DSFA

Spring 2021

Lecture 14

Probability

Announcements

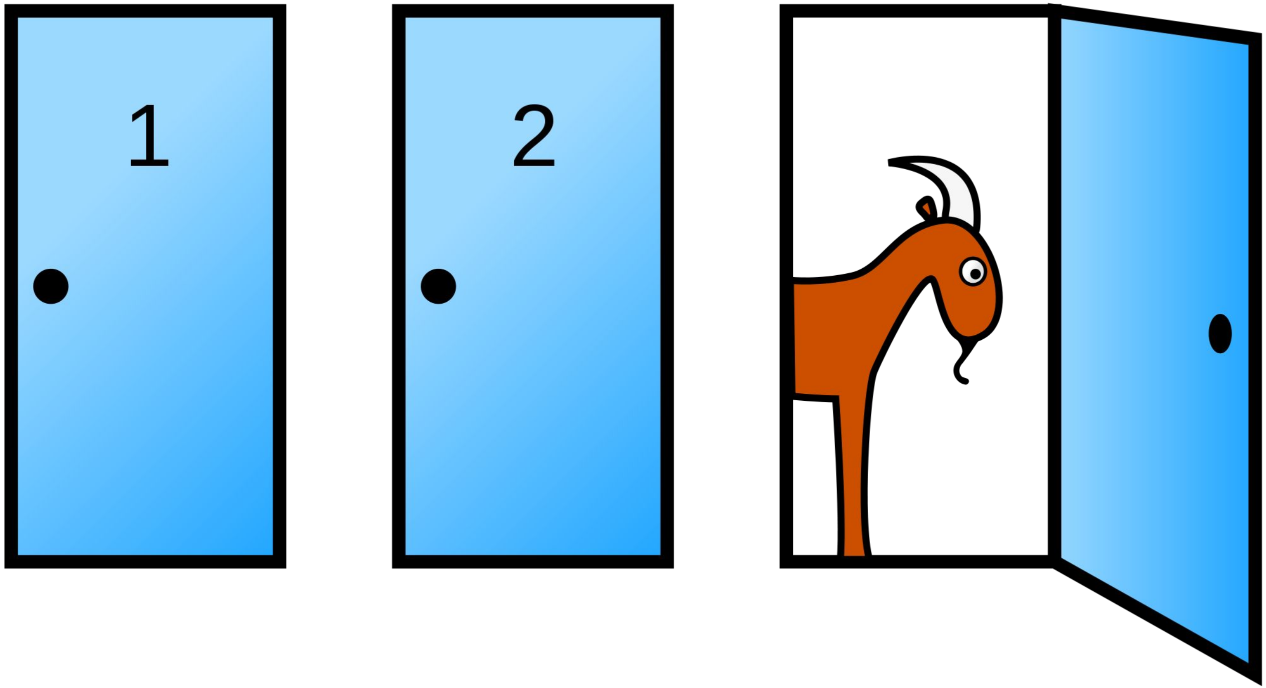
- Prelim 1: Tuesday, 8:30-10PM, here
- Lab: Wednesday-Thursday as usual
- Lecture: As usual
- Project 1, Part 1: due Friday 5:59PM

What's the best choice to win the game?

- Stay
- Switch
- It doesn't matter

The Monty Hall Problem

Monty Hall Problem



Game Show Problem

(This material in this article was originally published in PARADE magazine in 1990 and 1991.)

