# L28. Divide and Conquer Algorithms

Binary Search
Merge Sort
Mesh Generation
Recursion

## Question

The Manhatten phone book has 1,000,000+ entries.

How is it possible to locate a name by examining just a tiny , tiny fraction of those entries?

## Answer: Repeated Halving

To find the page containing Derek Jeter's number…

```
while (Phone book is longer than 1 page)
   Open to the middle page.
   if  "Jeter" comes before the first name,
       Rip and throw away the 2nd half.
   else
       Rip and throw away the 1st half.
   end
end
```

## What Happens to Phone Book Length…

```
Original:     3000 pages
After 1 rip:  1500 pages
After 2 rips:  750 pages
After 3 rips:  375 pages
After 4 rips:  188 pages
After 5 rips:   94 pages

After 12 rips:   1 page
```

## Binary Search

The idea of repeatedly halving the size of the "search space" is the main idea behind the method of binary search.

An item in a sorted array of length n can be located with just $\log_2 n$ comparisons.

## Problem

Search a Sorted Array
For a Given Value

(We assume that the array has no repeated elements.)

## Binary Search: a = 70

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x: | 12 | 15 | 33 | 35 | 42 | 45 | 51 | 62 | 73 | 75 | 86 | 98 |

L: 1

Mid: 6

R: 12

x(Mid) <= a

So throw away the left half…

## Binary Search: a = 70

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x: | 12 | 15 | 33 | 35 | 42 | 45 | 51 | 62 | 73 | 75 | 86 | 98 |

L: 6

Mid: 9

R: 12

a < x(Mid)

So throw away the right half…

## Binary Search: a = 70

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x: | 12 | 15 | 33 | 35 | 42 | 45 | 51 | 62 | 73 | 75 | 86 | 98 |

L: 6

Mid: 7

R: 9

x(Mid) <= a

So throw away the left half…

## Binary Search: a = 70

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x: | 12 | 15 | 33 | 35 | 42 | 45 | 51 | 62 | 73 | 75 | 86 | 98 |

L: 7

Mid: 8

R: 9

x(Mid) <= a

So throw away the left half…

## Binary Search: a = 70

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x: | 12 | 15 | 33 | 35 | 42 | 45 | 51 | 62 | 73 | 75 | 86 | 98 |

L: 8

Mid: 8

R: 9

Done because R-L = 1

```
  function L = BinarySearch(a,x)
% x is a row n-vector with x(1) < ... < x(n)
% x(1) <= a <= x(n)

% x(L) <= a <= x(L+1)

  L = 1;  R = length(x);
% x(L) <= a <= x(R)
  while R-L > 1
     mid = floor((L+R)/2);
%    Note that mid does not equal L or R.
     if a < x(mid)
%       x(L) <= a <= x(mid)
        R = mid;
     else
%       x(mid) <= a <== x(R)
        L = mid;
    end
end
```

## Next Problem

Sort the values in an array so that they are arranged from smallest t o biggest.

Chosen Method: Merge Sort

This technique is an excellent example of a divide and conquer procedure.

## Motivation

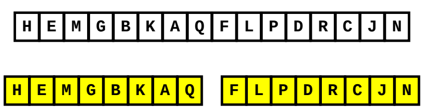
A plan if you have two "helpers":

Split the array and have each helper sort his/her half.

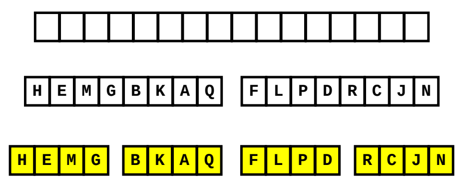
Merge the two sorted subarrays.

