

SPANNING TREES, INTRO. TO THREADS

Lecture 23 CS2110 — Fall 2013

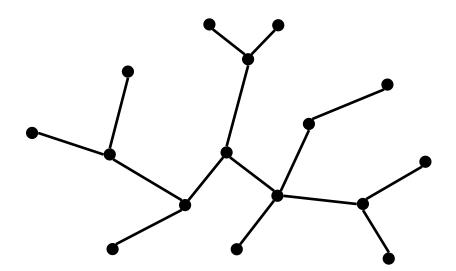
A lecture with two distinct parts

- □ Part I: Finishing our discussion of graphs
 - Today: Spanning trees
 - Definitions, algorithms (Prim's, Kruskal's)
 - Travelling salesman problem

- Part II: Introduction to the idea of threads
 - Why do we need them?
 - What is a thread?

Undirected Trees

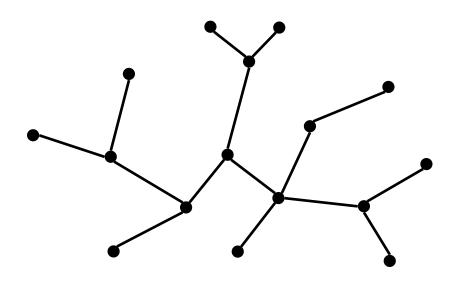
 An undirected graph is a tree if there is exactly one simple path between any pair of vertices



Facts About Trees

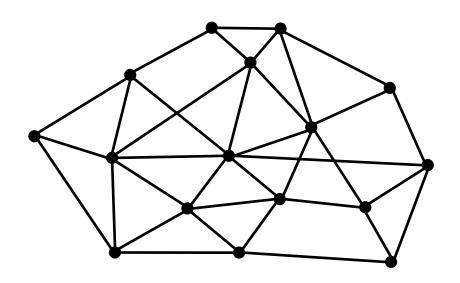
- |E| = |V| 1
- connected
- no cycles

In fact, any two of these properties imply the third, and imply that the graph is a tree



Spanning Trees

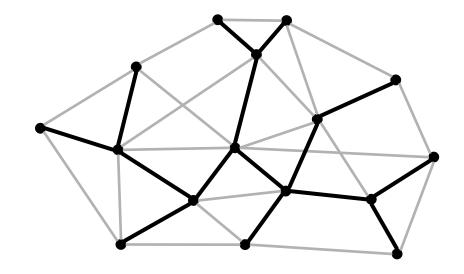
A *spanning tree* of a connected undirected graph (V,E) is a subgraph (V,E') that is a tree



Spanning Trees

A *spanning tree* of a connected undirected graph (V,E) is a subgraph (V,E') that is a tree

- Same set of vertices V
- E' ⊆ E
- (V,E') is a tree

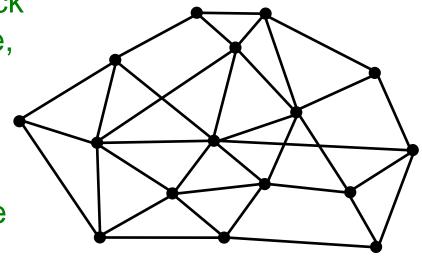


A subtractive method

Start with the whole graph – it is connected

 If there is a cycle, pick an edge on the cycle, throw it out – the graph is still connected (why?)

 Repeat until no more cycles

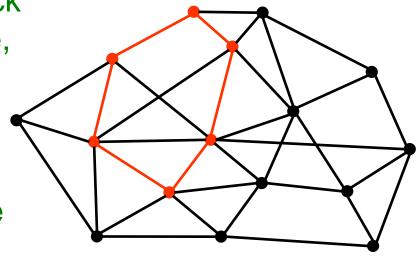


A subtractive method

Start with the whole graph – it is connected

 If there is a cycle, pick an edge on the cycle, throw it out – the graph is still connected (why?)

 Repeat until no more cycles

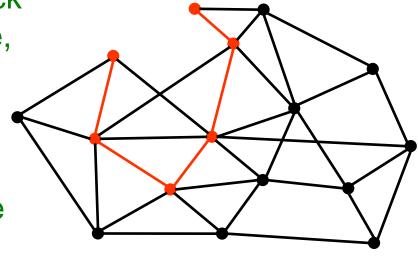


A subtractive method

Start with the whole graph – it is connected

 If there is a cycle, pick an edge on the cycle, throw it out – the graph is still connected (why?)

