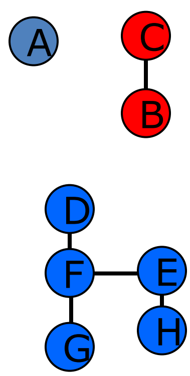
Breadth-first and depth-first traversal

CS1114
http://cs1114.cs.cornell.edu

Cornell University
Computer Science

Blobs are components!

A	0	0	0	0	0	0	0	B	0
0	0	0	0	0	0	0	0	C	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	D	0	0	0	0	0
0	0	0	E	F	G	0	0	0	0
0	0	0	H	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0



Finding blobs

1	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Finding blobs

1	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Blobs are connected components!

Finding components

1. Pick a 1 to start with, where you don't know which component it is in
   - When there aren't any, you're done
2. Give it a new component color
3. Assign the same component color to each pixel that is part of the same component
   - Basic strategy: color any neighboring 1's, have them color their neighbors, and so on

Strategy for finding components

- For each vertex we visit, we color its neighbors and remember that we need to visit them at some point
  - Need to keep track of the vertices we still need to visit in a todo list
  - After we visit a vertex, we'll pick one of the vertices in the todo list to visit next

- This is also called *graph traversal*

Stacks and queues

- Two ways of representing a "todo list"
- Stack: Last In First Out (LIFO)
  - (Think cafeteria trays)
  - The newest task is the one you'll do next
- Queue: First In First Out (FIFO)
  - (Think a line of people at the cafeteria)
  - The oldest task is the one you'll do next

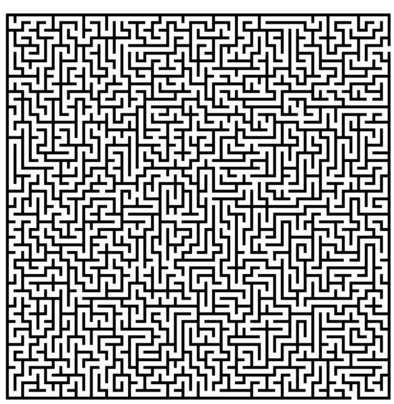
Stacks

- Two operations:
- Push: add something to the top of the stack
- Pop: remove the thing on top of the stack

Queue

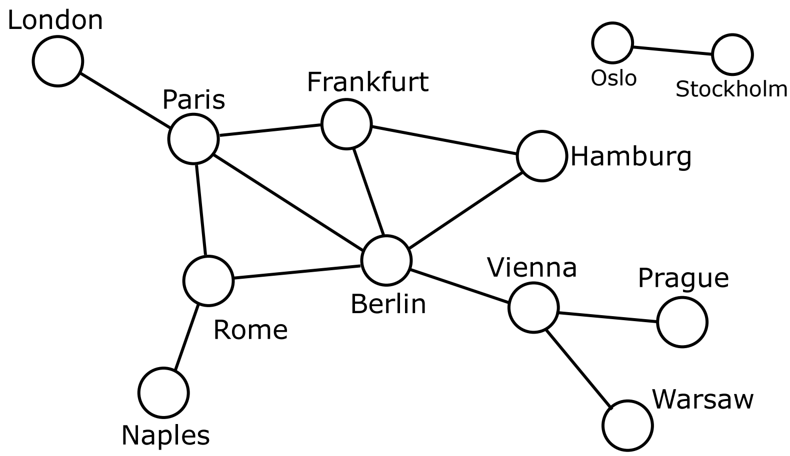
- Two operations:
- Enqueue: add something to the end of the queue
- Dequeue: remove something from the front of the queue

Graph traversal



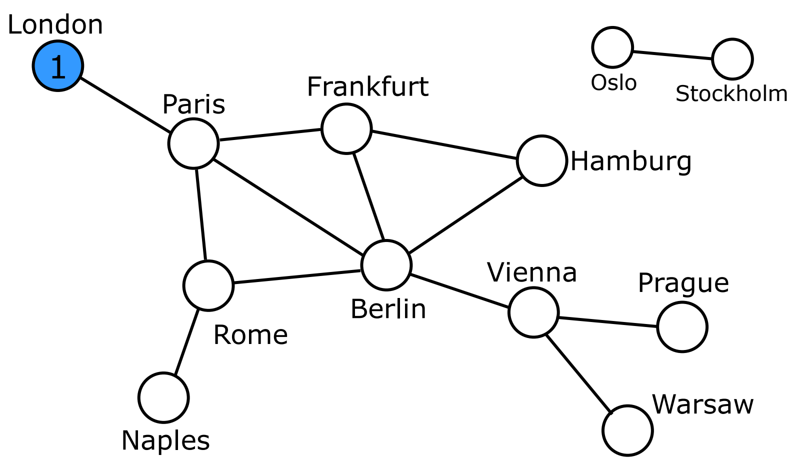
- Suppose you're in a maze
- What strategy can you use to find the exit?

