Lecture 14

Probability
Announcements

- Project 1: Due tonight at 9 pm
- Prelim 1: Thursday, 7:30 pm
The Monty Hall Problem
Monty Hall Problem
Probability
Probability

- Lowest value: 0
  - Chance of event that is impossible
- Highest value: 1 (or 100%)
  - Chance of event that is certain

If an event has chance 70%, then the chance that it doesn’t happen is
- 100% - 70% = 30%
- 1 - 0.7 = 0.3
Equally Likely Outcomes

Assuming all outcomes are equally likely, the chance of an event A is:

\[ P(A) = \frac{\text{number of outcomes that make A happen}}{\text{total number of outcomes}} \]
Multiplication Rule

Chance that two events $A$ and $B$ both happen

$$= P(A \text{ happens}) \times P(B \text{ happens } \textit{given that } A \text{ has happened})$$

- The answer is \textit{less than or equal to} each of the two chances being multiplied
- The more conditions you have to satisfy, the less likely you are to satisfy them all

(Demo)
Fraction of a Fraction

Stage 1:
- \( \frac{1}{3} \)
- \( \frac{2}{3} \)

Stage 2:
- \( \frac{1}{2} \)
- \( \frac{1}{2} \)

The winner: \( \frac{1}{2} \) of \( \frac{1}{3} \)
Addition Rule

If event $A$ can happen in exactly one of two ways, then

$$P(A) = P(\text{first way}) + P(\text{second way})$$

- The answer is *greater than or equal to* the chance of each individual way.
Example: At Least One Head

- In 3 tosses:
  - Any outcome except TTT
  - $P(TTT) = \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = \frac{1}{8}$
  - $P(\text{at least one head}) = 1 - P(TTT) = \frac{7}{8} = 87.5\%$

- In 10 tosses:
  - $1 - (\frac{1}{2})^{10}$
  - 99.9\%

(Demo)