CS/ENGRD 2110 Object-Oriented Programming and Data Structures

Spring 2012
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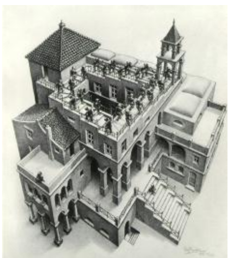
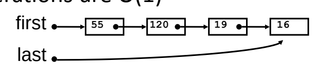


Lecture 17: Heaps and Priority Queues

Stacks and Queues as Lists

- Stack (LIFO) implemented as list
  - insert (i.e. push) to, extract (i.e. pop) from front of list
- Queue (FIFO) implemented as list
  - insert (i.e. add) on back of list, extract (i.e. poll) from front of list
- All operations are O(1)



Priority Queue

- ADT Definition
  - data items are **`Comparable`**
  - *lesser* elements (as determined by **`compareTo()`**) have *higher* priority
  - **`extract()`** returns the element with the highest priority
    - i.e. least in the **`compareTo()`** ordering
  - break ties arbitrarily
    - alternatively could break ties FIFO, but lets keep it simple

Priority Queue Examples

- Scheduling jobs to run on a computer
  - default priority = arrival time
  - priority can be changed by operator
- Scheduling events to be processed by an event handler
  - priority = time of occurrence
- Airline check-in
  - first class, business class, coach
  - FIFO within each class

`java.util.PriorityQueue<E>`

```
boolean add(E e) {...} //insert an element (insert)
void clear() {...} //remove all elements
E peek() {...} //return min element without removing
               //(null if empty)
E poll() {...} //remove min element (extract)
               //(null if empty)
int size() {...}
```

Priority Queues as Lists

- Maintain as *unordered* list (i.e. queue)
  - **`insert()`** puts new element at front – O(1)
  - **`extract()`** must search the list – O(n)
- Maintain as *ordered* list
  - **`insert()`** must search the list – O(n)
  - **`extract()`** gets element at front – O(1)
- In either case, O(n^2) to process n elements
- Can we do better?

Important Special Case

- Fixed (and small) number of p priority levels
  – Queue within each level
  – Example: airline check-in

- insert() – insert in appropriate queue – O(1)
- extract() – must find a nonempty queue – O(p)

Heaps

- A *heap* is a concrete data structure that can be used to implement priority queues
- Gives better complexity than either ordered or unordered list implementation:
  – insert(): O(log n)
  – extract(): O(log n)
  → O(n log n) to process n elements

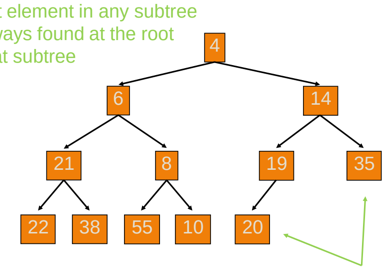
NOTE: Do not confuse with heap memory, where the Java virtual machine allocates space for objects – different usage of the word heap

Heap Invariant

- Binary tree with data at each node
- Satisfies the Heap Order Invariant:

The least (highest priority) element of any subtree is found at the root of that subtree.

Least element in any subtree is always found at the root of that subtree



But it is possible to have smaller elements deeper in the tree!

Examples of Heaps

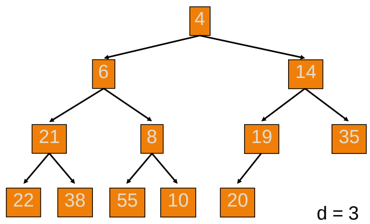
- Ages of people in family tree
  – parent is always older than children, but you can have an uncle who is younger than you

- Salaries of employees of a company
  – bosses generally make more than subordinates, but a VP in one subdivision may make less than a Project Supervisor in a different subdivision

Balanced Heaps

- Two restrictions:
  – Any node of depth < d – 1 has exactly 2 children, where d is the height of the tree
    - implies that any two maximal paths (path from a root to a leaf) are of length d or d – 1, and the tree has at least 2d nodes
  – All maximal paths of length d are to the left of those of length d – 1

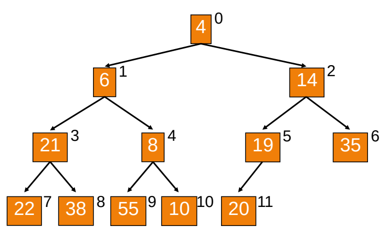
A Balanced Heap



Store in an ArrayList

- Elements of the heap are stored in the array in order, going across each level from left to right, top to bottom
- The children of the node at array index n are found at 2n + 1 and 2n + 2
- The parent of node n is found at (n – 1)/2