Graph traversal



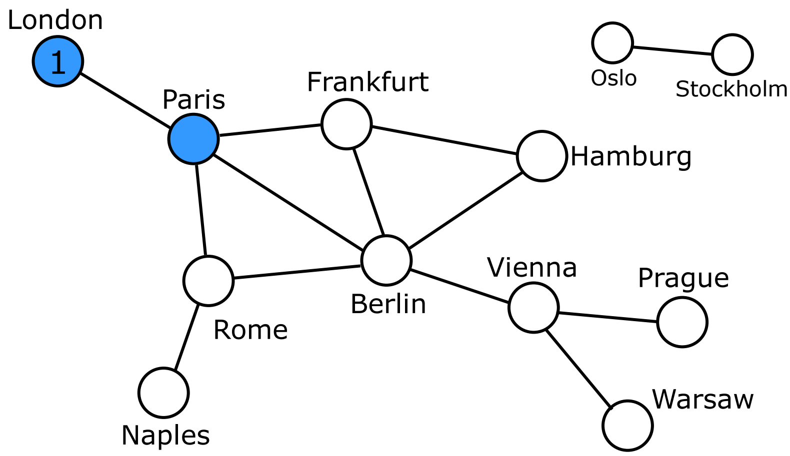
Pick a node to start with

Graph traversal (stack)



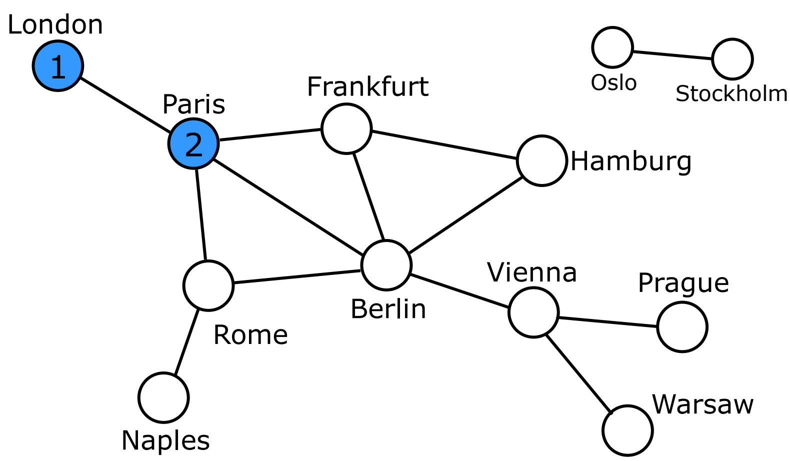
Current node: London
Todo list: [ ]

Graph traversal (stack)



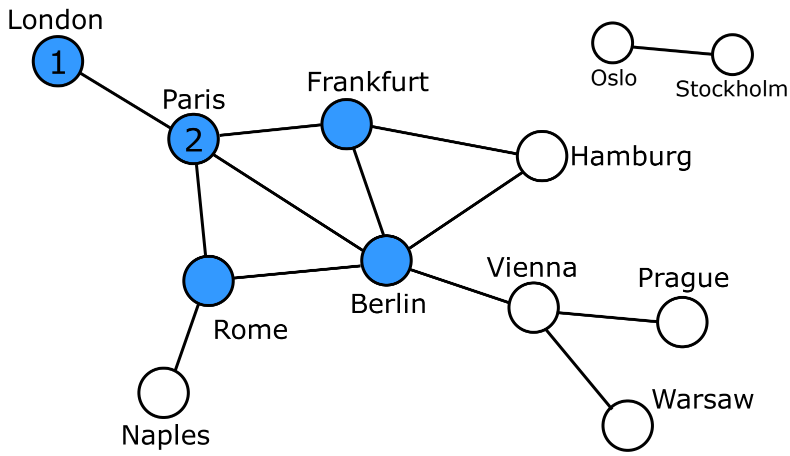
Current node: London
Todo list: [Paris]