**And what if those two helpers each had two sub-helpers? Etc**

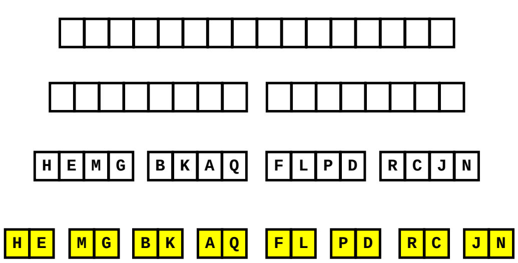
## Subdivide the Sorting Task



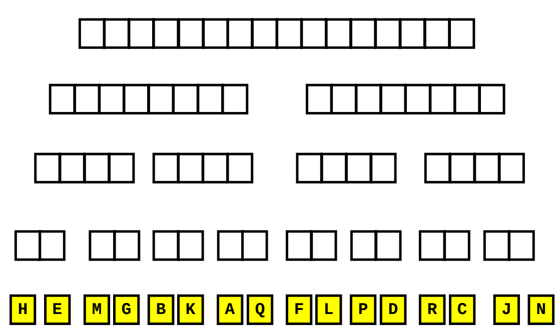

## Subdivide Again



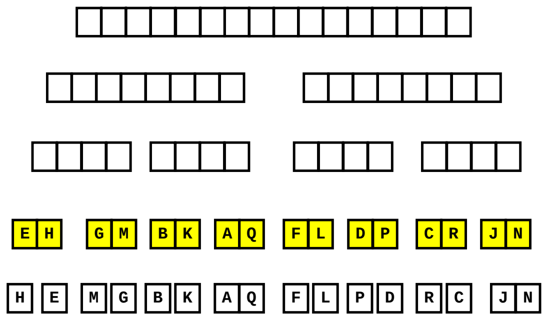

## And Again



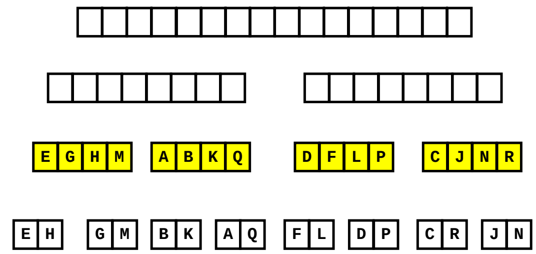

## And One Last Time



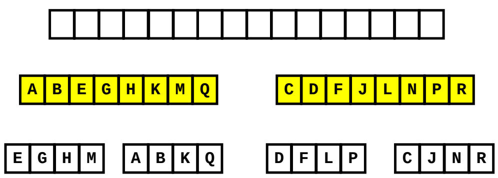
# Now Merge



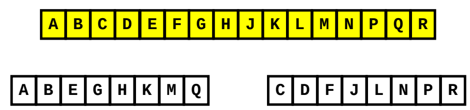

# And Merge Again




# And Again




# And One Last Time




# Done!


```
  function y = MergeSort(x)
% x is a column n-vector.
% y is a column n-vector consisting
% of the values in x sorted
% from smallest to largest.

  n = length(x);
  if n==1
      y = x;
  else
      m  = floor(n/2);
      y1 = MergeSort(x(1:m));
      y2 = MergeSort(x(m+1:n));
      y  = Merge(y1,y2);
  end
```

## Important Sub-Problem

Merging Two Sorted Arrays
Into a
Single Sorted Array

## Example

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 12 | 33 | 35 | 45 | | | | | |
| 15 | 42 | 55 | 65 | 75 | | | | |
| 12 | 15 | 33 | 35 | 42 | 45 | 55 | 65 | 75 |