Store in an ArrayList



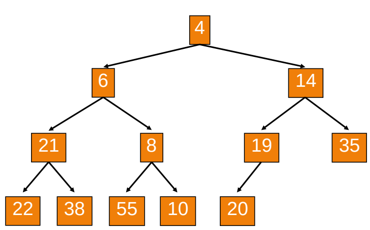
children of node n are found at 2n + 1 and 2n + 2

insert()

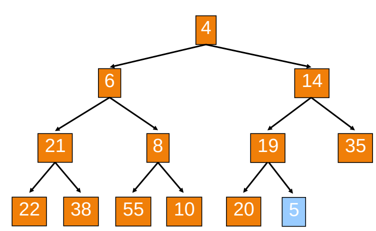
- Put the new element at the end of the array
- If this violates heap order because it is smaller than its parent, swap it with its parent
- Continue swapping it up until it finds its rightful place

→ The heap invariant is maintained!

insert() Example



insert() Example



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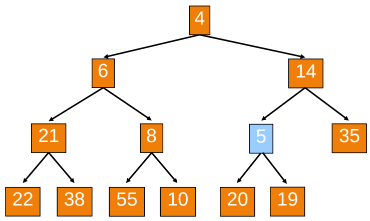
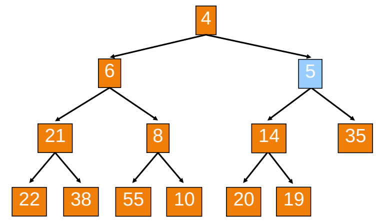


insert() Example



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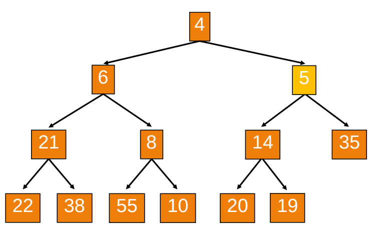
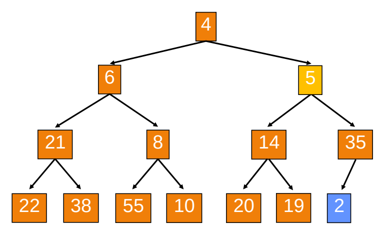
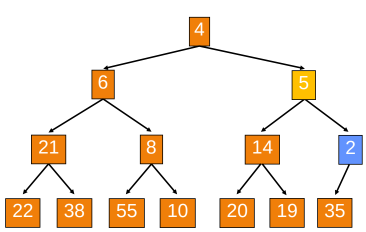


insert() Example



insert() Example



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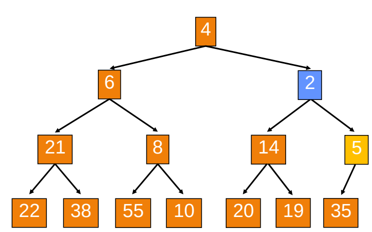

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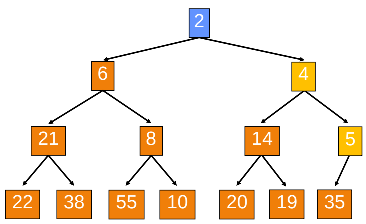

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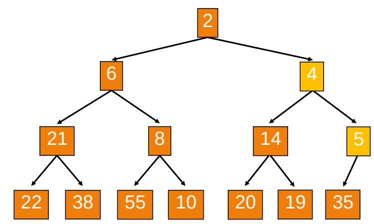


Analysis of insert()

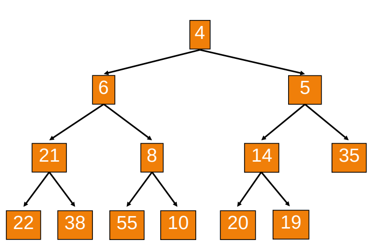
- Time is O(log n), since the tree is balanced
  - At most log(d) swaps up the tree before invariant is restored
  - size of tree is exponential as a function of depth d ⇔ depth of tree is logarithmic as a function of size n
  - Each insertion is finished after at most d <= log(n) swaps

extract()

- Remove the least element – it is at the root
- This leaves a hole at the root – fill it in with the last element of the array
- If this violates heap order because the root element is too big, swap it down with the smaller of its children
- Continue swapping it down until it finds its rightful place

→ The heap invariant is maintained!