Graph traversal (stack)



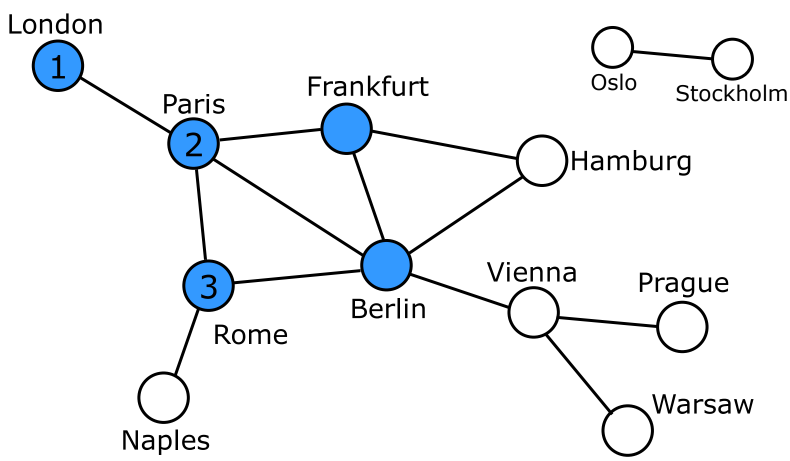
Current node: Paris
Todo list: [ ]

Graph traversal (stack)



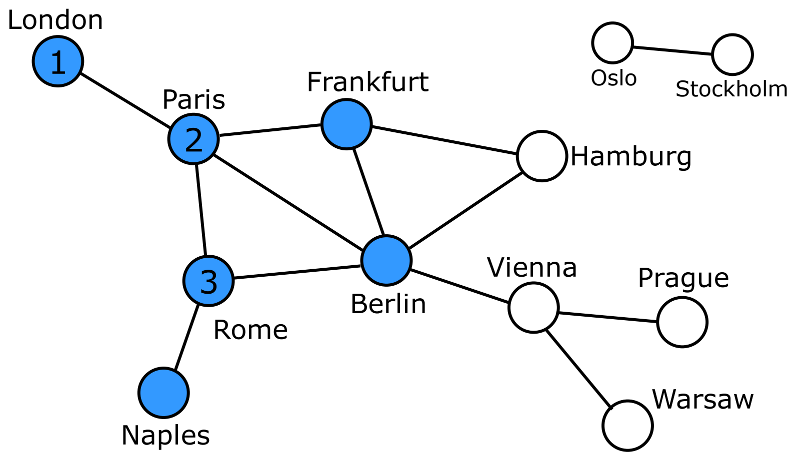
Current node: Paris
Todo list: [Frankfurt, Berlin, Rome]

Graph traversal (stack)



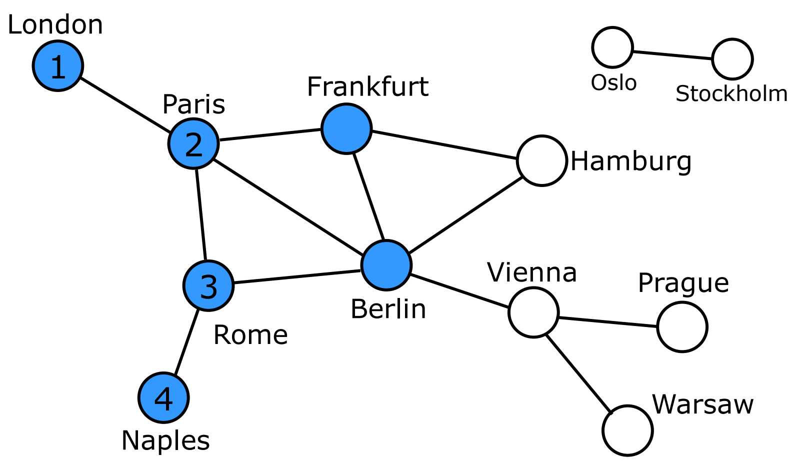
Current node: Rome
Todo list: [Frankfurt, Berlin]

Graph traversal (stack)