 Repeat until no more cycles



- Start with no edges there are no cycles
- If more than one connected component, insert an edge between them – still no cycles (why?)
- Repeat until only one component

- Start with no edges there are no cycles
- If more than one connected component, insert an edge between them – still no cycles (why?)
- Repeat until only one component

- Start with no edges there are no cycles
- If more than one connected component, insert an edge between them – still no cycles (why?)
- Repeat until only one component

- Start with no edges there are no cycles
- If more than one connected component, insert an edge between them – still no cycles (why?)
- Repeat until only one component

- Start with no edges there are no cycles
- If more than one connected component, insert an edge between them – still no cycles (why?)
- Repeat until only one component

- Start with no edges there are no cycles
- If more than one connected component, insert an edge between them – still no cycles (why?)
- Repeat until only one component

Minimum Spanning Trees

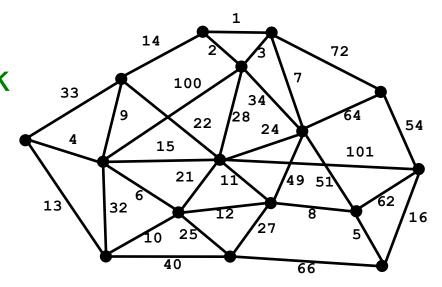
- Suppose edges are weighted, and we want a spanning tree of *minimum cost* (sum of edge weights)
- Some graphs have exactly one minimum spanning tree. Others have multiple trees with the same cost, any of which is a minimum spanning tree

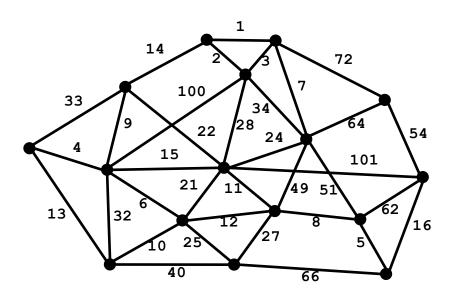
Minimum Spanning Trees

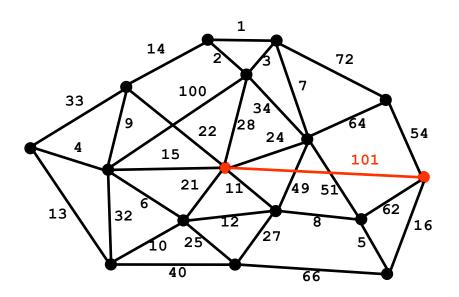
 Suppose edges are weighted, and we want a spanning tree of *minimum cost* (sum of edge weights)

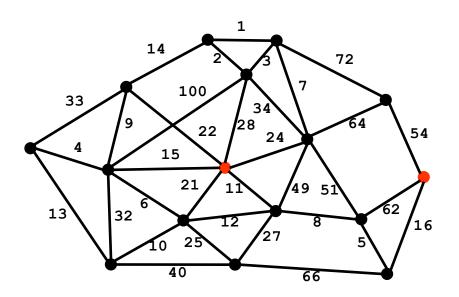
 Useful in network routing & other applications