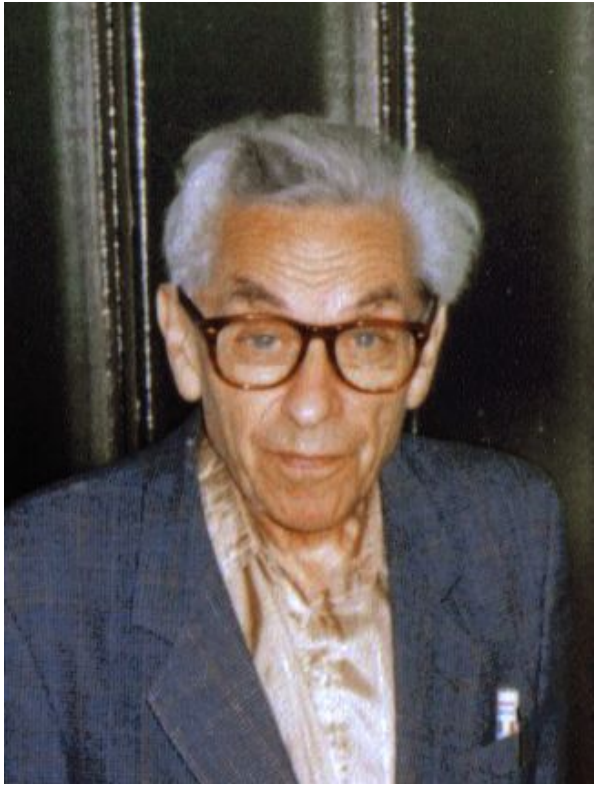
Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say #1, and the host, who knows what's behind the doors, opens another door, say #3, which has a goat. He says to you, "Do you want to pick door #2?" Is it to your advantage to switch your choice of doors?
Craig F. Whitaker
Columbia, Maryland

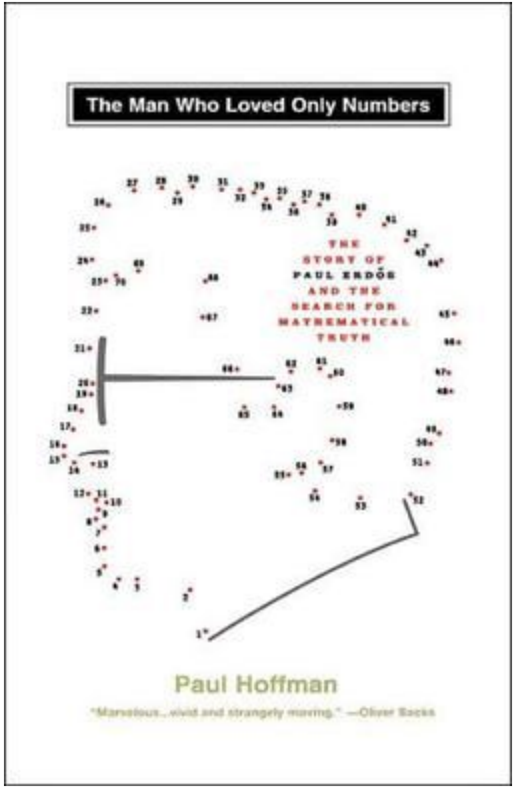
Yes; you should switch. The first door has a 1/3 chance of winning, but the second door has a 2/3 chance. Here's a good way to visualize what happened. Suppose there are a million doors, and you pick door #1. Then the host, who knows what's behind the doors and will always avoid the one with the prize, opens them all except door #777,777. You'd switch to that door pretty fast, wouldn't you?

Since you seem to enjoy coming straight to the point, I'll do the same. You blew it! Let me explain. If one door is shown to be a loser, that information changes the probability of either remaining choice, neither of which has any reason to be more likely, to 1/2. As a professional mathematician, I'm very concerned with the general public's lack of mathematical skills. Please help by confessing your error and in the future being more careful.
Robert Sachs, Ph.D.
George Mason University

We've received thousands of letters, and of the people who performed the experiment by hand as described, the results are close to unanimous: you win twice as often when you change doors. Nearly 100% of those readers now believe it pays to switch.