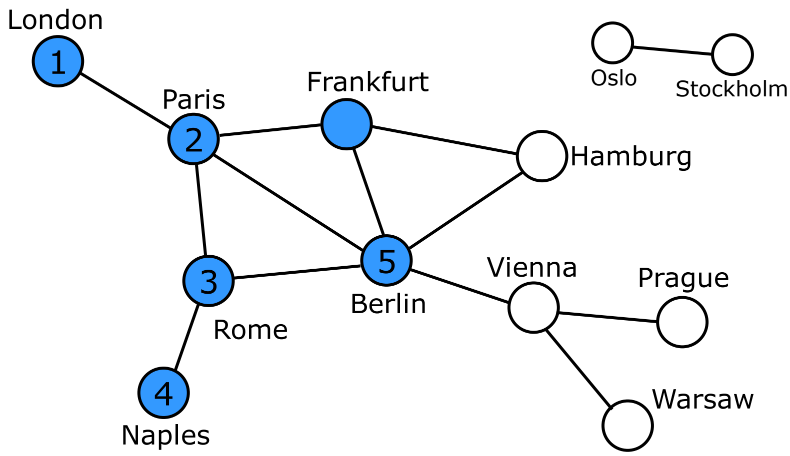
Current node: Rome
Todo list: [Frankfurt, Berlin, Naples]

Graph traversal (stack)



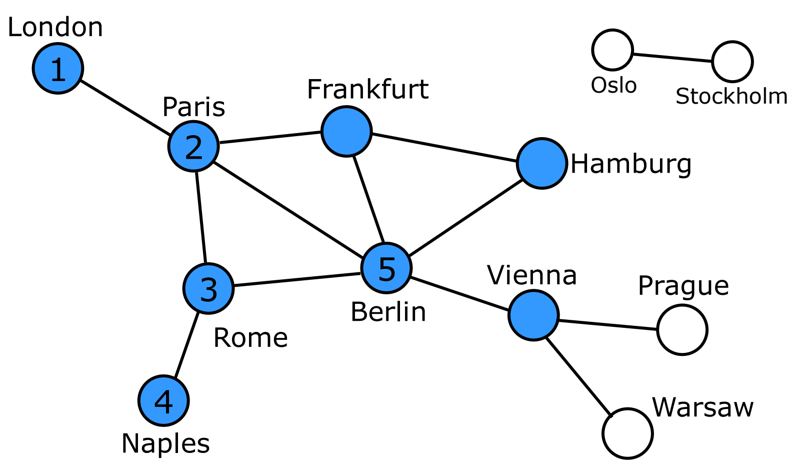
Current node: Naples
Todo list: [Frankfurt, Berlin]

Graph traversal (stack)



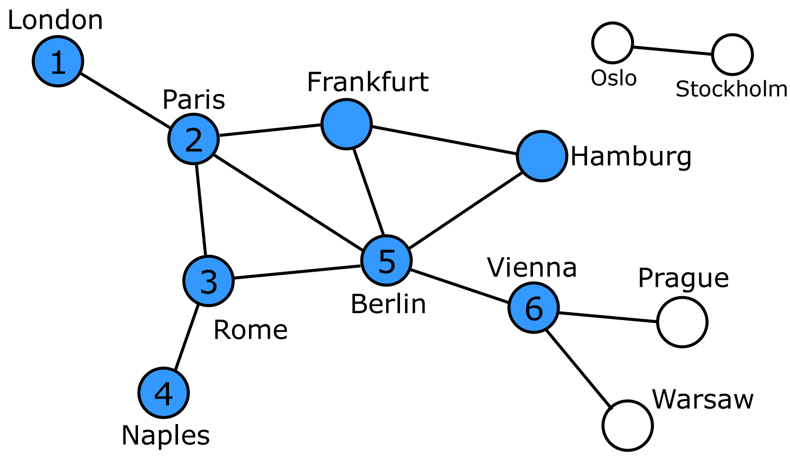
Current node: Berlin
Todo list: [Frankfurt]

Graph traversal (stack)



