

L20. More on 2D Arrays

Operations

Subscripting

Functions* & 2D Arrays

*Will see two new things.

Boolean-valued functions

Functions that have a function as a parameter.

Two Applications

A commercial setting that involves cost arrays, inventory arrays, and purchase orders.

A setting that requires the visualization of a function of two variables $f(x,y)$ via contour plotting.

A Cost/Inventory Setting

A company has 3 factories that make 5 different products.

The cost of making a product varies from factory to factory.

The inventory varies from factory to factory.

Problems

A customer submits a purchase order that is to be filled by a single factory.

1. How much would it cost a factory to fill the order?
2. Does a factory have enough inventory to fill the order?
3. Among the factories that can fill the order, who can do it most cheaply?

Cost Array

C:

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

The value of $c(i, j)$ is what it costs factory i to make product j .

Inventory Array

Inv:

38	5	99	34	42
82	19	83	12	42
51	29	21	56	87

The value of $\text{Inv}(i, j)$ is the inventory in factory i of product j .

Purchase Order

PO:

1	0	12	29	5
---	---	----	----	---

The value of $PO(j)$ is the number product j 's that the customer wants

How Much Does it Cost for
Each Factory to Process
a Purchase order?

C:

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

PO:

1	0	12	29	5
---	---	----	----	---

For
factory 1:

$$1*10 + 0*36 + 12*22 + 29*15 + 5*62$$

C:

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

j = 1

PO:

1	0	12	29	5
---	---	----	----	---

For
factory 1:

```

s = 0;
for j=1:5
    s = s + C(1,j) * PO(j)
end

```

C:

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

j = 2

PO:

1	0	12	29	5
---	---	----	----	---

For
factory 1:

```
s = 0;  
for j=1:5  
    s = s + C(1,j) * PO(j)  
end
```

C:

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

$j = 3$

PO:

1	0	12	29	5
---	---	----	----	---

For
factory 1:

```

s = 0;
for j=1:5
    s = s + C(1,j) * PO(j)
end

```

C:

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

j = 4

PO:

1	0	12	29	5
---	---	----	----	---

For
factory 1:

```
s = 0;  
for j=1:5  
    s = s + C(1,j) * PO(j)  
end
```

C:

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

j = 5

PO:

1	0	12	29	5
---	---	----	----	---

For
factory 1:

```
s = 0;  
for j=1:5  
    s = s + C(1,j) * PO(j)  
end
```

C:

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

PO:

1	0	12	29	5
---	---	----	----	---

For
factory 2:

```
s = 0;  
for j=1:5  
    s = s + C(2,j)*PO(j)  
end
```

C:

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

PO:

1	0	12	29	5
---	---	----	----	---

For
factory i:

```
s = 0;  
for j=1:5  
    s = s + C(i,j)*PO(j)  
end
```


Encapsulate...

```
function TheBill = iCost(i,C,PO)
% The cost when factory i fills
% the purchase order
nProd = length(PO)
TheBill = 0;
for j=1:nProd
    TheBill = TheBill + C(i,j)*PO(j);
end
```

Finding the Cheapest

C:

10	36	22	15	62	1019
12	35	20	12	66	930
13	37	21	16	59	1040

PO:

1	0	12	29	5
---	---	----	----	---



As computed
by iCost

Finding Cheapest: Initialization

C:	10	36	22	15	62	1019	Can we do better?
	12	35	20	12	66	930	
	13	37	21	16	59	1040	
PO:	1	0	12	29	5		
iBest:	0					minBill:	inf

A Note on "inf"

A special value that can be regarded as + infinity.

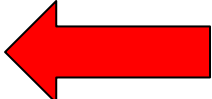
`x = 10 / 0` assigns inf to x

`y = 1 + x` assigns inf to y

`z = 1 / x` assigns zero to z

`w < inf` is always true if w is numeric

Improvement at $i = 1$

C:	10	36	22	15	62	1019	
	12	35	20	12	66	930	
	13	37	21	16	59	1040	
PO:	1	0	12	29	5		

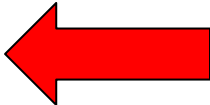
iBest:

1

minBill:

1019

Improvement at $i = 2$

C:	10	36	22	15	62	1019	
	12	35	20	12	66	930	
	13	37	21	16	59	1040	
PO:	1	0	12	29	5		

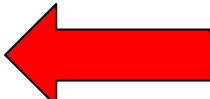
iBest:

2

minBill:

930

No Improvement at $i = 3$

C:	10	36	22	15	62	1019	
	12	35	20	12	66	930	
	13	37	21	16	59	1040	
PO:	1	0	12	29	5		

iBest:

2

minBill:

930

Finding the Cheapest

```
iBest = 0; minBill = inf;
for i=1:nFact
    iBill = iCost(i,C,PO);
    if iBill < minBill
%       Found an Improvement
        iBest = i; minBill = iBill;
    end
end
```


Inventory Considerations

What if a factory lacks the inventory to fill the purchase order?