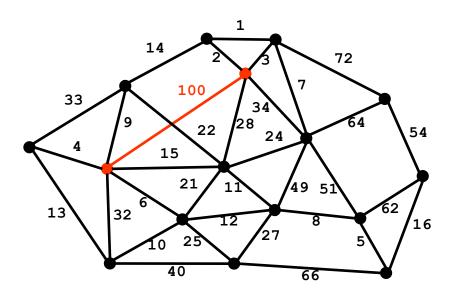
 For example, to stream a video

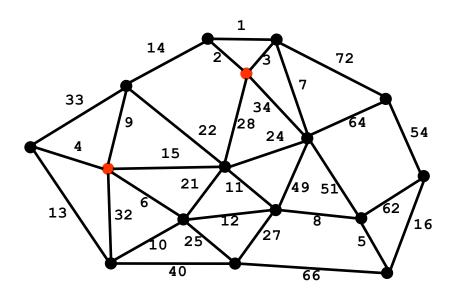


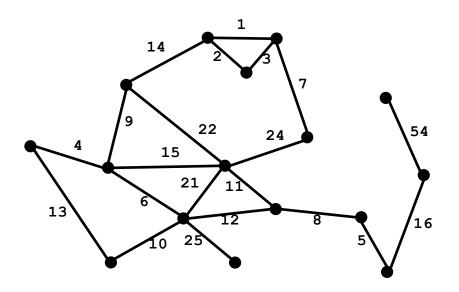


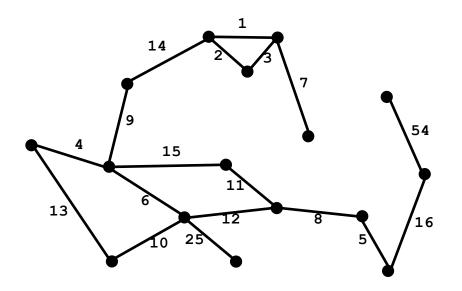


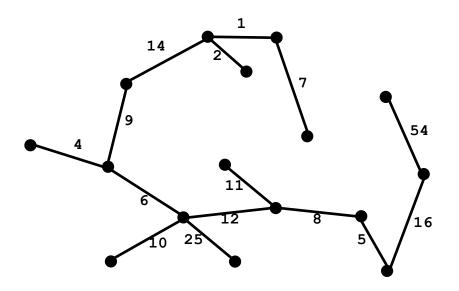




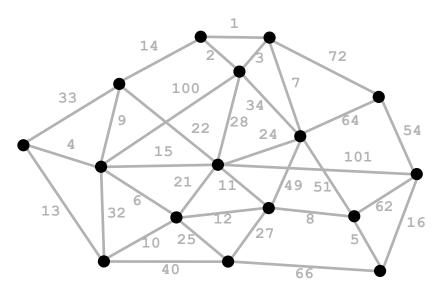




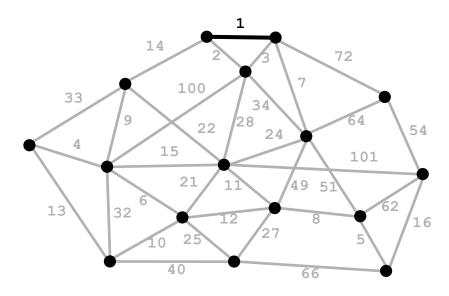




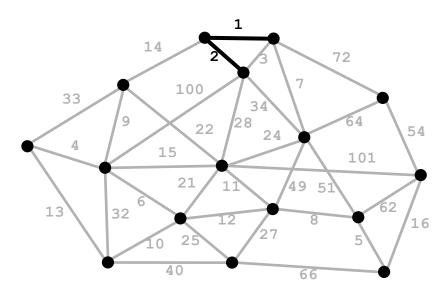
B. Find a min weight edge – if it forms a cycle with edges already taken, throw it out, otherwise keep it



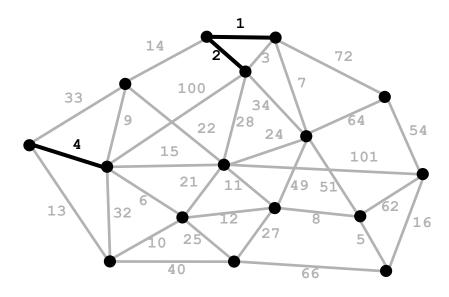
B. Find a min weight edge – if it forms a cycle with edges already taken, throw it out, otherwise keep it



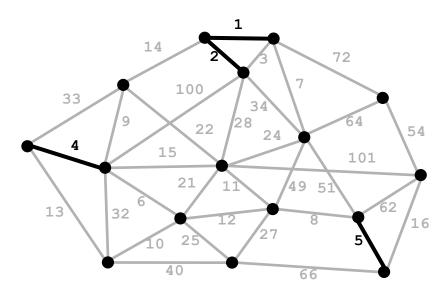
B. Find a min weight edge – if it forms a cycle with edges already taken, throw it out, otherwise keep it



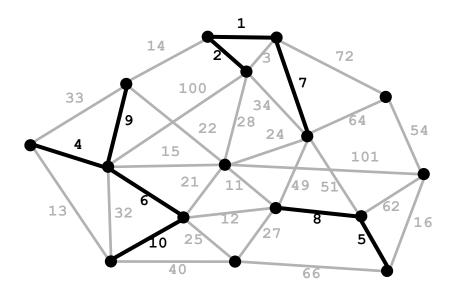
B. Find a min weight edge – if it forms a cycle with edges already taken, throw it out, otherwise keep it



