Course Review
&
A Few
Unanswered
Questions

Lecture 25
CS2110 – Fall 2008
Announcements

• Final Exam
  ▪ Thursday, Dec 18
  ▪ 2 - 4:30pm
  ▪ Uris Auditorium

• Review Sessions
  ▪ Wednesday, Dec 17
  ▪ 7:30 - 9pm and 9 - 10:30pm
  ▪ Upson B17
  ▪ Both sessions the same
  ▪ Maybe an extra one Tuesday of next week - watch web site for an announcement

• For exam conflicts:
  ▪ Notify Kelly Patwell today
  ▪ You must provide
    ▪ Your entire exam schedule
    ▪ Include the course numbers

• Definition of exam conflict:
  ▪ Two exams at the same time or
  ▪ Three or more exams within 24 hours

• A5 due Monday, Dec 8, 11:59pm
  ▪ Sorry, no more extensions
Announcements

• Check the course website for additional announcements as the final exam approaches
  ▪ Consulting ends this week
  ▪ Office hours continue until Final Exam
    ✷ There may be changes (TAs have exams, too)
    ✷ Any changes will be announced on the course website

• Jealous of the glamorous life of a CS consultant?
  ▪ We're recruiting next-semester consultants for CS1110 and CS2110
  ▪ Interested students should fill out an application, available in 303 Upson
Course Evaluations

• Worth one assignment point
  ▪ Will count as 1% of your course grade
  ▪ This is a regular point, not a bonus point
  ▪ Anonymity
    ♦ We get a list of who completed the course evaluations and a list of responses, but no link between names & responses

• Open now

• Closes Wednesday, December 10, midnight

• http://www.engineering.cornell.edu/CourseEval
  ▪ This link also appears on the CS2110 announcements page
Course Overview

• Programming concepts
   We use Java, but the goal is to understand the ideas rather than to become a Java expert
  ▪ Recursion
  ▪ Object-Oriented Programming
  ▪ Interfaces
  ▪ Graphical User Interfaces (GUIs)

• Data structure concepts
   The goal here is to develop skill with a set of tools that are widely useful
  ▪ Induction
  ▪ Asymptotic analysis (big-O)
  ▪ Arrays, Trees, and Lists
  ▪ Searching & Sorting
  ▪ Stacks & Queues
  ▪ Priority Queues
  ▪ Sets & Dictionaries
  ▪ Graphs
Programming Concepts

• Recursion
  ▪ Stack frames
  ▪ Exceptions

• Object-oriented programming
  ▪ Classes and objects
  ▪ Primitive vs. reference types
  ▪ Dynamic vs. static types
  ▪ Subtypes and Inheritance
    ▷ Overriding
    ▷ Shadowing
    ▷ Overloading
    ▷ Upcasting & downcasting
  ▪ Inner & anonymous classes

• Interfaces
  ▪ Type hierarchy vs. class hierarchy
  ▪ The Comparable interface
  ▪ Iterators & Iterable

• GUIs
  ▪ Components, Containers, & Layout Managers
  ▪ Events & listeners
Data Structure Concepts

- Induction
- Grammars & parsing
- Asymptotic analysis (big-O)
  - Solving recurrences
  - Lower bounds on sorting
- Basic building blocks
  - Arrays
  - Lists
    - Singly- and doubly-linked
  - Trees
    - Binary Search Trees (BSTs)
- Searching
  - Linear- vs. binary-search
- Sorting
  - Insertion-, Selection-, Merge-, Quick-, and Heapsort
- Useful ADTs (& implementations)
  - Stacks & Queues
    - Arrays & lists
  - Priority Queues
    - Heaps
    - Array of queues
  - Sets & Dictionaries
    - Bit vectors (for Sets)
    - Arrays & lists
    - Hashing & Hashtables
    - BSTs (& balanced BSTs)
  - Graphs...
Overview of Graphs

• Mathematical definition of a graph (directed, undirected)
• Representations
  ▪ Adjacency matrix
  ▪ Adjacency list
• Topological sort
• Coloring & planarity
• Searching (BFS & DFS)
• Dijkstra’s shortest path algorithm
• Minimum Spanning Trees (MSTs)
  ▪ Prim’s algorithm (growing a single tree)
  ▪ Kruskal’s algorithm (build a forest by adding edges in order)
Some Unsolved Problems
Complexity of Bounded-Degree Euclidean MST

• The Euclidean MST (Minimum Spanning Tree) problem:
  ▪ Given n points in the plane, determine the MST
  ▪ Can be solved in $O(n \log n)$ time by first building the Delaunay Triangulation

• Bounded-degree version:
  ▪ Given n points in the plane, determine a MST where each vertex has degree $\leq d$
    ▪ Known to be NP-hard for $d=3$ [Papadimitriou & Vazirani 84]
    ▪ $O(n \log n)$ algorithm for $d=5$ or greater
      • Can show Euclidean MST has degree $\leq 5$
    ▪ Unknown for $d=4$
Complexity of Euclidean MST in $\mathbb{R}^d$

• Given $n$ points in dimension $d$, determine the MST
  ▪ Is there an algorithm with runtime close to the $\Omega(n \log n)$ lower bound?

