Computation, Information, and Intelligence (ENGRI/CS/INFO/COGST 172)
2/2: **Games and minimax values**

Topics: games — a model of problems in which not all the actions are under the control of a single entity; play based on minimax values.

I. Finite two-player zero-sum games with perfect information Players alternate turns.

(a). Both players know the full specification.

(b). Both players also know the *outcome* function o, whose domain is on dead states, and whose range is a set of numeric values indicating the "final score" of Player 1.

(c). Player 2's "final score" in any dead state s is $-o(s)$, and both players want positive final scores.

(d). The game is guaranteed to terminate.

II. Evaluation functions Each player may employ an *evaluation function* on states, where $f_i(s)$ indicates Player i's *estimate of the final score* that Player 1 will receive *if* both players *act optimally* starting from state s.

We assume that if s is a dead state, then $f_1(s) = f_2(s) = o(s)$.

We will often use f as shorthand for f_1.

III. Example evaluation functions

(a). Problem: win a game of checkers. States: legal board positions (plus move counts, to prevent infinite games).

Function: Let $n_1(s)$ and $n_2(s)$ be the number of pieces you and your opponent, respectively, have in the board position corresponding to state s, with kings counting double.

$$f(s) = \begin{cases} +100 & \text{s is a state in which you have won} \\ -100 & \text{s is a state in which you have lost} \\ n_1(s) - n_2(s) & \text{otherwise} \end{cases}$$

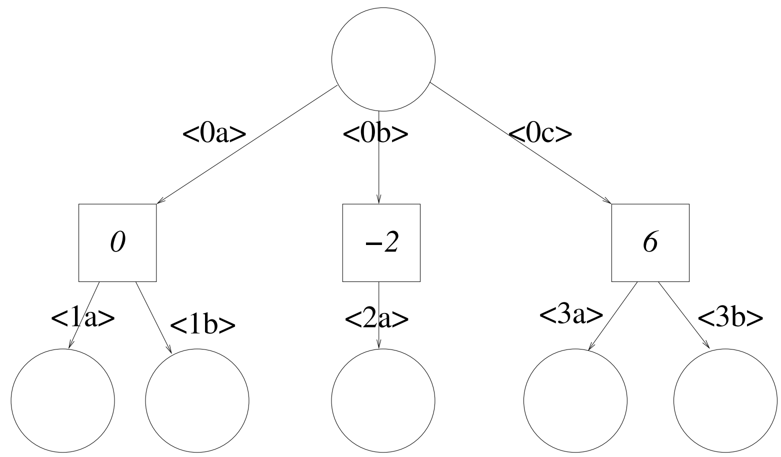
(b). Problem: begin and maintain a stock portfolio for the month with the aim of making a profit. States: portfolio and the prices of each item of the portfolio.

Function: the value of the portfolio, except that when it's not the end of the month, you add \$30 for every technology stock held.

IV. Example game tree (Player 1's perspective) We omit the state labels on the nodes for clarity. ◯ and □ indicate that in the state labeling the node, it is Player 1 or Player 2's turn to move, respectively.

The numbers in italics indicate the value of f_1 on the states labeling the game-tree nodes in question.

We are omitting the values of f_1 on the nodes that are labeled with dead states (i.e., the *leaves* of the tree) in order to make a point.



V. Minimax value of a (game tree) node N: the value of the outcome function for the (dead) state that results if, starting from the state that labels node N, both players play optimally until the end of the game.

VI. The tree from above, with the outcome values revealed