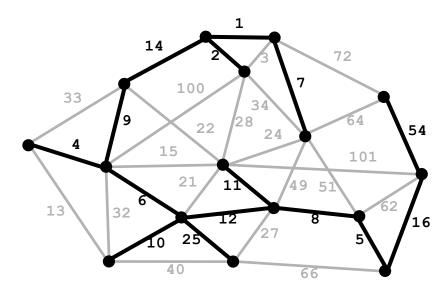
B. Find a min weight edge – if it forms a cycle with edges already taken, throw it out, otherwise keep it



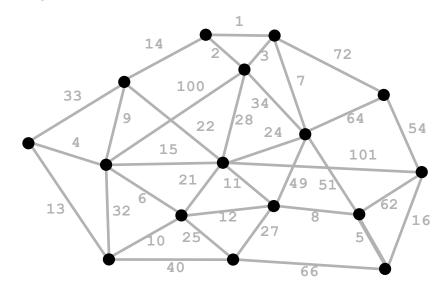
B. Find a min weight edge – if it forms a cycle with edges already taken, throw it out, otherwise keep it



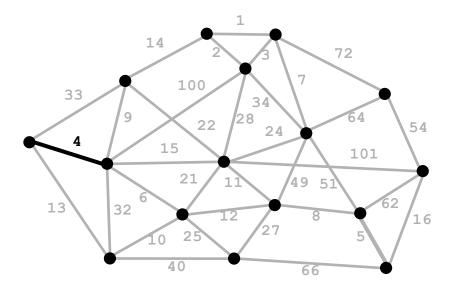
B. Find a min weight edge – if it forms a cycle with edges already taken, throw it out, otherwise keep it



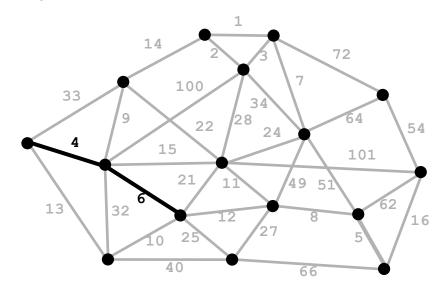
C. Start with any vertex, add min weight edge extending that connected component that does not form a cycle



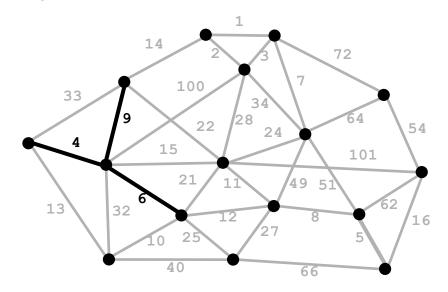
C. Start with any vertex, add min weight edge extending that connected component that does not form a cycle



C. Start with any vertex, add min weight edge extending that connected component that does not form a cycle

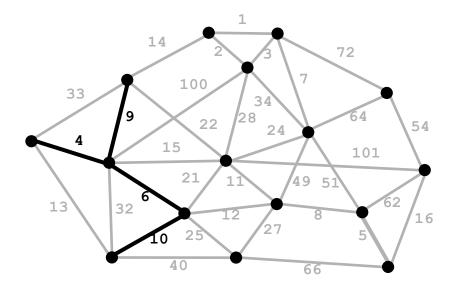


C. Start with any vertex, add min weight edge extending that connected component that does not form a cycle



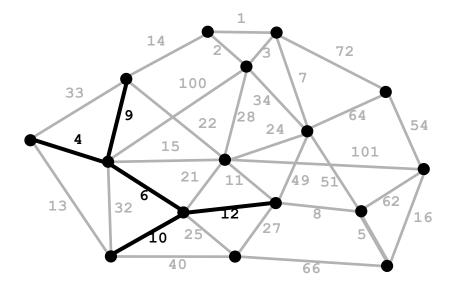
C. Start with any vertex, add min weight edge extending that connected component that does not form a cycle

Prim's algorithm (reminiscent of Dijkstra's algorithm)