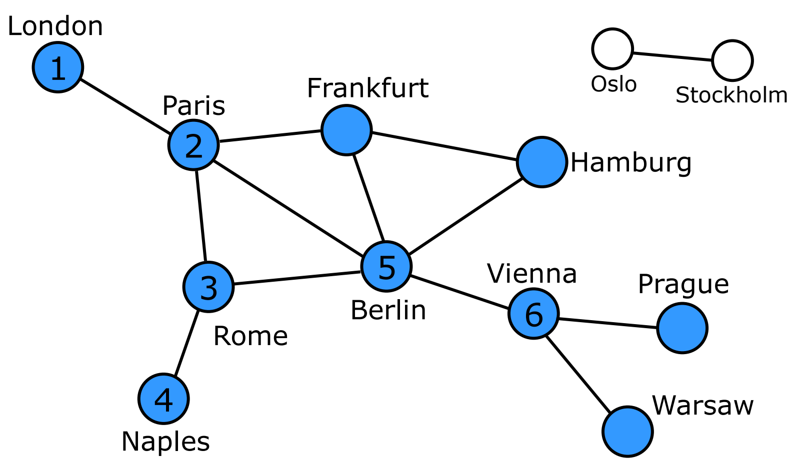
Current node: Berlin
Todo list: [Frankfurt, Hamburg, Vienna]

Graph traversal (stack)



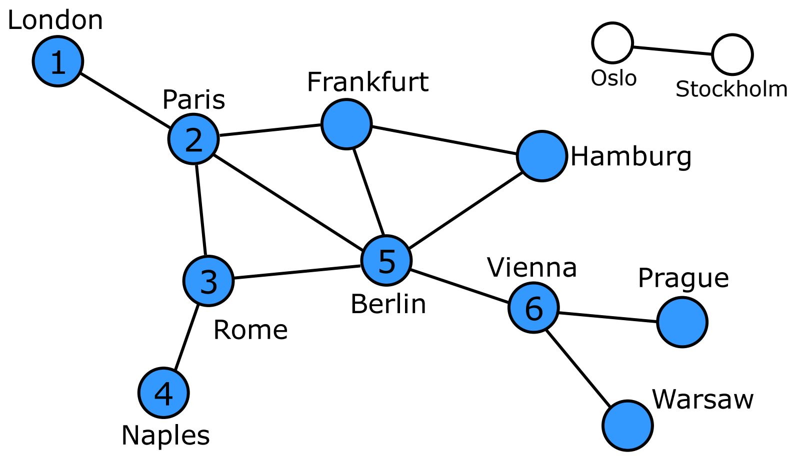
Current node: Vienna
Todo list: [Frankfurt, Hamburg]

Graph traversal (stack)



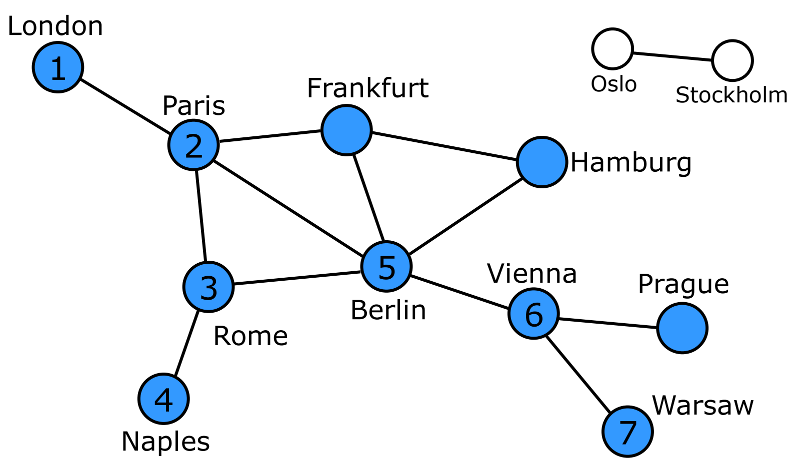
Current node: Vienna
Todo list: [Frankfurt, Hamburg, Prague, Warsaw]

Graph traversal (stack)



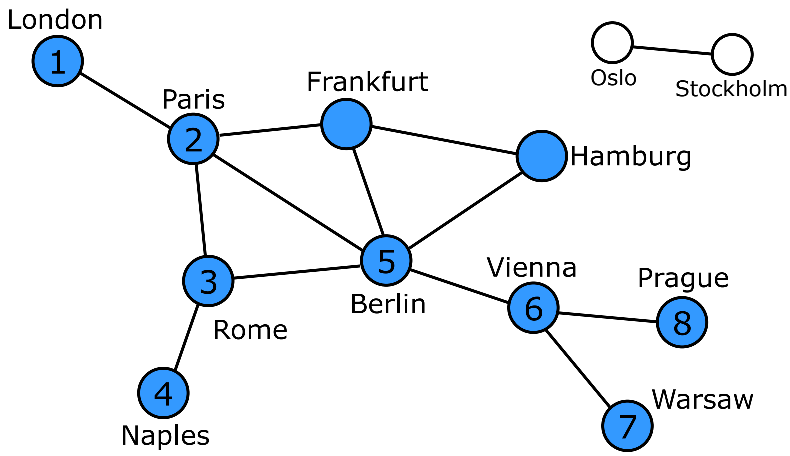
Current node: Vienna
Todo list: [Frankfurt, Hamburg, Prague, Warsaw]

