Write your name and Cornell netid. There are 5 questions on 10 numbered pages. Check now that you have all the pages. Write your answers in the boxes provided. Use the back of the pages for workspace. Ambiguous answers will be considered incorrect. The exam is closed book and closed notes. Do not begin until instructed. You have 90 minutes. Good luck!

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score</td>
<td>/20</td>
<td>/20</td>
<td>/20</td>
<td>/25</td>
<td>/15</td>
<td></td>
</tr>
<tr>
<td>Grader</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. (20 points) Here’s code for Quicksort. The code was on our web site and implements exactly the algorithm we discussed in class; it hasn’t been changed and it works exactly the way it should.

```
public class QuickSort<T extends Comparable<T>> implements Sorter<T> {
    static Random rand = new Random();

    public void sort(T[] x) {
        sort(x, 0, x.length);
    }

    // sort the portion of Comparable array x between
    // lo (inclusive) and hi (exclusive) in place
    // does not touch other parts of x
    private void sort(T[] x, int lo, int hi) {
        // base case
        if (hi <= lo + 1) return;

        // Pick the middle element as the pivot
        T pivot = x[(lo+hi)/2];

        // partition the elements about the pivot
        int i = lo;
        int j = hi - 1;
        while (true) {
            // advance indices
            while (i < j && x[i].compareTo(pivot) < 0) i++;
            while (i < j && x[j].compareTo(pivot) > 0) j--;
            // done?
            if (i == j) break;
            // swap
            T tmp = x[i];
            x[i] = x[j];
            x[j] = tmp;
        }

        // recursively sort the partition elements
        sort(x, lo, j);
        sort(x, j, hi);
    }
}
```

(a) 10 points. In class we analyzed the complexity of this algorithm and showed that the typical complexity is $O(n \log n)$. But we also discussed a “worst case” complexity bound. Explain what conditions provoke the worst-case complexity, and what that complexity is. You must explain your answer.
Hint: For parts (b) and (c), the specific way of picking the pivot element matters. In the version of Quicksort given above, this is the line that selects the pivot: \[ \texttt{pivot} = x[\lfloor (\texttt{lo}+\texttt{hi})/2 \rfloor]; \]

(b) 5 points. What would be the complexity of this version of Quicksort if it runs only on pre-sorted arrays (arrays that are already in the correct sorted order). Briefly explain your answer: a correct answer without a good explanation will lose points.

(c) 5 points. How would you construct inputs that cause Quicksort to exhibit the worst-case complexity that you gave us in part (a). Hint: tell us how to do it in words, then illustrate your solution for \( n=5 \). You don’t need to come up with actual code for building worst-case vectors.
2. (20 points) True or false?

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>T</td>
<td>F</td>
<td>In Java, it is possible to write code that can only be type-checked when the program is actually running.</td>
</tr>
<tr>
<td>b</td>
<td>T</td>
<td>F</td>
<td>If two classes ( A ) and ( B ) implement the same interface ( I ), then ( A ) is a subclass of ( B ), or ( B ) is a subclass of ( A ).</td>
</tr>
<tr>
<td>c</td>
<td>T</td>
<td>F</td>
<td>If ( \text{String s1 = &quot;Cleopatra&quot;} ), and ( \text{String s2 = &quot;Cleo&quot;+&quot;patra&quot;} ), then ( \text{s1.equals(s2)} ) returns true.</td>
</tr>
<tr>
<td>d</td>
<td>T</td>
<td>F</td>
<td>In Java, it is perfectly legal to have multiple definitions for the same method that have different types of arguments.</td>
</tr>
<tr>
<td>e</td>
<td>T</td>
<td>F</td>
<td>If ( \text{myArray} ) is of type ( \text{Array&lt;T&gt;} ), then the elements of ( \text{myArray} ) all have the same dynamic type.</td>
</tr>
<tr>
<td>f</td>
<td>T</td>
<td>F</td>
<td>Suppose class ( \text{foo} ) defines method ( m ), subclass ( \text{foobar} ) extends ( \text{foo} ) and does not override ( m ). Then if ( \text{fb} ) is a ( \text{foobar} ) object, ( \text{fb.m()} ) is legal and will runs ( m ) as defined in class ( \text{foo} ).</td>
</tr>
<tr>
<td>g</td>
<td>T</td>
<td>F</td>
<td>Given ( \text{int a = something} ), and ( \text{double b = something else} ), then ( ((\text{double}) a &lt; b) ) and ( (a &lt; (\text{int}) b) ) are equivalent expressions.</td>
</tr>
<tr>
<td>h</td>
<td>T</td>
<td>F</td>
<td>If ( \text{B} ) is a subclass of ( \text{A} ) and ( \text{b} ) is an instance of ( \text{B} ), then ( ((\text{B}(A)b == b) ) is true.</td>
</tr>
<tr>
<td>i</td>
<td>T</td>
<td>F</td>
<td>If an application requires ( O(1) ) cost for a particular method, that property would be specified as part of the type signature and will then be automatically checked by Java.</td>
</tr>
<tr>
<td>j</td>
<td>T</td>
<td>F</td>
<td>If any thread terminates by returning from its top-level method, the program in which that thread was running will terminate too, even if other threads were still active.</td>
</tr>
<tr>
<td>k</td>
<td>T</td>
<td>F</td>
<td>Java uses ( \text{synchronize, wait and notify} ) as tools for threads to coordinate their actions.</td>
</tr>
<tr>
<td>l</td>
<td>T</td>
<td>F</td>
<td>When using the predefined Java ( \text{HashMap&lt;T1,T2&gt;} ), the programmer must supply two types, one describing the type of the key and one describing the type of the value.</td>
</tr>
<tr>
<td>m</td>
<td>T</td>
<td>F</td>
<td>If ( \text{foo} ) is of type ( \text{HashMap&lt;Animal,ArrayList&lt;Gene&gt;&gt;} ) then one might use ( \text{foo} ) to store and retrieve the list of genes associated with some animal.</td>
</tr>
<tr>
<td>n</td>
<td>T</td>
<td>F</td>
<td>If a recursive method lacks a base case, Java will discover this when you try to compile the method and will force you to fix the code before it can be executed.</td>
</tr>
<tr>
<td>o</td>
<td>T</td>
<td>F</td>
<td>A race condition arises if two or more threads share (reading and writing) an object without synchronization.</td>
</tr>
<tr>
<td>p</td>
<td>T</td>
<td>F</td>
<td>A deadlock can only occur if two or more threads end up waiting for one-another in a “cycle”.</td>
</tr>
<tr>
<td>q</td>
<td>T</td>
<td>F</td>
<td>An exception handler is a special block of code that runs when some an exception of a particular class is thrown. It must fix whatever caused the exception, because when it completes, the operation that caused the exception will be retried.</td>
</tr>
<tr>
<td>r</td>
<td>T</td>
<td>F</td>
<td>If a class includes two methods with the same name but different type signatures, and one method calls the other, we would say that the method is recursive.</td>
</tr>
<tr>
<td>s</td>
<td>T</td>
<td>F</td>
<td>In class we discussed the use of induction to prove things about concurrently executing threads. In particular, using induction, we can prove that a traffic circle in which cars on the circle have priority over cars trying to enter will not be subject to deadlock.</td>
</tr>
<tr>
<td>t</td>
<td>T</td>
<td>F</td>
<td>The role of induction in a proof is analogous to the role of recursion in an algorithm.</td>
</tr>
</tbody>
</table>
3. (20 points) Suppose you are given a binary search tree class.

```java
public class BST<T extends Comparable<T>> {
    T nodeValue; // Value for this node
    BST<T> left;  // Left child, or null if none
    BST<T> right; // Right child, or null if none

    // usual code for methods Add and Lookup ...
}
```