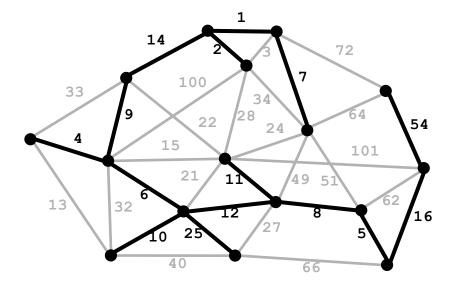
C. Start with any vertex, add min weight edge extending that connected component that does not form a cycle

Prim's algorithm (reminiscent of Dijkstra's algorithm)

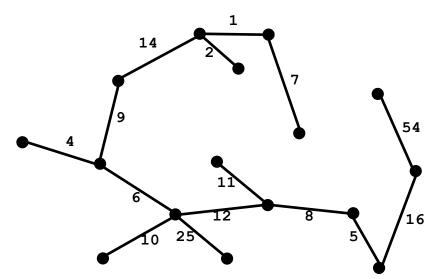


C. Start with any vertex, add min weight edge extending that connected component that does not form a cycle

Prim's algorithm (reminiscent of Dijkstra's algorithm)



• When edge weights are all distinct, or if there is exactly one minimum spanning tree, the 3 algorithms all find the identical tree



Prim's Algorithm

```
prim(s) {
   D[s] = 0; mark s; //start vertex
   while (some vertices are unmarked) {
      v = unmarked vertex with smallest D;
      mark v;
      for (each w adj to v) {
         D[w] = min(D[w], c(v,w));
      }
   }
}
```

- O(n²) for adj matrix
- While-loop is executed n times
- For-loop takes O(n) time

- \square O(m + n log n) for adj list
 - Use a PQ
 - Regular PQ produces time O(n + m log m)
 - Can improve to O(m + n log n) using a fancier heap

- □ These are examples of Greedy Algorithms
- □ The Greedy Strategy is an algorithm design technique
 - □ Like Divide & Conquer
- ☐ Greedy algorithms are used to solve optimization problems
 - The goal is to find the best solution
- □ Works when the problem has the greedy-choice property
 - A global optimum can be reached by making locally optimum choices

- Example: the Change Making Problem: Given an amount of money, find the smallest number of coins to make that amount
- Solution: Use a Greedy Algorithm
- Give as many large coins as you can
- This greedy strategy produces the optimum number of coins for the US coin system
- Different money system ⇒greedy strategy may fail
- Example: old UK system

Similar Code Structures

```
while (some vertices are
          unmarked) {
    v = best of unmarked
         vertices;
    mark v;
    for (each w adj to v)
          update w;
}
```

- Breadth-first-search (bfs)
- -best: next in queue
- -update: D[w] = D[v]+1
- Dijkstra's algorithm
- -best: next in priority queue
- -update: D[w] = min(D[w], D[v]+c(v,w))
- Prim's algorithm
- -best: next in priority queue
- -update: D[w] = min(D[w], c(v,w))

here c(v,w) is the $v\rightarrow w$ edge weight

Traveling Salesman Problem

- Given a list of cities and the distances between each pair, what is the shortest route that visits each city exactly once and returns to the origin city?
 - Basically what we want the butterfly to do in A6! But we don't mind if the butterfly revisits a city (Tile), or doesn't use the very shortest possible path.
 - The true TSP is very hard (NP complete)... for this we want the <u>perfect</u> answer in all cases, and can't revisit.
 - Most TSP algorithms start with a spanning tree, then "evolve" it into a TSP solution. Wikipedia has a lot of information about packages you can download...



THREADS: WHO NEEDS 'EM?

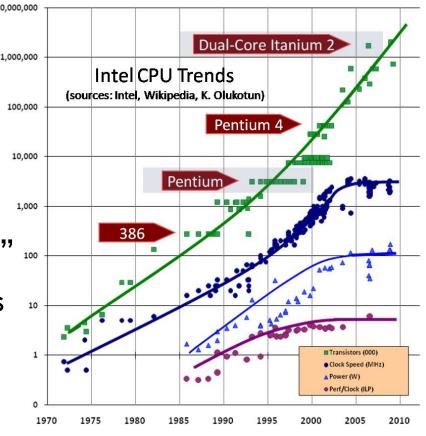
Introduction to the concept...