Such a factory should be excluded from the find-the-cheapest computation.

Who Can Fill the Order?

	38	5	99	34	42	Yes
Inv:	82	19	83	12	42	No
	51	29	21	56	87	Yes
PO:	1	0	12	29	5	

Because $12 < 29$

Wanted: A True/False Function



`B` is "true" if factory `i` can fill the order.

`B` is "false" if factory `i` cannot fill the order.

Boolean Operations in Matlab

SO FAR we have indicated that expressions like

`a <= x && x <= b`

`abs(y) > 10`

are either TRUE or FALSE.

The 0-1 Secret

In reality, expressions like

`a <= x && x <= b`

`abs(y) > 10`

render the value "1" if they are TRUE and "0" if they are FALSE.

Example

```
>> x = 8; y = 7;
```

```
>> B = x < y
```

```
B =
```

```
0
```

```
>> B = x > y
```

```
B =
```

```
1
```

A Boolean-Valued Function

```
function B = Overlap(a,b,c,d)
% B is true if intervals [a,b]
% and [c,d] intersect.
% Otherwise B is false.
% Assume a<b and c<d.

abToLeft    = b < c;
abToRight   = d < a;
B = ~(abToLeft || abToRight);
```



Using Overlap

```
s = 0;
for k=1:100
    a = rand; b = a + rand;
    c = rand; d = c + rand;
    if Overlap(a,b,c,d)
        s = s+1;
    end
end
probOverlap = s/100
```


Back to Inventory Problem

	38	5	99	34	42
Inv:	82	19	83	12	42
	51	29	21	56	87
PO:	1	0	12	29	5

Initialization

Inv:

38	5	99	34	42
82	19	83	12	42
51	29	21	56	87

B: 1

PO:

1	0	12	29	5
---	---	----	----	---

Still True...

Inv:

38	5	99	34	42
82	19	83	12	42
51	29	21	56	87

B: 1

PO:

1	0	12	29	5
---	---	----	----	---

`B = B && (Inv(2,1) >= PO(1))`

Still True...

Inv:

38	5	99	34	42
82	19	83	12	42
51	29	21	56	87

B: 1

PO:

1	0	12	29	5
---	---	----	----	---

```
B = B && ( Inv(2,2) >= PO(2) )
```

Still True...

Inv:

38	5	99	34	42
82	19	83	12	42
51	29	21	56	87

B: 1

PO:

1	0	12	29	5
---	---	----	----	---

```
B = B && ( Inv(2,3) >= PO(3) )
```

No Longer True...

Inv:

38	5	99	34	42
82	19	83	12	42
51	29	21	56	87

B: 0

PO:

1	0	12	29	5
---	---	----	----	---

```
B = B && ( Inv(2,4) >= PO(4) )
```

Encapsulate...

```
function B = iCanDo(i,Inv,PO)
% B is true if factory i can fill
% the purchase order. Otherwise, false
nProd = length(PO);
j = 1;
B = 1;
while j<=nProd && B
    B = B && ( Inv(i,j) >= PO(j) );
    j = j+1;
end
```

Back To Finding the Cheapest

```
iBest = 0; minBill = inf;
```

```
for i=1:nFact
```

```
    iBill = iCost(i,C,PO);
```

```
    if iBill < minBill
```

```
        % Found an Improvement
```

```
        iBest = i; minBill = iBill;
```

```
    end
```

```
end
```

Don't bother with this unless sufficient inventory.