## Merge

x: | 12 | 33 | 35 | 45 |    ix: 1

y: | 15 | 42 | 55 | 65 | 75 |    iy: 1

z: | | | | | | | | | |    iz: 1

```
x(ix) <= y(iy)    ???
```

## Merge

x: | 12 | 33 | 35 | 45 |    ix: 1

y: | 15 | 42 | 55 | 65 | 75 |    iy: 1

z: | 12 | | | | | | | | |    iz: 1

```
x(ix) <= y(iy)    yes
```

## Merge

x: | 12 | 33 | 35 | 45 |    ix: 2

y: | 15 | 42 | 55 | 65 | 75 |    iy: 1

z: | 12 | | | | | | | | |    iz: 2

```
x(ix) <= y(iy)    ???
```

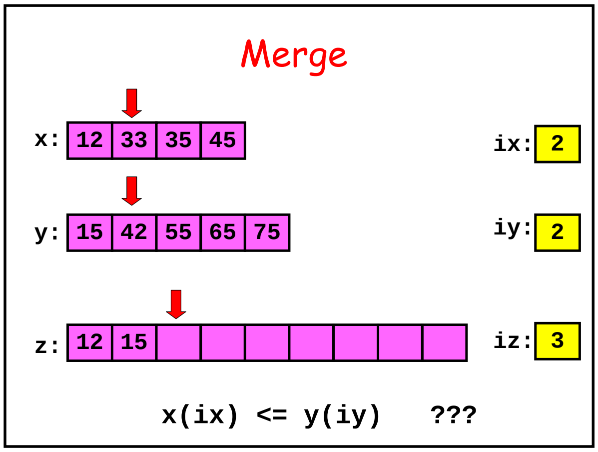
## Merge

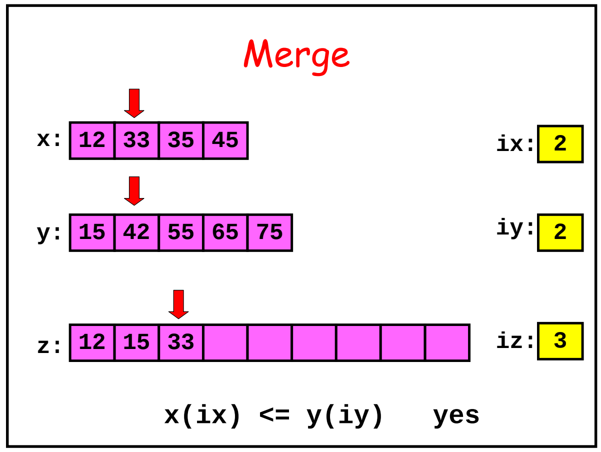
x: | 12 | 33 | 35 | 45 |    ix: 2

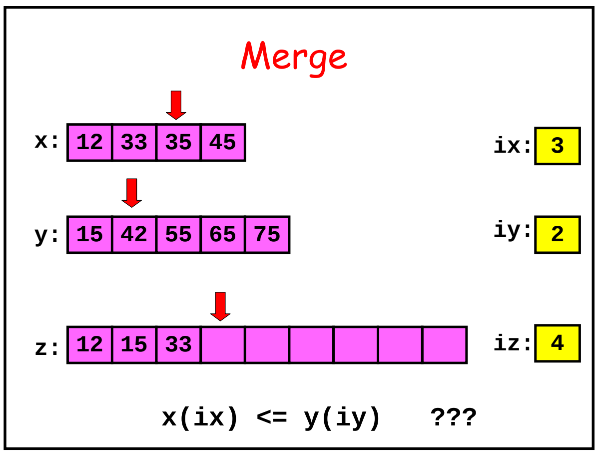
y: | 15 | 42 | 55 | 65 | 75 |    iy: 1

