## Lecture 11

Probability

## Announcements

- Prelim 1 tonight
- 7:30-9PM
- Room by last name:
- A-Q, Goldwin Smith G64
- R-Z, Goldwin Smith 142
- You can bring one sheet of notes, double-sided, made yourself
- You will be provided with a list of table functions
- Assigned seating; please arrive a few minutes early
- Project 1 Part 2 due next Friday, 3/6, 5:59PM

Recipes


## $21 / 4$ cups all-purpose flour

## 1 teaspoon baking soda

## 1 teaspoon salt

1 cup ( 2 sticks) butter, softened

## $3 / 4$ cup granulated sugar

## $3 / 4$ cup packed brown sugar

## 1 teaspoon vanilla extract

## 2 large eggs

2 cups (12-oz. pkg.) NESTLÉ ${ }^{\text {© }}$ TOLL
HOUSE ${ }^{\oplus}$ Semi-Sweet Chocolate Morsels

## 1 cup chopped nuts

PREHEAT oven to $375^{\circ} \mathrm{F}$.

COMBINE flour, baking soda and salt in small bowl. Beat butter, granulated sugar, brown sugar and vanilla extract in large mixer bowl until creamy. Add eggs, one at a time, beating well after each addition. Gradually beat in flour mixture. Stir in morsels and nuts. Drop by rounded tablespoon onto ungreased baking sheets.

BAKE for 9 to 11 minutes or until golden brown. Cool on baking sheets for 2 minutes; remove to wire racks to cool completely.

PAN COOKIE VARIATION: Preheat oven to $350^{\circ}$ F. Grease $15 \times 10$-inch jelly-roll pan. Prepare dough as above. Spread into prepared pan. Bake for 20 to 25 minutes or until golden brown. Cool in pan on wire rack. Makes 4 dozen bars.

## SLICE AND BAKE COOKIE VARIATION:

PREPARE dough as above. Divide in half; wrap in waxed paper. Refrigerate for 1 hour or until firm. Shape each half into 15 -inch log; wrap in wax paper. Refrigerate for 30 minutes.* Preheat oven to $375^{\circ}$ F. Cut into $1 / 2$-inch-thick slices; place on ungreased baking sheets. Bake for 8 to 10 minutes or until golden brown. Cool on baking sheets for 2 minutes; remove to wire racks to cool completely. Makes about 5 dozen cookies.

## Recipe instructions

COMBINE flour, baking soda and salt in small bowl. Beat butter, granulated sugar, brown sugar and vanilla extract in large mixer bowl until creamy. Add eggs, one at a time, beating well after each addition. Gradually beat in flour mixture. Stir in morsels and nuts. Drop by rounded tablespoon onto ungreased baking sheets.

BAKE for 9 to 11 minutes or until golden brown. Cool on baking sheets for 2 minutes; remove to wire racks to cool completely.

## Algorithm

Rules or a recipe for performing computation

Ideas we see in cookie recipe:

- Iteration: do something many times
- Conditionals: decide whether something is true, and maybe do something different
- Variability or randomness: some tasks might not be completely predictable


## Random Selection

## Random Selection

np.random. choice

- Selects at random
- with replacement
- from an array
- a specified number of times
np.random. choice (some_array, sample_size)
(Demo)


## Control Statements

## Control Statements

These statements control the sequence of computations that are performed in a program

- The keywords if and for begin control statements
- The purpose of if is to define computations that can choose different behaviors
- The purpose of for is to perform a computation for every element in a collection
(Demo)


## The Monty Hall Problem

Monty Hall Problem

A. Switch?
B. Stay?
C. Doesn't matter

## Probability

## Probability

- Lowest value: 0
- Chance of event that is impossible
- Highest value: 1 (or 100\%)
- Chance of event that is certain
- If an event has chance $70 \%$, then the chance that it doesn't happen is
- $100 \%-70 \%=30 \%$
- $1-0.7=0.3$


## Equally Likely Outcomes

Assuming all outcomes are equally likely, the chance of an event $A$ is:
number of outcomes that make A happen
$P(A)=$ total number of outcomes
(Demo)

## Multiplication Rule

Chance that two events $A$ and $B$ both happen
= $\mathrm{P}(\mathrm{A}$ happens $)$
$x \mathrm{P}(B$ happens given that $A$ has happened $)$

- The answer is less than or equal to each of the two chances being multiplied
(Demo)


## Fraction of a Fraction



## Addition Rule

If event $A$ can happen in exactly one of two ways, then

$$
P(A)=P(\text { first way })+P(\text { second way })
$$

The answer is greater than or equal to the chance of each individual way

## Example: At Least One Head

- In 3 tosses:
- Any outcome except TTT
- $\mathrm{P}(\mathrm{TTT})=(1 / 2) \times(1 / 2) \times(1 / 2)=(1 / 2)^{3}=1 / 8$
- $P($ at least one head $)=1-P($ TTT $)=7 / 8=87.5 \%$
- In 10 tosses:
- $P($ TTTTTTTTTT $)=(1 / 2)^{10}$
- $P($ at least one head $)=1-(1 / 2)^{10}=99.90 \%$


## Addition Rule

Chance that either $A$ or $B$ (inclusive)

$$
P(A \text { or } B)=P(A)+P(B)-P(A \text { and } B)
$$

Simplifies to $P(A$ or $B)=P(A)+P(B)$ if $A$ and $B$ are disjoint (mutually exclusive)

- The answer is greater than or equal to the chance of each individual way


## Example: Roll a pair of dice

$A$ = at least one 2
$B=$ sum less than or equal to 4
$C=$ double

What are $\mathrm{P}(A), \mathrm{P}(B)$ and $\mathrm{P}(C)$ ?
$\mathrm{P}(A$ and $B), \mathrm{P}(A$ or $B), \mathrm{P}(B$ and $C), \mathrm{P}(B$ or $C)$ ?