Back To Finding the Cheapest

```
iBest = 0; minBill = inf;
```

```
for i=1:nFact
```

```
    if iCanDo(i,Inv,PO)
```

```
        iBill = iCost(i,C,PO);
```

```
        if iBill < minBill
```

```
%
```

```
            Found an Improvement
```

```
            iBest = i; minBill = iBill;
```



```
        end
```

```
    end
```

```
end
```

```
function [iBest,minBill] = ...
    Cheapest(C,Inv,PO)
[nFact,nProd] = size(C);
iBest = 0; minBill = inf;
for i=1:nFact
    if iCanDo(i,Inv,PO)
        iBill = iCost(i,C,PO);
        if iBill < minBill
            iBest = i; minBill = iBill;
        end
    end
end
end
```

Finding the Cheapest

	10	36	22	15	62	1019	Yes
C:	12	35	20	12	66	930	No
	13	37	21	16	59	1040	Yes
PO:	1	0	12	29	5		
						As computed by iCost	As computed by iCanDo

New Problem

Visualizing a function of the form

$$z = f(x,y).$$

Think of z as an elevation which depends on the coordinates x and y of the location.

Sample Elevation Function

```
function z = Elev(x,y)
```

```
    r1 = (x-1)^2 + 3*(y-1.5)^2;
```

```
    r2 = 2*(x+2)^2 + (y-.5)^2;
```

```
    r3 = (x-.5)^2 + 7*y^2;
```

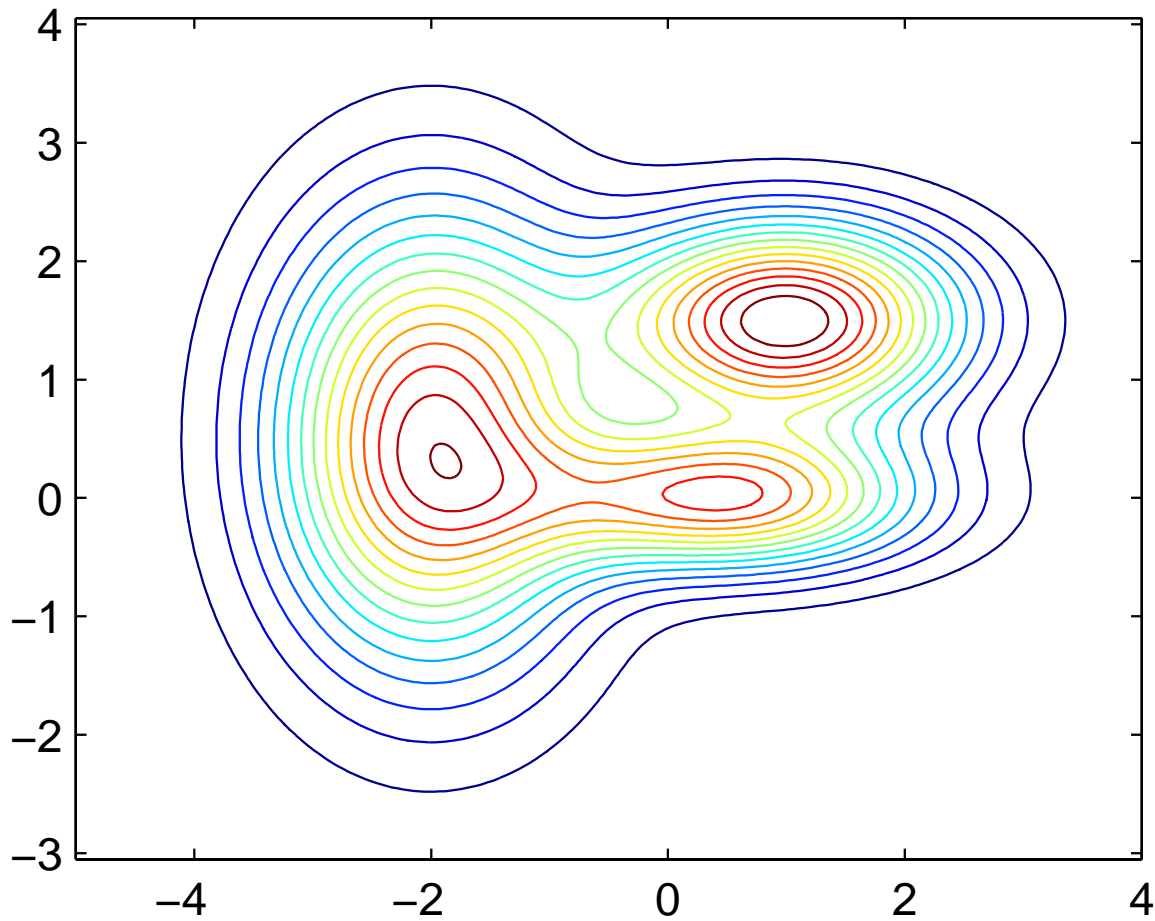
```
    z = 100*exp(-.5*r1) + ...
```

```
        90*exp(-.3*r2) + ...
```

```
        80*exp(-.4*r3);
```