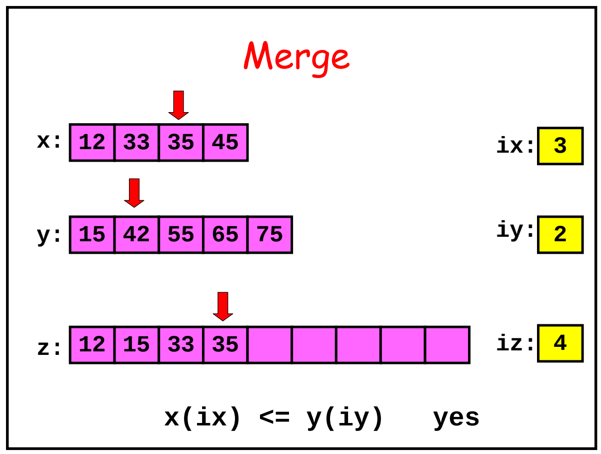
z: | 12 | 15 | | | | | | | |    iz: 2

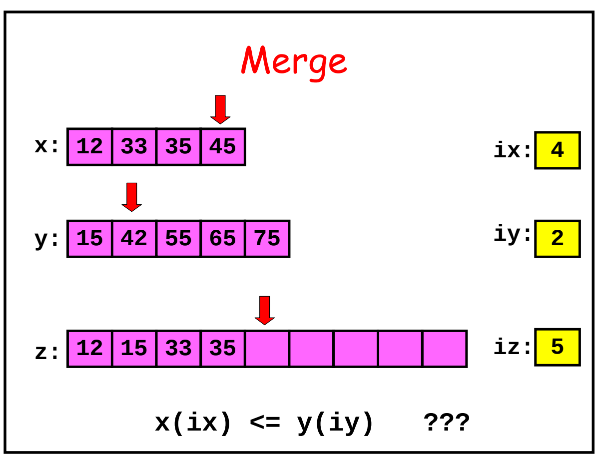
```
x(ix) <= y(iy)    no
```

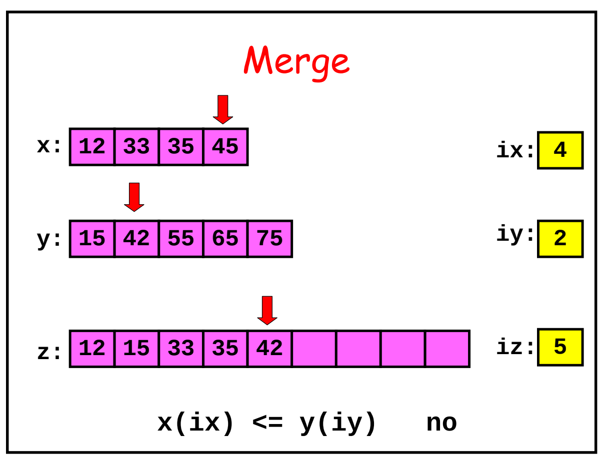

Merge
x: 12 33 35 45
ix: 2
y: 15 42 55 65 75
iy: 2
z: 12 15
iz: 3
x(ix) <= y(iy) ???

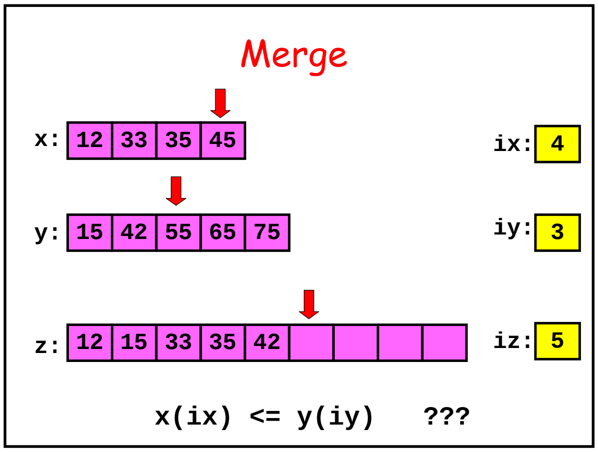

Merge
x: 12 33 35 45
ix: 2
y: 15 42 55 65 75
iy: 2
z: 12 15 33
iz: 3
x(ix) <= y(iy) yes


Merge
x: 12 33 35 45
ix: 3
y: 15 42 55 65 75
iy: 2
z: 12 15 33
iz: 4
x(ix) <= y(iy) ???


Merge
x: 12 33 35 45
ix: 3
y: 15 42 55 65 75
iy: 2
z: 12 15 33 35
iz: 4
x(ix) <= y(iy) yes


Merge
x: 12 33 35 45
ix: 4
y: 15 42 55 65 75
iy: 2
z: 12 15 33 35
iz: 5
x(ix) <= y(iy) ???