Paul Erdős




The Man Who Loved Only Numbers
THE STORY OF PAUL ERDŐS AND THE SEARCH FOR MATHEMATICAL TRUTH
Paul Hoffman
"Marvelous...vivid and strangely moving." —Oliver Sacks

Probability

Probability

- Lowest value: 0
  - Chance of event that is impossible
- Highest value: 1 (or 100%)
  - Chance of event that is certain

- If an event has chance 70%, then the chance that it doesn't happen is
  - 100% - 70% = 30%
  - 1 - 0.7 = 0.3

Equally Likely Outcomes

Assuming all outcomes are equally likely, the chance of an event A is:

$$P(A) = \frac{\text{number of outcomes that make A happen}}{\text{total number of outcomes}}$$

(Demo)

Probability of drawing Green then Red

1/6

1/3

1/2

None of the above

Probability of drawing Red either first or second

1/6

1/3

1/2

None of the above

Multiplication Rule

Chance that two events *A* and *B* both happen

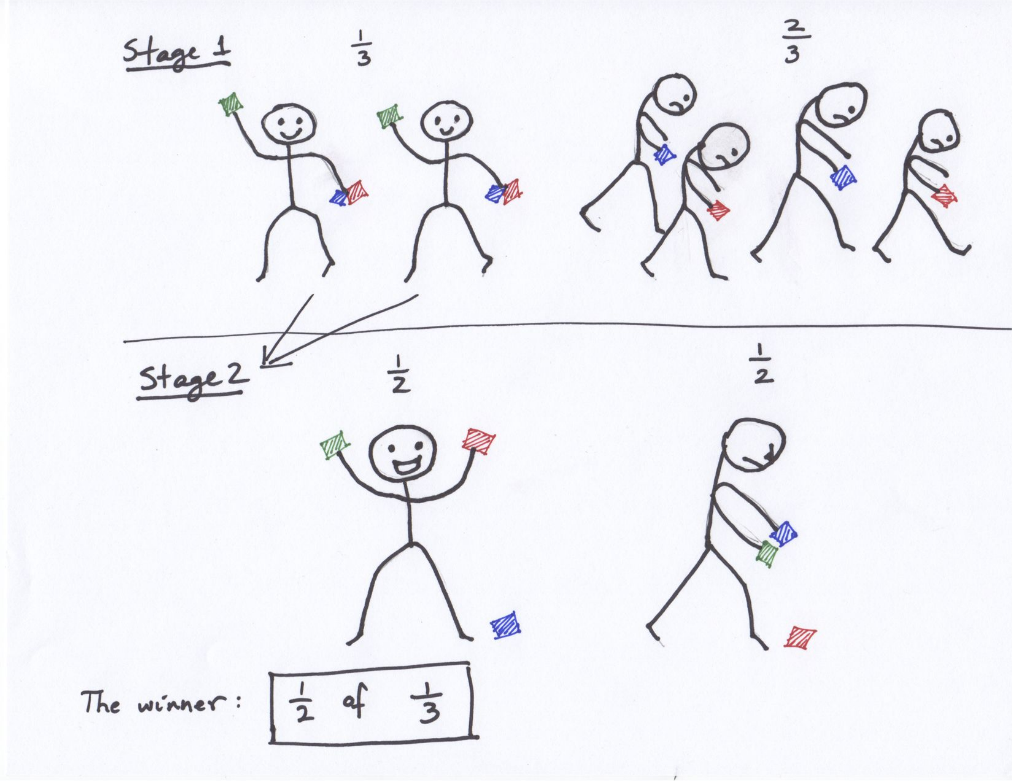
= P(*A* happens)

x P(*B* happens **given that** *A* has happened)

- The answer is *less than or equal to* each of the two chances being multiplied
- The more conditions you have to satisfy, the less likely you are to satisfy them all

(Demo)

Fraction of a Fraction



Addition Rule

If event *A* can happen in *exactly one* of two ways, then

$$P(A) = P(\text{first way}) + P(\text{second way})$$

- The answer is *greater than or equal to* the chance of each individual way

Example: At Least One Head

- In 3 tosses:
  - Any outcome *except* TTT
  - P(TTT) = (½) x (½) x (½) = $(½)^3$ = ⅛
  - P(at least one head) = 1 - P(TTT) = ⅞ = 87.5%

- In 10 tosses:
  - P(TTTTTTTTTT) = $(½)^{10}$
  - P(at least one head) = 1 - $(½)^{10}$ = 99.90%

(Demo)