Lecture 11

Probability
Announcements

● Prelim 1 tonight
  ○ 7:30-9PM
  ○ Room by last name:
    ■ A-Q, Goldwin Smith G64
    ■ R-Z, Goldwin Smith 142
  ○ You can bring one sheet of notes, double-sided, made yourself
  ○ You will be provided with a list of table functions
  ○ Assigned seating; please arrive a few minutes early
● Project 1 Part 2 due next Friday, 3/6, 5:59PM
Recipes
INGREDIENTS

2 1/4 cups all-purpose flour
1 teaspoon baking soda
1 teaspoon salt
1 cup (2 sticks) butter, softened
3/4 cup granulated sugar
3/4 cup packed brown sugar
1 teaspoon vanilla extract
2 large eggs
2 cups (12-oz. pkg.) NESTLÉ® TOLL HOUSE® Semi-Sweet Chocolate Morsels
1 cup chopped nuts

IN THIS RECIPE

INSTRUCTIONS

PREHEAT oven to 375°F.

COMBINE flour, baking soda and salt in small bowl. Beat butter, granulated sugar, brown sugar and vanilla extract in large mixer bowl until creamy. Add eggs, one at a time, beating well after each addition. Gradually beat in flour mixture. Stir in morsels and nuts. Drop by rounded tablespoon onto ungreased baking sheets.

BAKE for 9 to 11 minutes or until golden brown. Cool on baking sheets for 2 minutes; remove to wire racks to cool completely.


SLICE AND BAKE COOKIE VARIATION:

PREPARE dough as above. Divide in half; wrap in waxed paper. Refrigerate for 1 hour or until firm. Shape each half into 15-inch log; wrap in wax paper. Refrigerate for 30 minutes.

Preheat oven to 375°F. Cut into 1/2-inch-thick slices; place on ungreased baking sheets. Bake for 8 to 10 minutes or until golden brown. Cool on baking sheets for 2 minutes; remove to wire racks to cool completely. Makes about 5 dozen cookies.
Recipe instructions

COMBINE flour, baking soda and salt in small bowl. Beat butter, granulated sugar, brown sugar and vanilla extract in large mixer bowl until creamy. Add eggs, one at a time, beating well after each addition. Gradually beat in flour mixture. Stir in morsels and nuts. Drop by rounded tablespoon onto ungreased baking sheets.

BAKE for 9 to 11 minutes or until golden brown. Cool on baking sheets for 2 minutes; remove to wire racks to cool completely.
Algorithm

Rules or a recipe for performing computation

Ideas we see in cookie recipe:

- **Iteration:** do something many times
- **Conditionals:** decide whether something is true, and maybe do something different
- **Variability or randomness:** some tasks might not be completely predictable
Random Selection
Random Selection

np.random.choice

- Selects at random
- with replacement
- from an array
- a specified number of times

np.random.choice(some_array, sample_size)

(Demo)
Control Statements
Control Statements

These statements control the sequence of computations that are performed in a program

- The keywords `if` and `for` begin control statements
- The purpose of `if` is to define computations that can choose different behaviors
- The purpose of `for` is to perform a computation for every element in a collection

(Demo)
The Monty Hall Problem
Monty Hall Problem
A. Switch?
B. Stay?
C. Doesn’t matter
Probability
Probability

- Lowest value: 0
  - Chance of event that is impossible
- Highest value: 1 (or 100%)
  - Chance of event that is certain

- If an event has chance 70%, then the chance that it doesn’t happen is
  - $100\% - 70\% = 30\%$
  - $1 - 0.7 = 0.3$
Equally Likely Outcomes

Assuming all outcomes are equally likely, the chance of an event A is:

\[ P(A) = \frac{\text{number of outcomes that make A happen}}{\text{total number of outcomes}} \]
Multiplication Rule

Chance that two events $A$ and $B$ both happen

$$= P(A \text{ happens}) $$

$$\times P(B \text{ happens given that } A \text{ has happened})$$

- The answer is *less than or equal to* each of the two chances being multiplied

(Demo)
Fraction of a Fraction

Stage 1

Stage 2

The winner: $\frac{1}{2} \cdot \frac{1}{3}$
Addition Rule

If event $A$ can happen in *exactly one* of two ways, then

$$P(A) = P(\text{first way}) + P(\text{second way})$$

- The answer is *greater than or equal to* the chance of each individual way.
Example: At Least One Head

- In 3 tosses:
  - Any outcome except TTT
  - $P(\text{TTT}) = \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$
  - $P(\text{at least one head}) = 1 - P(\text{TTT}) = \frac{7}{8} = 87.5\%$

- In 10 tosses:
  - $P(\text{TTTTTTTTTT}) = \left(\frac{1}{2}\right)^{10}$
  - $P(\text{at least one head}) = 1 - \left(\frac{1}{2}\right)^{10} = 99.90\%$

(Demo)
Addition Rule

Chance that either $A$ or $B$ (inclusive)

\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

Simplifies to \( P(A \text{ or } B) = P(A) + P(B) \) if $A$ and $B$ are disjoint (mutually exclusive)

- The answer is \textit{greater than or equal to} the chance of each individual way
Example: Roll a pair of dice

$A =$ at least one 2
$B =$ sum less than or equal to 4
$C =$ double

What are $P(A)$, $P(B)$ and $P(C)$?

$P(A \text{ and } B)$, $P(A \text{ or } B)$, $P(B \text{ and } C)$, $P(B \text{ or } C)$?