Merge
x: 12 33 35 45
ix: 4
y: 15 42 55 65 75
iy: 2
z: 12 15 33 35 42
iz: 5
x(ix) <= y(iy) no


Merge
x: 12 33 35 45
y: 15 42 55 65 75
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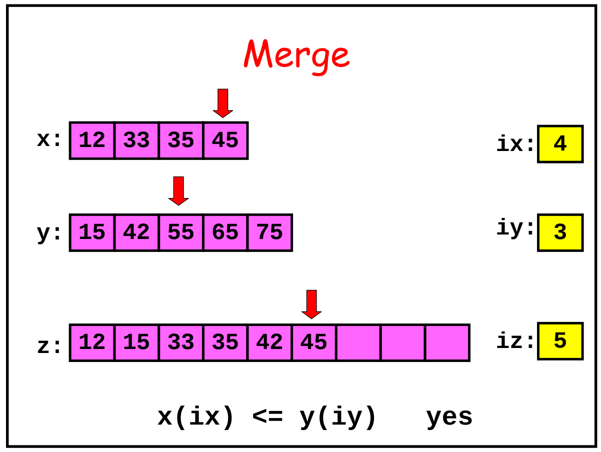
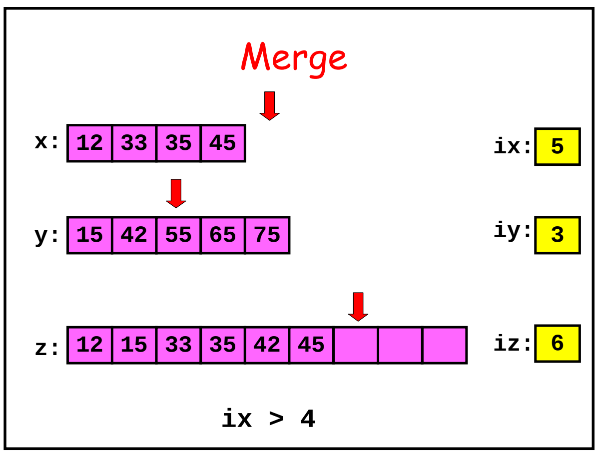
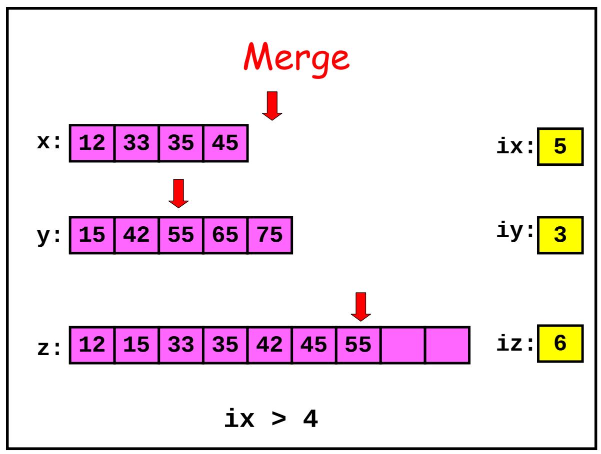
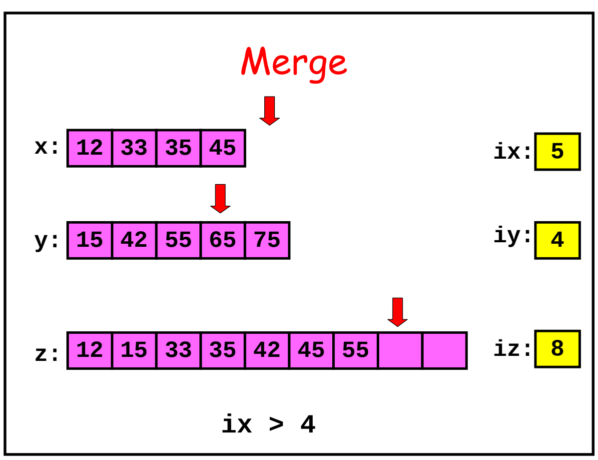
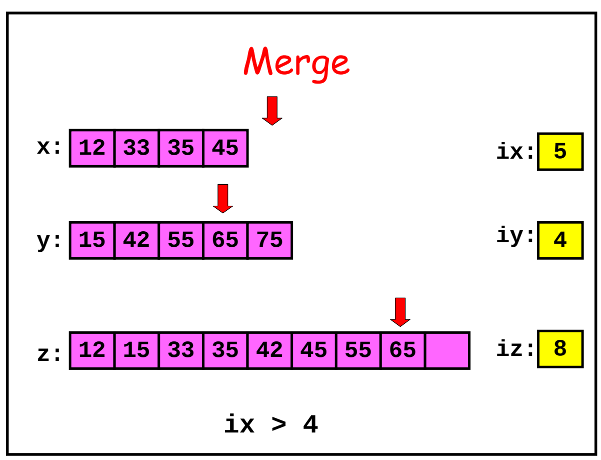
ix: 4
iy: 3
iz: 5
x(ix) <= y(iy) ???


Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12 15 33 35 42 45
ix: 4
iy: 3
iz: 5
x(ix) <= y(iy) yes


Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12 15 33 35 42 45
ix: 5
iy: 3
iz: 6
ix > 4


Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12 15 33 35 42 45 55
ix: 5
iy: 3
iz: 6
ix > 4


Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12 15 33 35 42 45 55
ix: 5
iy: 4
iz: 8
ix > 4


Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12 15 33 35 42 45 55 65
ix: 5
iy: 4
iz: 8
ix > 4

## Merge

x: | 12 | 33 | 35 | 45 |  ix: 5

y: | 15 | 42 | 55 | 65 | 75 |  iy: 5

z: | 12 | 15 | 33 | 35 | 42 | 45 | 55 | 65 | |  iz: 9

ix > 4

## Merge

x: | 12 | 33 | 35 | 45 |  ix: 5

y: | 15 | 42 | 55 | 65 | 75 |  iy: 5

z: | 12 | 15 | 33 | 35 | 42 | 45 | 55 | 65 | 75 |  iz: 9

ix > 4

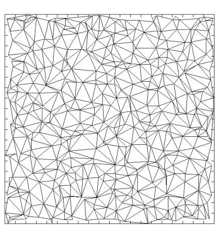
```
function z = Merge(x,y)
n = length(x); m = length(y);
z = zeros(1,n+m);
ix = 1; iy = 1;
for iz=1:(n+m)
    if ix > n
        z(iz) = y(iy);  iy = iy+1;
    elseif iy>m
        z(iz) = x(ix); ix = ix + 1;
    elseif x(ix) <= y(iy)
        z(iz) = x(ix); ix = ix + 1;
    else
        z(iz) = y(iy);iy = iy + 1;
    end
end
```

## New Problem

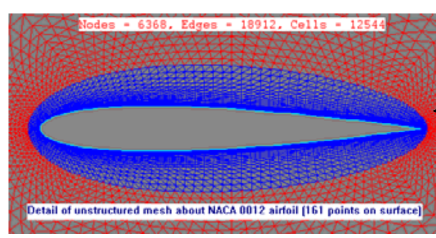
## Recursive Mesh Generation

## Divide and Conquer Methods Also Show Up in Geometric Situations

Chop a Region
up into triangles
with smnaller
triangles
in “areas of interest”.