Graph traversal (stack)



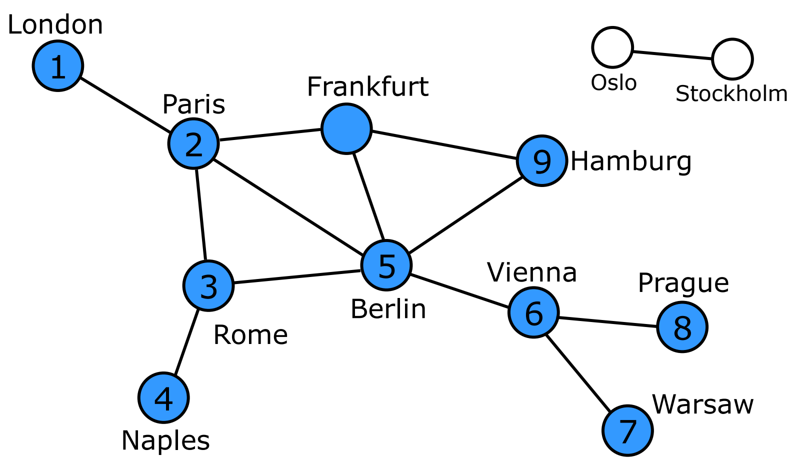
Current node: Warsaw
Todo list: [Frankfurt, Hamburg, Prague]

Graph traversal (stack)



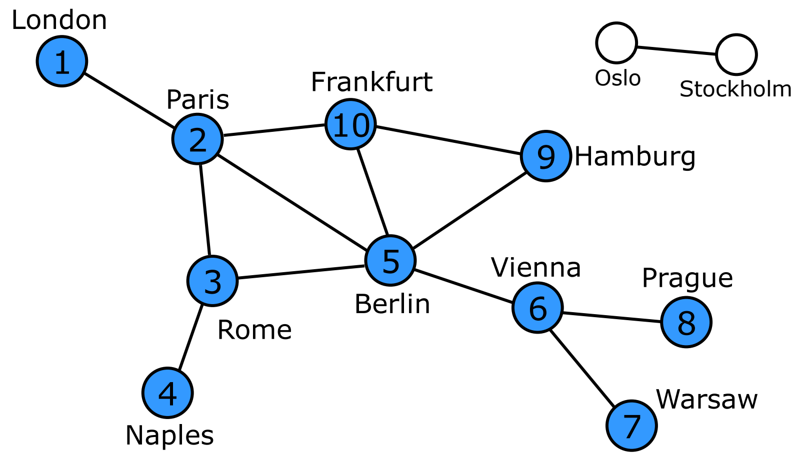
Current node: Prague
Todo list: [Frankfurt, Hamburg]

Graph traversal (stack)



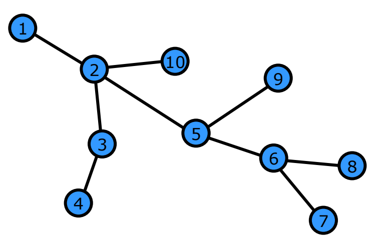
Current node: Hamburg
Todo list: [Frankfurt]

Graph traversal (stack)



Current node: Frankfurt
Todo list: [ ]

Depth-first search (DFS)



- Call the starting node the *root*
- We traverse paths all the way until we get to a dead-end, then backtrack (until we find an unexplored path)