a) (10 points) Write a new method `public int size()` that is called on a BST node, and counts the number of nodes in the subtree rooted at “this”. Don’t add extra class variables, but you can use additional helper methods, or local variables, as needed. **Example: if size is called on a leaf, it returns 1; if called on the root of a tree, size returns the number of nodes in the entire tree.**
b) (10 points) Write a method `public List<T> toList()` that is called on a BST node, and returns a list of node values for the subtree rooted at “this”, in sorted order. Again, don’t add class variables, but helper methods and local variables are fine. Hint: It is not necessary to implement a sorting algorithm!
4. (25 points) Here’s a map for part of the Paris subway system. Assume that the full map is represented as a directed graph and that each edge is labeled by how long the train takes. For example, Concorde-Invalides normally takes 2min 10secs.

a) (10 points) In class we learned about Dijkstra’s shortest path algorithm. Suppose that you were designing the computer program to recommend routes. The program knows where the user is currently located (the station where he/she is posing the question), and where that person wants to go. How could you apply Dijkstra’s algorithm to solve this problem? How does that algorithm work, and why does it solve our problem? Make sure to tell us how we would obtain the actual route to follow!
b) (15 points) Our map (and hence our graph) lacks any information about the time needed to switch trains. Suppose we are given a correspondence delays table, as shown to the right. Describe a simple way of modifying the graph representing the metro map, that lets us continue to use the same algorithm as in (a) but will now take correspondence delays into account. *Hint: your solution will add some extra nodes and some extra edges.* Make sure to tell us which nodes to add and which edges to add and what weight to put on the edges. Notice that the table isn’t “symmetric” (this is because trains are more frequent on some lines).
5. (10 points) You developed your genome phylogeny application as a single-threaded application, even though you own a 4-core computer. It needed 10 minutes to compute a Phylogeny graph.

Using a debugging tool you discover that the program is spending 8 minutes just reading the data files. The number of files is the same as in cs2110 (40), but the files themselves are much, much larger: the DNA contains billions of codons. In contrast, DNA in our cs2110 files had about 50,000 codons.

To fully leverage your computer, you decide to use threads:

1. The main thread first creates 40 threads, one for each Animal.
2. These threads read the Animal data files in parallel; the main thread waits for them to finish.
3. As each thread reads the file, it parses the animal's DNA, storing each parsed gene into a `TreeSet<Gene>` collection. This `TreeSet` is shared, hence the threads use `synchronize` when accessing it.
4. When finished, the Animal-file-reader threads terminate and the main thread then resumes and finishes the computation.

Your code only uses synchronization statements for step 3, and has no bugs or race conditions.

You were hoping to see the 8 minutes drop to 2, and hence for your whole program to run in about 4 minutes. However, it actually runs a little slower with threads than it originally did without them!

a) (5 points). Your kid brother comments out all the `synchronize()` statements and now your program finishes quite a bit faster than the original 8 minutes. What does this tell you about why it was running so slowly? Was your brother’s idea a good one? Note: zero credit for “Yes” or “No” as an answer. We want to know why!
b) (5 points). Now we want you to generalize what you told us in part (a). In general, suppose that two concurrently active threads both execute synchronize(something) { ... }. Describe some conditions under which the synchronize statement can safely be removed.

c) (5 points). Now take (a) and (b) into account and tell, briefly, how to modify the solution so that it would actually speed up from multi-core parallelism. *Hint: for full credit, your proposal won’t change the number of threads but will instead focus on the problem causing the slowdown.*