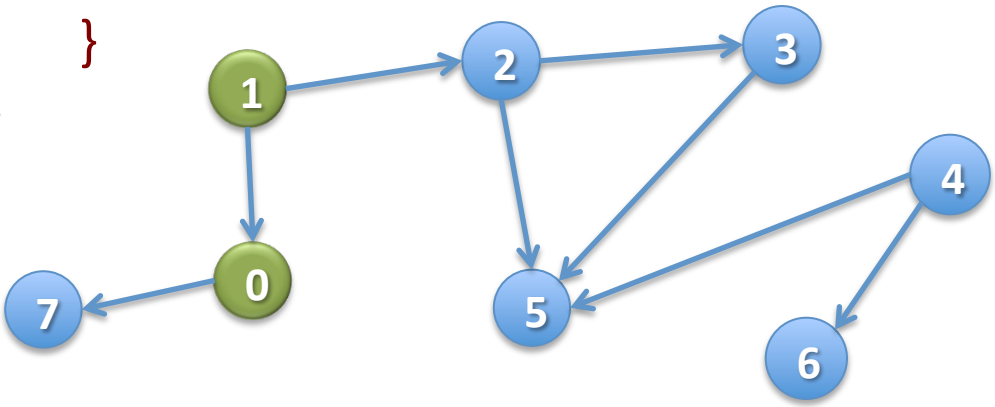
Call bfs(1)

Iteration 1



2

Queue q

Breadth-First Search

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            u= q.popFirst();
            if (u has not been visited) {
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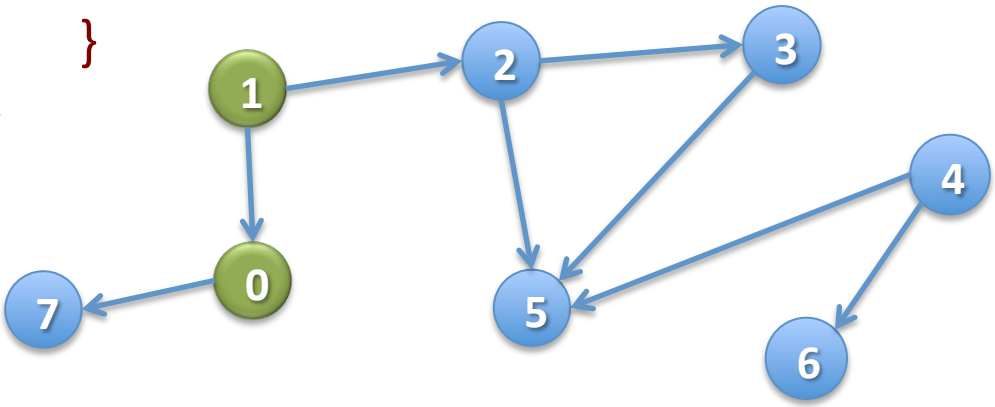
      }
}
```

Call bfs(1)

Iteration 1



2 7

Queue q

Breadth-First Search

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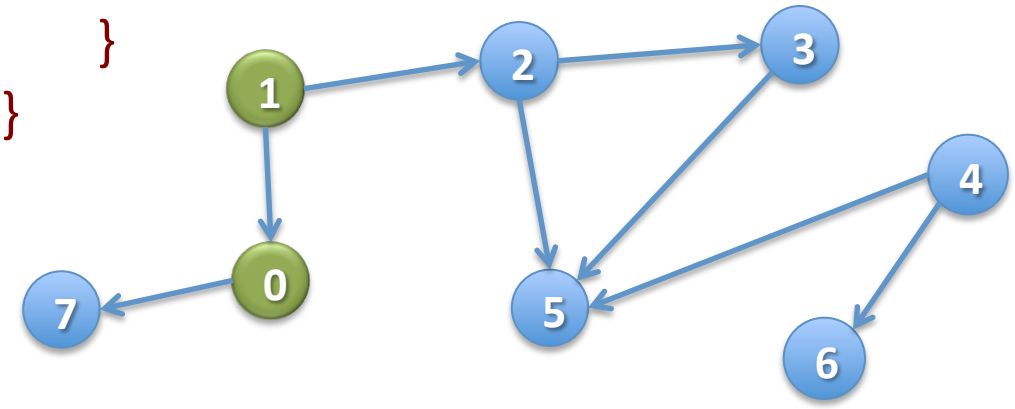
            u= q.popFirst();
            if (u has not been visited) {
                  visit u;
                  for each edge (u, v) leaving u:
                        q.append(v);
            }
      }
}
```

Call bfs(1)