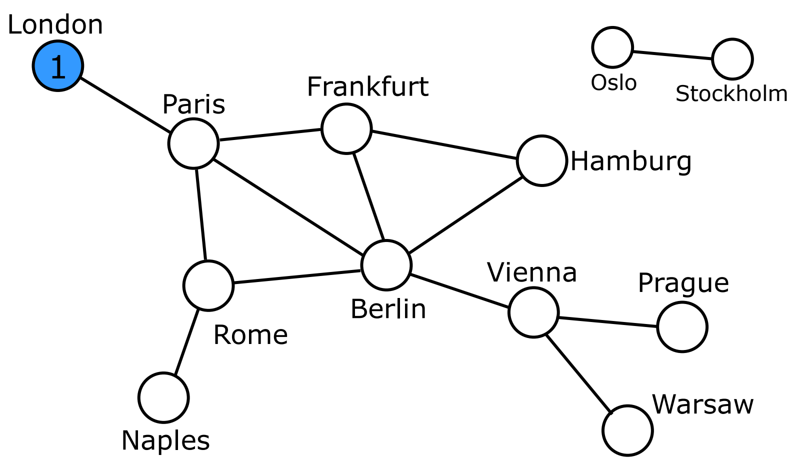
Another strategy

1. Explore all the cities that are one hop away from the root
2. Explore all cities that are two hops away from the root
3. Explore all cities that are three hops away from the root

   …

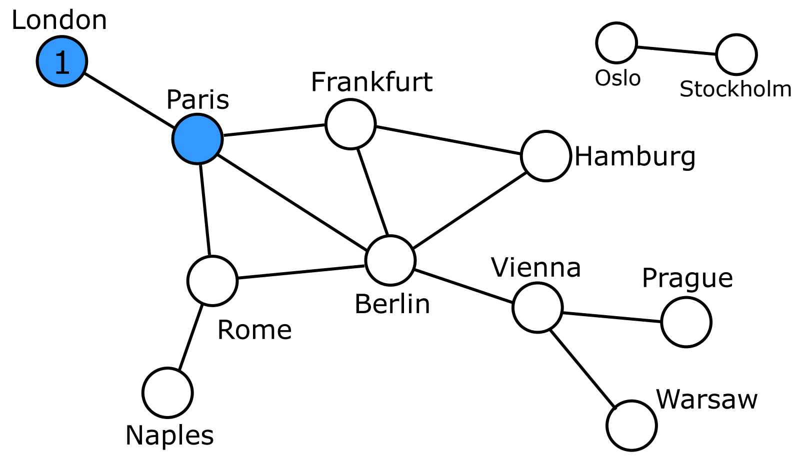
- This corresponds to using a *queue*

Graph traversal (queue)



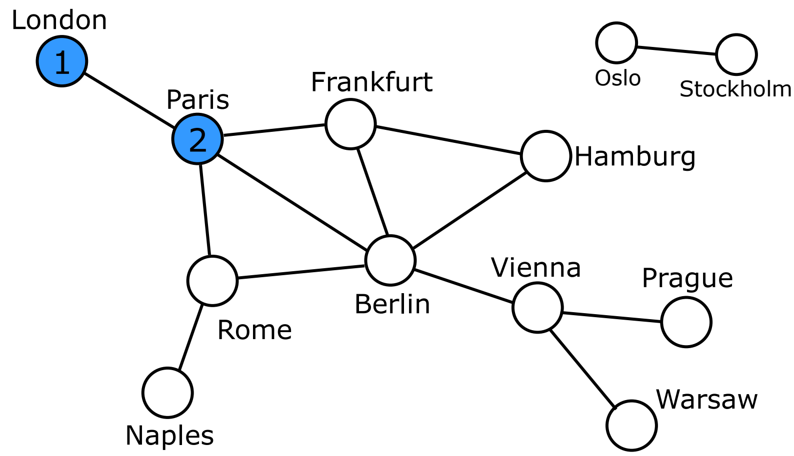
Current node: London
Todo list: [ ]

Graph traversal (queue)



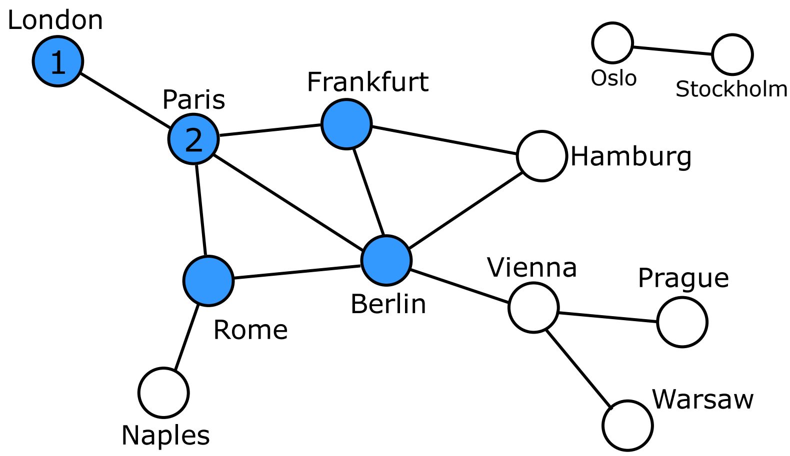
Current node: London
Todo list: [Paris]