extract() Example



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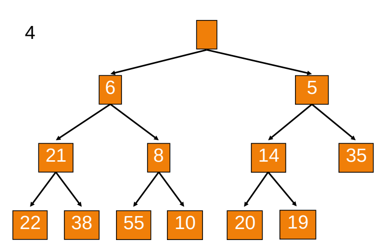

extract() Example

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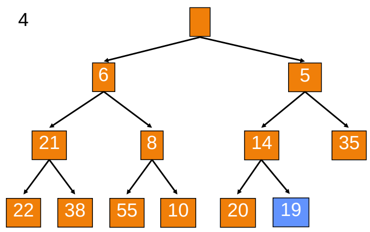

extract() Example

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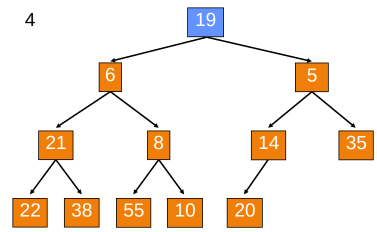

extract() Example

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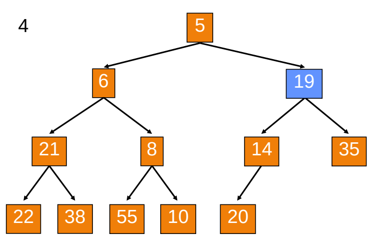

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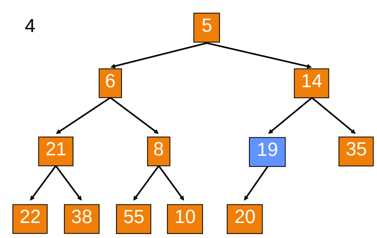

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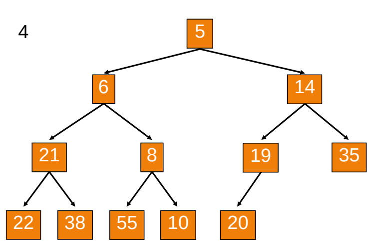

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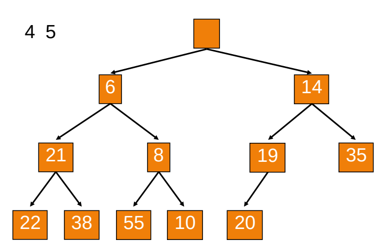

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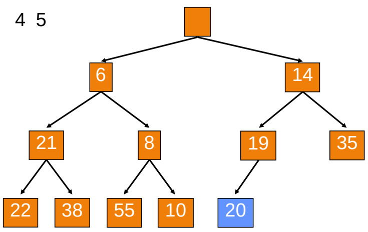

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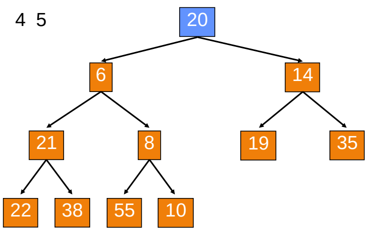

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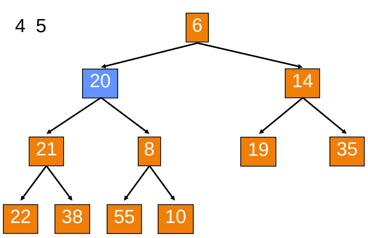

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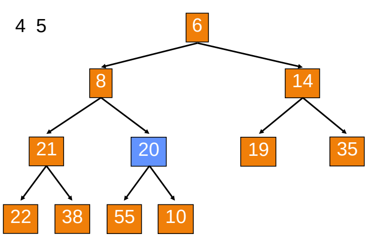

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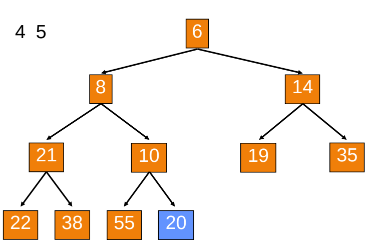

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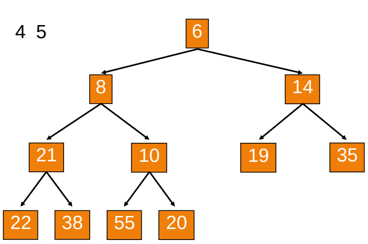

Analysis of extract()

- Time is O(log n), since the tree is balanced
  - At most log(d) swaps down towards the leaves of the tree before invariant is restored
  - size of tree is exponential as a function of depth d ⇔ depth of tree is logarithmic as a function of size n
  - Each extraction is finished after at most d <= log(n) swaps

HeapSort

- Given a Comparable[] array of length n
- Put all n elements into a heap – O(n log n)
- Repeatedly get the min and sequentially put into new array – O(n log n)