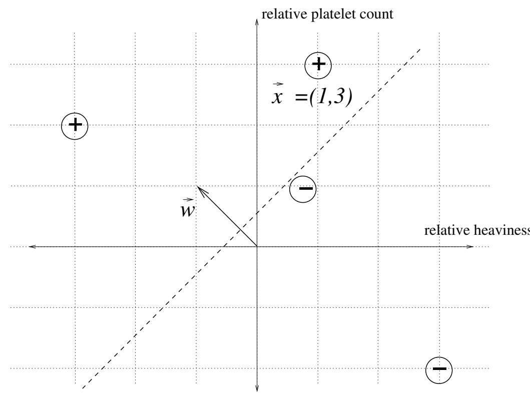
Computation, Information, and Intelligence (ENGRI/CS/INFO/COGST 172), Spring 2007
2/9/07: **Lecture aid — Perceptron characterizations; on-line learning**

Agenda: perceptrons as linear separators (a geometric perspective); formalizing the learning problem

I. Recall: perceptron functions Assume a fixed weight vector $\overrightarrow{w} = (w_1, \ldots, w_n)$ and a fixed threshold value T.

$$f_{\overrightarrow{w},T}(\overrightarrow{x}) \stackrel{\text{def}}{=} \begin{cases} +1, & \overrightarrow{w} \cdot \overrightarrow{x} \geq T \\ -1 & \text{o.w. (otherwise)} \end{cases} .$$

II. Geometric interpretation example Let $\overrightarrow{w} = (-1, 1), T = \frac{1}{2}$.



III. Half-plane characterization Ignoring cases of length-zero vectors, it's the case that

$$f_{\overrightarrow{w},T}(\overrightarrow{x}) = \begin{cases} +1, & \text{proj}(\overrightarrow{x}, \overrightarrow{w}) \geq \frac{T}{\text{length}(\overrightarrow{w})} \\ -1 & \text{otherwise} \end{cases} .$$

So the points for which the perceptron's function is +1 are precisely the "half-plane" consisting of vectors (points) whose projections onto $\overrightarrow{w}$ (determined by "dropping a perpendicular" onto $\overrightarrow{w}$) are at least $T/\text{length}\,(\overrightarrow{w})$.

IV. Practice questions

1. Suppose we have observed two patients, and have recorded the following two features ("symptoms") for them: heaviness relative to the average, and platelet count relative to the average. We are trying to figure out some rule to explain why one of them, $\overrightarrow{x}^{(1)} = (-2, -4)$ was diagnosed as having a certain disease, and the other, $\overrightarrow{x}^{(2)} = (-1, 0)$, wasn't. Give a $\overrightarrow{w}$ and T corresponding to a perceptron function that correctly classifies the two patients with respect to the given diagnoses.

2. Suppose we now require that the rule be particularly simple, in that it is mandatory that $T = 0$ and the length of $\overrightarrow{w}$ is 1. Try the question above under these restrictions.

V. Conventions for the oracle's sequence of labeled examples We denote the instances by $\overrightarrow{x}^{(1)}, \overrightarrow{x}^{(2)}, \ldots, \overrightarrow{x}^{(i)}, \ldots$. Those that are given label +1 are known as *positive* examples; those that are given label -1 are known as *negative* examples. The reason for the superscripting is to avoid confusion between $\overrightarrow{x}^{(i)}$, a vector, and x_i, a vector component; the reaons for the parentheses is to avoid confusing i for an exponent.