The Multicore Trend

Moore's Law: Computer speeds and memory densities
 nearly double each year

 But we no longer are getting this speed purely by running a faster CPU clock

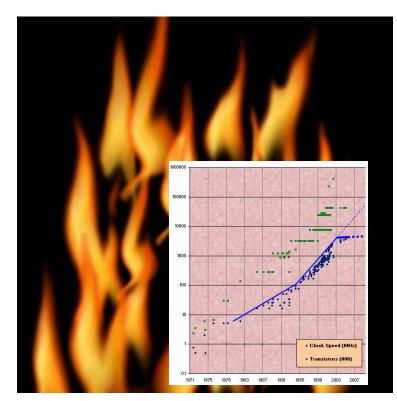
- □ CPU = "central processor unit" 100
- CPU clock roughly determines instructions / second for the computer



Issue: A fast computer runs hot

- Power dissipation rises as the <u>square of the CPU</u> <u>clock rate</u>
- Chips were heading towards melting down!

Multicore: with four
 CPUs (cores) on one chip,
 even if we run each at half
 speed we get more overall
 performance!



How a computer works

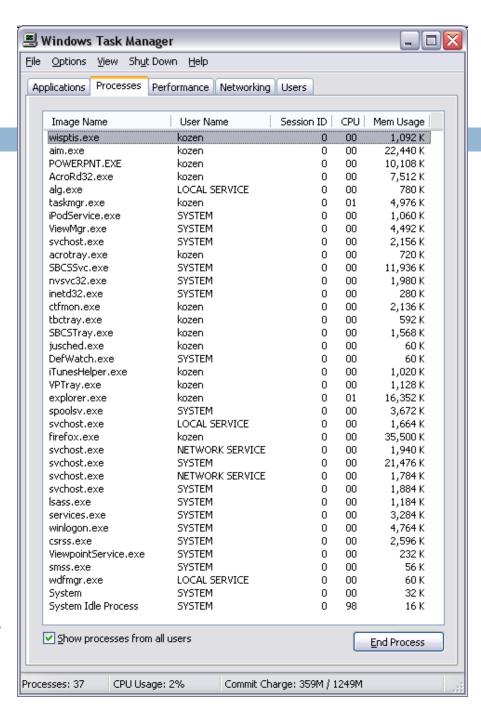
- Your program translates to machine instructions
- CPU has a pointer into the code: Program Counter
 - To execute an instruction, it fetches what the PC points to, decodes it, fetches the arguments, and performs the required action (such as add two numbers, then store at some location)
 - We call this a "thread of execution" or a "context of execution"
- □ One CPU == 1 thread, right? Well, not really....

Each program has its own thread!

- Earliest days: shared one CPU among many programs by just having it run a few instructions each, "round robin"
 - Program A gets to run 10,000 instructions
 - Then pause A, "context switch" to B, run 10,000 of B
 - □ Then pause B, context switch to C, run 10,000 for C...
- □ This makes one CPU seem like N (slower) CPUs
- With the new trend toward multicore we can have a lot of threads all concurrently active

Keeping those cores busy

- The operating system provides support for multiple "processes"
- In reality there there may be fewer processors than processes
- Processes are an illusion at the hardware level, lots of multitasking
 - memory subsystem
 - video controller
 - buses
 - instruction prefetching
- Virtualization can even let one machine create the illusion of many machines (they share disks, etc)



How is a Thread defined?

- A separate "execution" that runs within a single program and can perform a computational task independently and concurrently with other threads
- Many applications do their work in just a single thread: the one that called main() at startup
 - But there may still be extra threads...
 - ... Garbage collection runs in a "background" thread
 - GUIs have a separate thread that listens for events and "dispatches" upcalls
- Today: learn to create new threads of our own

What <u>is</u> a Thread in Java?

- A thread is a kind of object that "independently computes"
 - Has an associated stack and local variables (context)
 - Needs to be created, like any object
 - Then "started". This causes some method (like main()) to be invoked. It runs side by side with other thread in the same program and they see the same global data
- The actual execution could occur on distinct CPU cores, but Java could also simulate multiple cores.
 You can't really tell which approach Java is using

Concurrency

- Concurrency refers to a single program in which several threads are running simultaneously
 - Special problems arise: These threads literally access the same shared memory regions at the same time!
 - They are at risk of interfering with each other, e.g. if one thread is modifying a complex structure like a heap while another is trying to read it
- In cs2110 we focus on simple ways to use this model without bugs introduced by interference