Three Hills at (1,1.5), (-2,.5), (.5,0)

Its Contour Plot



Making a Contour Plot

```
x = linspace(-5,4,200);  
y = linspace(-2.5,6.5,200);  
A = zeros(200,200);  
for i=1:200  
    for j=1:200  
        A(i,j) = Elev(x(j),y(i));  
    end  
end
```

```
contour(x,y,A,15)
```

Set up a matrix of function evals

General Set-Up

```
function A = SetUp(f,xVals,yVals)
Nx = length(xVals);
Ny = length(yVals);
A = zeros(Ny,Nx);
for i=1:Ny
    for j=1:Nx
        A(i,j) = f(xVals(j),yVals(i));
    end
end
end
```

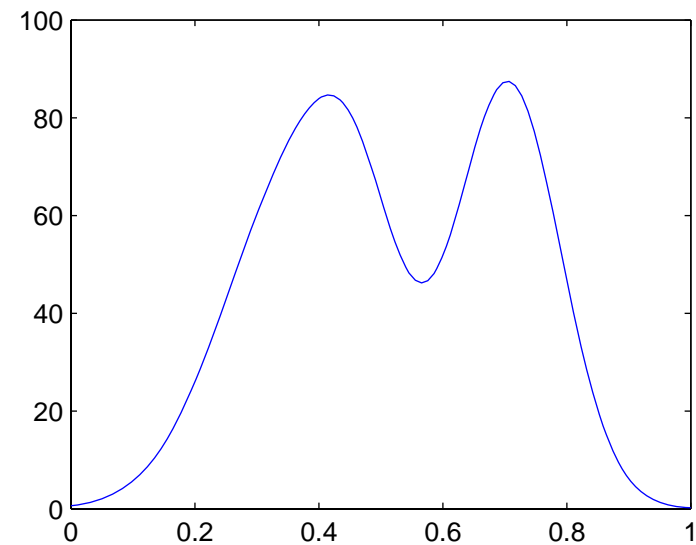
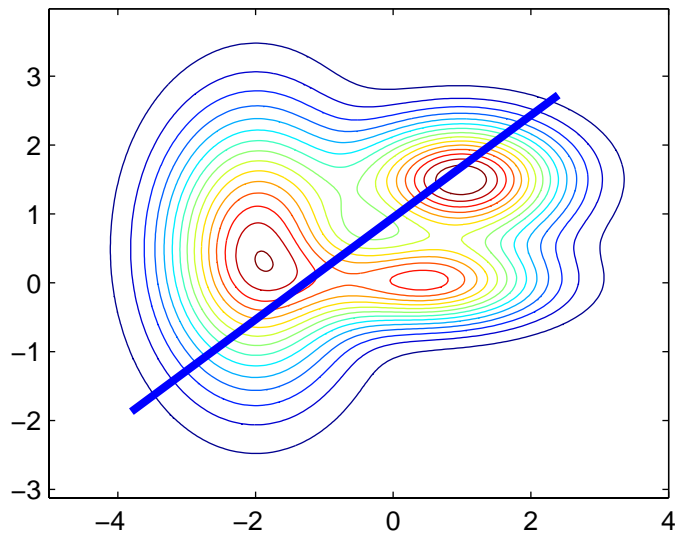

Calling SetUp

```
x = linspace(-5,4,200);  
y = linspace(-2.5,6.5,200);  
F = SetUp(@Elev,x,y);
```

Not just 'Elev'

The @ is required for function parameters.

Generating a Cross Section



Enter endpoints via `ginput`
Sample `Elev(x,y)` along the line segment

Mouse Input Via `ginput`

To draw a line segment connecting $(a(1),b(1))$ and $(a(2),b(2))$:

```
[a,b] = ginput(2);  
plot(a,b)
```

`[a,b] = ginput(n)` puts the mouseclick coords in length- n arrays `a` and `b`.

```
n = 100;  
t = linspace(0,1,n);  
x = linspace(a(1),a(2),n);  
y = linspace(b(1),b(2),n);  
for i=1:n  
% At "time" t(i) we are at (x(i),y(i)).  
% Compute elevation at time t(i).  
    f(i) = Elev(x(i),y(i));  
end  
figure  
plot(t,f)
```