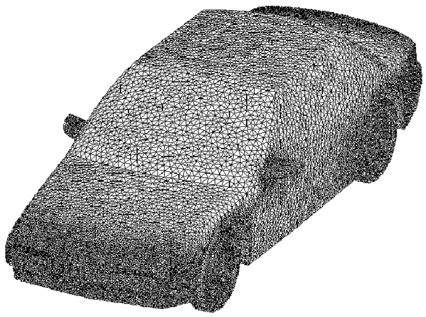
## Mesh Generation




An
Area
Of
Interest

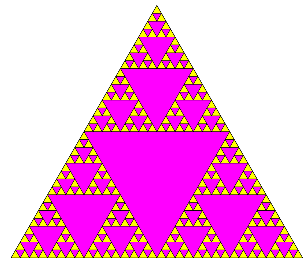
Step One in simulating flow around an airfoil is to generate a mesh and (say) estimate velocity at each mesh point.

## Mesh Generation in 3D



## Why is Mesh Generation a Divide & Conquer Process?

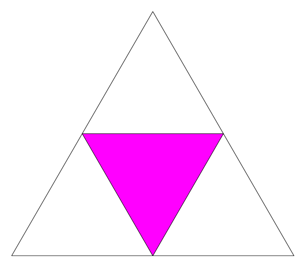
Let Us Draw this Graphic:



## The Basic Operation

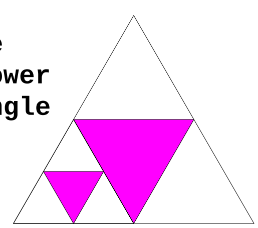
```
if the triangle is big enough

   Connect the midpoints.
   Color the interior triangle mauve.
else
   Color the whole triangle yellow.

end
```

## At the Start…

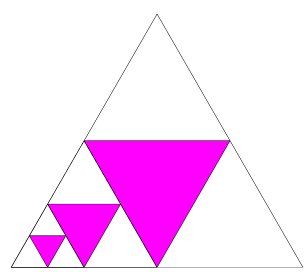


## Recur On this Idea

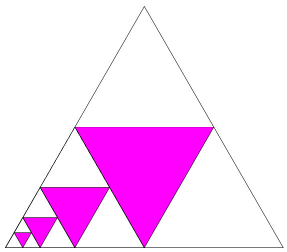
**Apply same idea to lower left triangle**

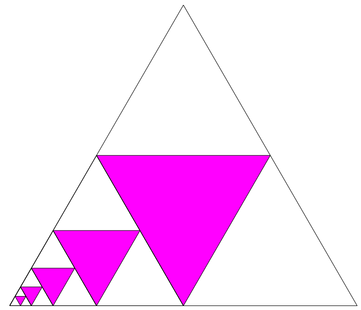


## Recur Again

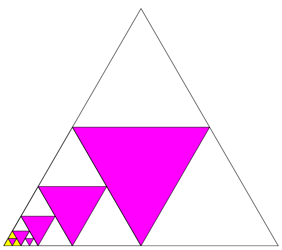


## Recursion And Again

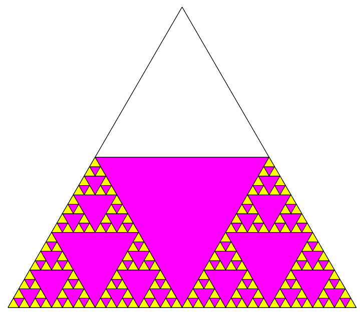




## Now, Climb Your Way Out.



## Etc



```
function MeshTriangle(x,y,L)

if L==0
   % No subdivision required...
   fill(x,y,'y','linewidth',1.5)
else
   % A subdivision is called for. Get midpoints…
   a = [(x(1)+x(2))/2 (x(2)+x(3))/2 (x(3)+x(1))/2];
   b = [(y(1)+y(2))/2 (y(2)+y(3))/2 (y(3)+y(1))/2];
   % Color the interior triangle magenta...
   fill(a,b,'m','linewidth',1.5)
   % Apply the process to the three "outer" triangles...
   MeshTriangle([x(1) a(1) a(3)],[y(1) b(1) b(3)],L-1)
   MeshTriangle([x(2) a(2) a(1)],[y(2) b(2) b(1)],L-1)
   MeshTriangle([x(3) a(3) a(2)],[y(3) b(3) b(2)],L-1)
end
```