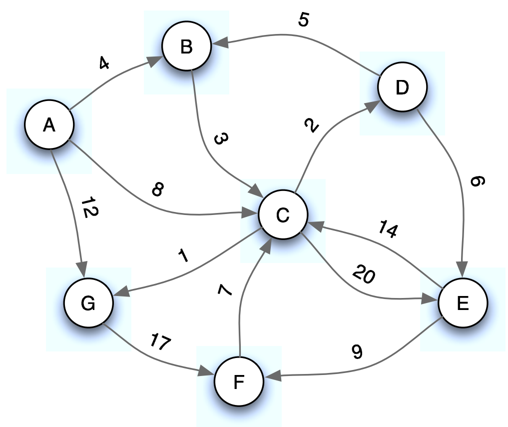
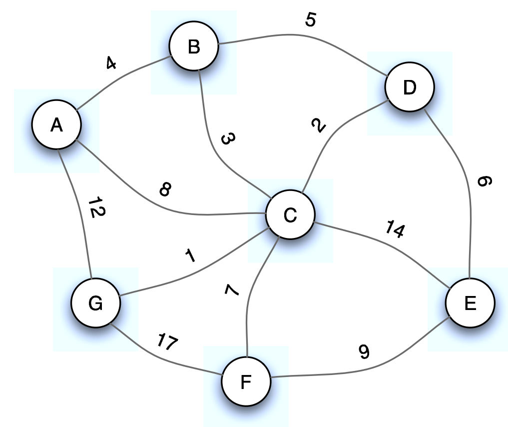
CS 211 Summer 2007, Review problem set #2

Consider the following two graphs:



Graph (a)



Graph (b)

1. What is the indegree of node C in graph (a)? What is its outdegree? What is the degree of node G in graph (b)? Indegree of C is 4. Outdegree is 2. Degree of G is 3.

2. Is graph (a) a dag? How do you know? How many edges must be removed before it becomes a dag? Not a dag, because there are cycles (e.g. C -> D -> E -> C). Three edges must be removed: (C,D), (C,E), (C,G).

3. How many simple cycles of path length 4 are there in graph (b)? 6 simple cycles.

4. What is the matrix representation of graph (a)? What is the adjacency list representation of graph (a)?

	A	B	C	D	E	F	G
A		1	1				1
B			1				
C				1	1		1
D		1			1		
E			1			1	
F			1				
G						1	

A: B -> C -> G
B: C
C: D -> E
D: B -> E
E: C -> F
F: C
G: F

5. How many edges are there in a spanning tree of graph (b)? How many minimum spanning trees of graph (b) are there?
   # edges in tree = (# nodes) - 1 = 6. There is only one MST for this graph because the edge weights are unique.

6. Is graph (b) complete? Is it bipartite? Is it planar?
   Not complete, not bipartitle. It is planar.

7. In what order would the nodes of graph (a) be traversed in a breadth-first search? What about a depth-first search?
   Many possible answers, e.g. BFS: A, B, C, G, D, E, F; DFS: A, B, C, D, E, F, G.

8. Use Dijkstra's algorithm to find the shortest path from A to every other node in graph (a).
   Shown in class.

9. Compute a minimum spanning tree of graph (b) using Prim's algorithm.
   Shown in class.

10. If an undirected connected graph has n nodes, what is the minimum number of edges that it has? What is the maximum number of edges? What is the maximum number of edges if the graph is directed?
    Since it's connected (i.e. each node is involved in at least one edge), it must at least have $n-1$ edges. The maximum number of edges in an undirected graph is:
    $$(n-1)+(n-2)+...+1=\frac{n(n-1)}{2}$$
    The maximum number of edges in a directed graph is twice as many, or $n(n-1)$.