NAME:_____________________________________

CU ID:____________________________________

Recitation instructor/time__________________________

You have one and a half hours to do this exam.

All programs in this exam must be written in Java. Excessively convoluted code will not be graded.

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Problem        Score

1

2

3

4

5

6

Total
1. (15 points)

(a) (10 points) Write a Java class method named *append* that takes two *non-empty* linked lists L1 and L2 as parameters, and updates the last cell of L1 so that it points to the first cell of L2, as shown in Figure 1 below. The method does not return anything. The ListCell class from lecture is reproduced at the end of the exam. You may NOT use the Java LinkedList class.

(b) (5 points) What is the asymptotic complexity of your algorithm, expressed as a function of $n_1$ and $n_2$ where $n_1$ is the number of elements in L1, and $n_2$ is the number of elements of L2 respectively? Justify your answer briefly.

![Figure 1: Appending two lists](image)
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2. (15 points)

Suppose \( f_1(n) = O(g(n)) \) and \( f_2(n) = O(g(n)) \). Answer the following questions.

(a) Consider the function \( h(n) = f_1(n) + f_2(n) \). Is \( h(n) = O(g(n)) \)?
Justify your answer formally using witness pairs \((k, N)\) as described in class.

(b) Consider the function \( k(n) = f_1(n) \cdot f_2(n) \). Is \( k(n) = O(g(n)) \)?
Justify your answer formally.
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3. (30 points)

(a) (10 points) Write a recursive class method that computes the number of nodes in a binary tree. Assume that the root of the tree is a parameter to the method. The TreeCell class from lecture is reproduced at the end of the exam.

(b) (2 points) Does your method do an in-order, post-order, or pre-order walk of the tree?

(c) (3 points) What is the asymptotic complexity of your method? Explain your answer briefly.

(d) (10 points) Write a recursive class method to print the values stored in a binary search tree. You may assume that the root of the tree is a parameter to the method. The values must be printed in \textit{descending} order — that is, the largest value must be printed first. You may assume that each data item has a \texttt{toString} method that returns a string representation of that item.

(e) (2 points) Does your method do an in-order, post-order, or pre-order walk of the tree?

(f) (3 points) What is the asymptotic complexity of your method? Explain your answer briefly.
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4. (Short answers) (20 points)

*Time* in the following questions refers to *worst-case asymptotic complexity.*

(a) Linear search of an array requires that the array be sorted. True or false.

(b) Linear search in a sorted array of n elements takes time _____.

(c) Binary search in an array requires that the array be sorted. True or false.

(d) Binary search in a sorted array of n elements takes time _____.

(e) Quick-sort of an array of n elements takes time _____.

(f) Merge-sort of an array of n elements takes time _____.

(g) Insertion into a sorted list of n elements takes time _____.

(h) Deletion from a sorted list of n elements takes time _____.

(i) Search in a (not necessarily balanced) binary search tree of n elements takes time _____.

(j) Deletion in a (not necessarily balanced) binary search tree of n elements take time _____.


5. (12 points) A binary tree is known to have $2^n - 1$ nodes where $n$ is some positive integer greater than 1, but nothing else is known about its structure.

(a) What is the smallest number of leaf nodes it can have?

(b) What is the largest number of leaf nodes it can have?

(c) What is the largest number of edges that can be there in a simple path from the root of the tree to a leaf?

(d) What is the smallest number of edges that can be there in a simple path from the root of the tree to a leaf?

Justify your answers.
6. (8 points) We know that a sorted array can contain values in either ascending or descending order. As defined in class, binary search trees contain values in “ascending” order — that is, the left sub-tree of a node contains values less than the value at the node, while the right sub-tree contains values greater than the value at that node.

Define a reverse binary search tree to be a binary search tree that contains values in “descending” order — that is, the left sub-tree of a node contains values greater than the value at that node, while the right sub-tree contains values less than the value at that node.

(a) Write a recursive class method to modify a binary search tree into the corresponding reverse binary search tree. Assume that the root of the tree is passed as a parameter to the method, and that the tree is represented using the TreeCell class given at the end of this exam.

(b) Does your method perform an in-order, post-order or pre-order walk of the tree?

![Binary Search Tree and Reverse Binary Search Tree](image)

**Binary Search Tree**  
**Reverse Binary Search Tree**
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class ListCell {
    protected Object datum;
    protected ListCell next;

    public ListCell(Object o, ListCell n) {
        datum = o;
        next = n;
    }

    // this is sometimes called the "car" method
    public Object getDatum() {
        return datum;
    }

    // this is sometimes called the "cdr" method
    public ListCell getNext() {
        return next;
    }

    // this is sometimes called the "rplaca" method
    public void setDatum(Object o) {
        datum = o;
    }

    // this is sometimes called the "rplacd" method
    public void setNext(ListCell l) {
        next = l;
    }

    public String toString() {
        String rString = datum.toString();
        if (next == null) return rString;
        else return rString + " " + next.toString();
    }
}

class TreeCell {
    protected Object datum;
    protected TreeCell left;
    protected TreeCell right;

    public TreeCell(Object i) {
        datum = i; // left and right are null by default
    }
public TreeCell (Object i, TreeCell l, TreeCell r) { 
    datum = i;
    left = l;
    right = r;
}

public void setDatum(Object o) {
    this.datum = o;
}

public Object getDatum() {
    return datum;
}

public void setLeft(TreeCell t) {
    this.left = t;
}

public TreeCell getLeft() {
    return left;
}

public void setRight(TreeCell t) {
    this.right = t;
}

public TreeCell getRight() {
    return right;
}

public String toString() {
    String lString, rString;
    if (left == null)
        lString = "()"
    else
        lString = left.toString();
    if (right == null)
        rString = "()"
    else
        rString = right.toString();
    return "(" + lString + " + " + datum + "+" + rString + ")";
}