• Can solve in time $O(n \log n)$ for $d=2$

• For large $d$, it appears that runtime approaches $O(n^2)$

• Best algorithms for general graphs run in time linear in $m$
  ▪ But for Euclidean distances on points, the number of edges is $n(n-1)/2$
O(n^2) Time for X+Y Sorting?

How long does it take to sort an n-by-n table of numbers?

- O(n^2 log n) because there are n^2 numbers in the table

- What if it’s an addition table?
  - Shouldn’t it be easier to sort than an arbitrary set of n^2 numbers?

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- There is a technique that uses just O(n^2) comparisons [Fredman 76]
  - But it uses O(n^2 log n) time to decide which comparisons to use [Lambert 92]

- This problem is closely related to the problem of sorting the vertices of a line arrangement
3SUM in Subquadratic Time?

- Given a set of $n$ integers, are there three that sum to zero?
  - $O(n^2)$ algorithms are easy (e.g., use a hashtable)
  - Are there better algorithms?

- This problem is closely related to many other problems [Gajentaan & Overmars 95]
  - Given $n$ lines in the plane, are there 3 lines that intersect in a point?
  - Given $n$ triangles in the plane, does their union have a hole?
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3-Colorability of Great-Circle Graphs?

• Build a graph by drawing great-circles on a sphere
  ▪ Create a vertex for each intersection
  ▪ Assume no three great circles intersect in a point

• Is the resulting graph 3-colorable?

• All arrangements for up to 11 great circles have been verified as 3-colorable

• For general circles on the sphere (or for circles on the plane) the graph can require 4 colors
Winning Strategies for the Parity Game?

- Played on a directed graph with nodes 0, 1, 2, ..., n−1
- Start with a pebble on node 0
- Players Steven and Todd alternately choose edges along which to push the pebble
- They play forever...
- Steven wins if the least-numbered vertex visited infinitely often is even
- Todd wins if the least-numbered vertex visited infinitely often is odd
- It is known that for any graph, either Steven or Todd has a winning strategy – but can you determine which?
- Equivalent to a major open problem in logic
The Big Question: Is P=NP?

• P is the class of problems that can be solved in polynomial time
  ▪ These problems are considered tractable
  ▪ Problems that are not in P are considered intractable

• NP represents problems that, for a given solution, the solution can be checked in polynomial time
  ▪ But finding the solution may be hard

• For ease of comparison, problems are usually stated as yes-or-no questions

• Examples
  ▪ Given a weighted graph G and a bound k, does G have a spanning tree of weight at most k?
    ❖ This is in P because we have an algorithm for the MST with runtime $O(m + n \log n)$

  ▪ Given graph G, does G have a Hamiltonian cycle (a simple cycle that visits all vertices)?
    ❖ This is in NP because, given a possible solution, we can check in polynomial time that it’s a cycle and that it visits all vertices exactly once
Current Status: P vs. NP

- It’s easy to show that $P \subseteq NP$
- Most researchers believe that $P \neq NP$
  - But at present, no proof
  - We do have a large collection of NP-complete problems
    - If any NP-complete problem has a polynomial time algorithm, then they all do

- A problem B is *NP-complete* if
  1. it is in NP
  2. any other problem in NP reduces to it efficiently

- Thus by making use of an imaginary fast subroutine for B, any problem in NP could be solved in polynomial time
  - the Boolean satisfiability problem is NP-complete [Cook 1971]
  - many useful problems are NP-complete [Karp 1972]
  - By now thousands of problems are known to be NP-complete
Some NP-Complete Problems

• Graph coloring: Given graph G and bound k, is G k-colorable?
• Planar 3-coloring: Given planar graph G, is G 3-colorable?
• Traveling salesperson: Given weighted graph G and bound k, is there a cycle of cost \( \leq k \) that visits each vertex at least once?
• Hamiltonian cycle: Given graph G, is there a cycle that visits each vertex exactly once?
• Knapsack: Given a set of items i with weights \( w_i \) and values \( v_i \), and numbers W and V, does there exist a subset of at most W items whose total value is at least V?

• What if you really need an algorithm for an NP-complete problem?
  ▪ Some special cases can be solved in polynomial time
    ✷ If you’re lucky, you have such a special case
  ▪ Otherwise, once a problem is shown to be NP-complete, the best strategy is to start looking for an approximation

• For a while, a new proof showing a problem NP-complete was enough for a paper
  ▪ Nowadays, no one is interested unless the result is somehow unexpected
Good luck on the final!
Thanks for an enjoyable semester!
Have a great winter break!