Iteration 2



2 7

Queue q

Breadth-First Search

```
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      Queue q= (u);
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                        q.append(v);
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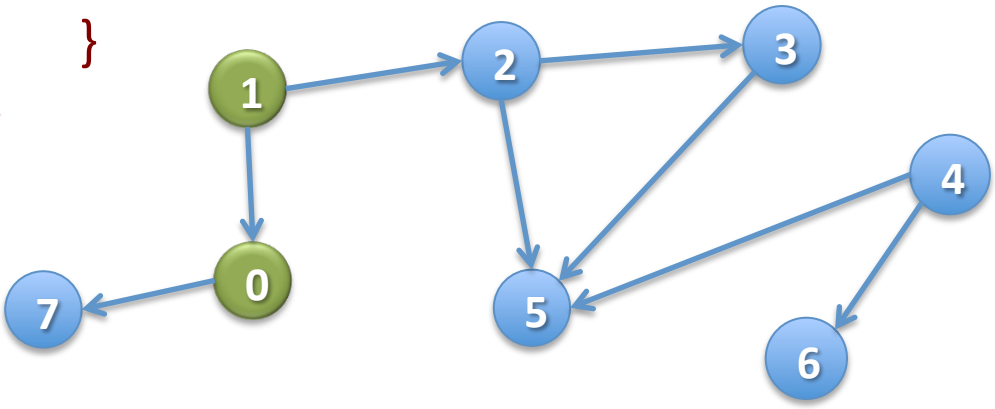
Call bfs(1)

Iteration 2



7

Queue q

Breadth-First Search

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/** Node u is unvisited. Visit all nodes REACHABLE from u. */
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      while q is not empty) {
            u= q.popFirst();
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                        q.append(v);
            }
      }
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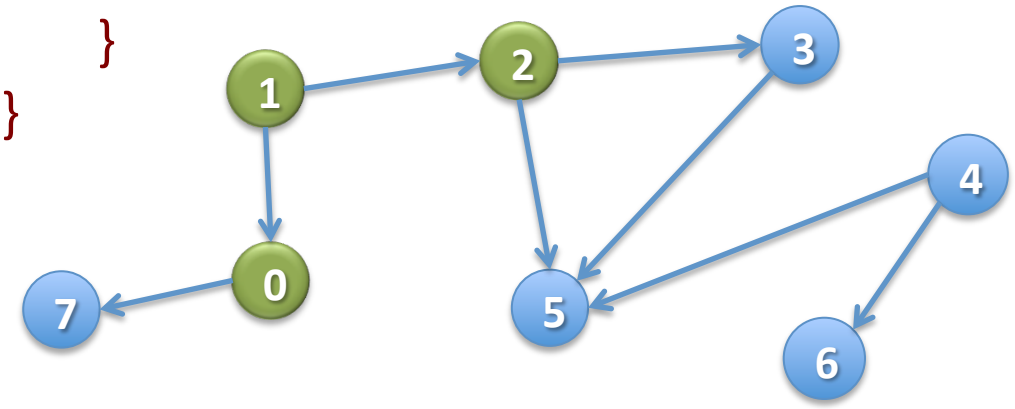
Call bfs(1)

Iteration 2



7

Queue q

Breadth-First Search

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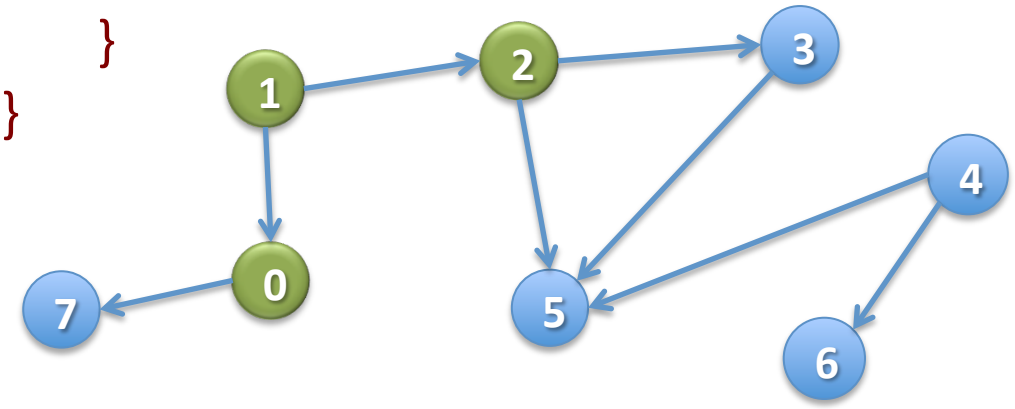
                        q.append(v);
            }
      }
}
```

Call bfs(1)

Iteration 2

Breadth first:
(1) Node u
(2) All nodes 1 edge from u
(3) All nodes 2 edges from u
(4) All nodes 3 edges from u
…



7 3 5

Queue q