Topics: games — a model of problems in which not all the actions are under the control of a single entity; play based on minimax values.

I. Finite two-player zero-sum games with perfect information Players alternate turns.

(a). Both players know the full specification.

(b). Both players also know the outcome function $o$, whose domain is on dead states, and whose range is a set of numeric values indicating the “final score” of Player 1.

(c). Player 2’s “final score” in any dead state $s$ is $-o(s)$, and both players want positive final scores.

(d). The game is guaranteed to terminate.

II. Evaluation functions Each player may employ an evaluation function on states, where $f_i(s)$ indicates Player $i$’s estimate of the final score that Player 1 will receive if both players act optimally starting from state $s$.

We assume that if $s$ is a dead state, then $f_1(s) = f_2(s) = o(s)$.

We will often use $f$ as shorthand for $f_1$.

III. Example evaluation functions


Function: Let $n_1(s)$ and $n_2(s)$ be the number of pieces you and your opponent, respectively, have in the board position corresponding to state $s$, with kings counting double.

$$f(s) = \begin{cases} 
+100 & \text{s is a state in which you have won} \\
-100 & \text{s is a state in which you have lost} \\
n_1(s) - n_2(s) & \text{otherwise}
\end{cases}$$

(b). Problem: begin and maintain a stock portfolio for the month with the aim of making a profit. States: portfolio and the prices of each item of the portfolio.

Function: the value of the portfolio, except that when it’s not the end of the month, you add $30 for every technology stock held.
IV. Example game tree (Player 1’s perspective) We omit the state labels on the nodes for clarity. ○ and □ indicate that in the state labeling the node, it is Player 1 or Player 2’s turn to move, respectively.

The numbers in italics indicate the value of $f_1$ on the states labeling the game-tree nodes in question.

We are omitting the values of $f_1$ on the nodes that are labeled with dead states (i.e., the leaves of the tree) in order to make a point.

V. Minimax value of a (game tree) node $N$: the value of the outcome function for the (dead) state that results if, starting from the state that labels node $N$, both players play optimally until the end of the game.

VI. The tree from above, with the outcome values revealed