Optimization revisited



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http://www.cs.cornell.edu/courses/cs1114/



Administrivia

- Assignments:
 - A6 due tomorrow (demo slots available)
- Prelim 3 next Thursday
 - Review in class on Tuesday

Administrivia

- Final projects
 - Demo session: Tuesday, May 15, 1:30-3:30
 - For the demo session, prepare a 7-minute demo for your project
 - Sign-up slots available soon
 - Best demo will receive the "Best Demo" prize



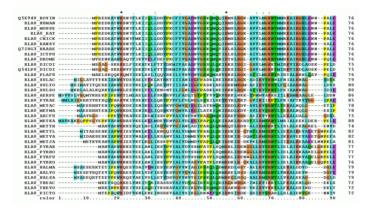
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Optimization problems

- Very important concept for many different areas of science and engineering
- City planning
 - How do I figure out what bus routes to put in Ithaca?
 - How do I decide where to put the next hospital?

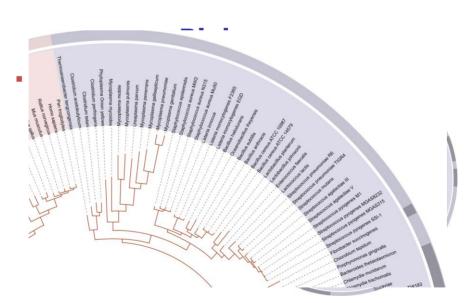
Biology

DNA sequence alignment





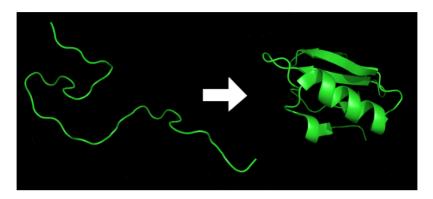
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Optimal Phylogenic tree

Biology

Protein folding





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Games

- What is the optimal strategy for a given game?
 - Rock / paper / scissors?
 - Chess?
 - Is there a winning strategy for white?



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Computer Vision



Optimization problems

- We've seen several optimization-type problems in this class:
 - k-means
 - k-centers
 - Fitting a line to a set of points
 - Coloring a graph with the minimum number of colors / fitting animals into the minimum set of cages
 - Finding the minimum spanning tree of a graph
 - Others?

Optimization problems

- Key elements:
 - 1. A set of valid solutions
 - Line fitting: all possible lines
 - Minimum spanning tree: all possible spanning trees of a graph
 - k-means: all possible sets of k means



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Optimization problems

- Key elements:
 - 2. A way to measure how good/bad a solution is (an *objective function*):
 - · Line fitting: sum of squared residuals
 - Minimum spanning tree: sum of edge weights
 - *k*-means: sum of squared distances from each input point to its assigned mean
 - Sometimes coming up with a good objective function is very difficult in itself (e.g., protein folding)

k-means objective function

- Find the centers that minimize the sum of squared distances to the points
- Objective function:

Given input points $x_1, x_2, x_3, \ldots, x_n$, find the clusters $C_1, C_2, \ldots C_k$ and the cluster centers $\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_k$ that minimize

$$\sum_{j=1}^{k} \sum_{x_i \in C_j} |x_i - \bar{x}_j|^2$$



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Optimization algorithms

- For any given optimization problem, we would like to come up with a (hopefully efficient) algorithm
 - That ideally finds the global minimum of the objective function

Optimization algorithms

- Example: minimum spanning tree (MST)
 - First algorithm: Borůvka's algorithm (1926)
 - For constructing an efficient electrical network for Moravia
 - Prim's algorithm
 - Kruskal's algorithm



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Optimization algorithms

- Possible algorithm types:
 - Greedy algorithms (e.g. Prim and Kruskal)
 - Iterative algorithms (e.g. bubble sort, gradient descent)
 - Guess-and-check (e.g. RANSAC)
- Some algorithms return the global optimum, others just return a "good" answer



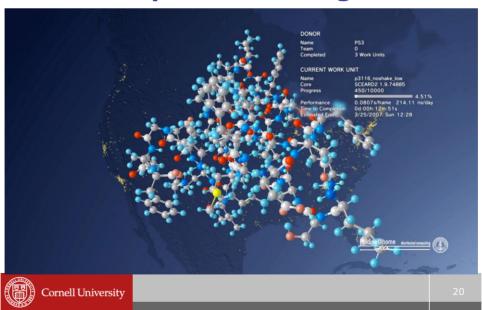
Another optimization problem

- Dense box
 - One statement: Given a set of 2D points, compute a good bounding box, that is not too big and contains most of the points
 - What objective function should we use?

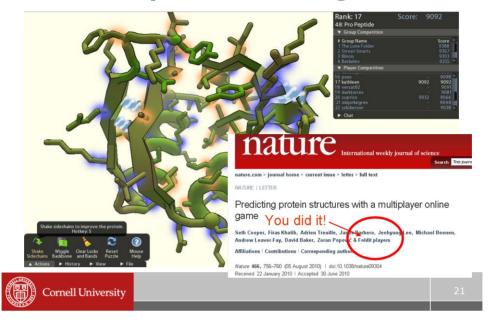


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Parallel optimization algorithms



Human optimization algorithms



Questions?