Graph traversal (queue)



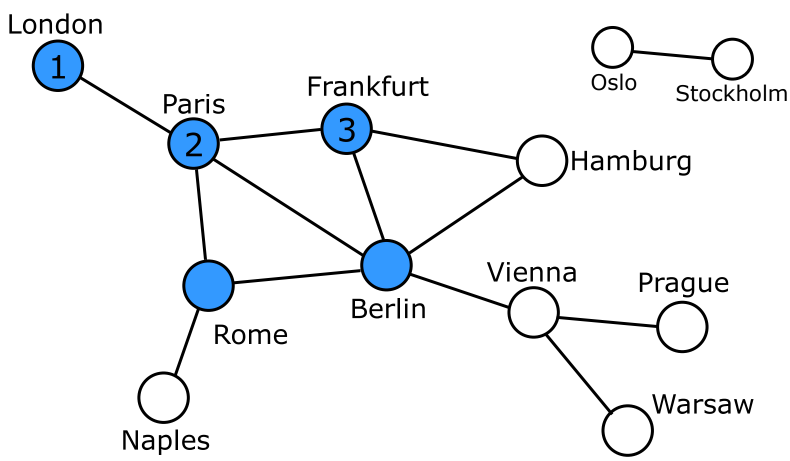
Current node: Paris
Todo list: [ ]

Graph traversal (queue)



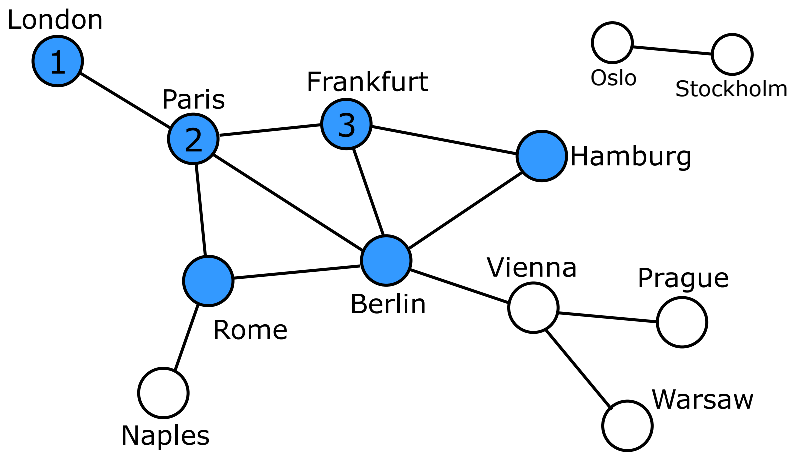
Current node: Paris
Todo list: [Frankfurt, Berlin, Rome]

Graph traversal (queue)



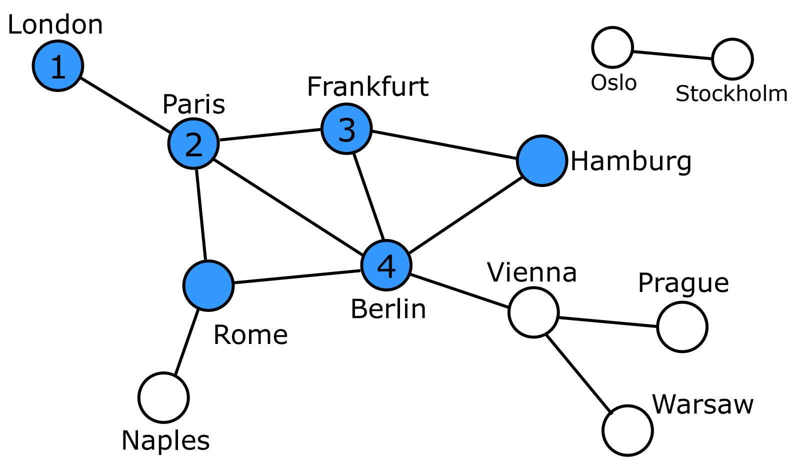
Current node: Frankfurt
Todo list: [Berlin, Rome]

Graph traversal (queue)



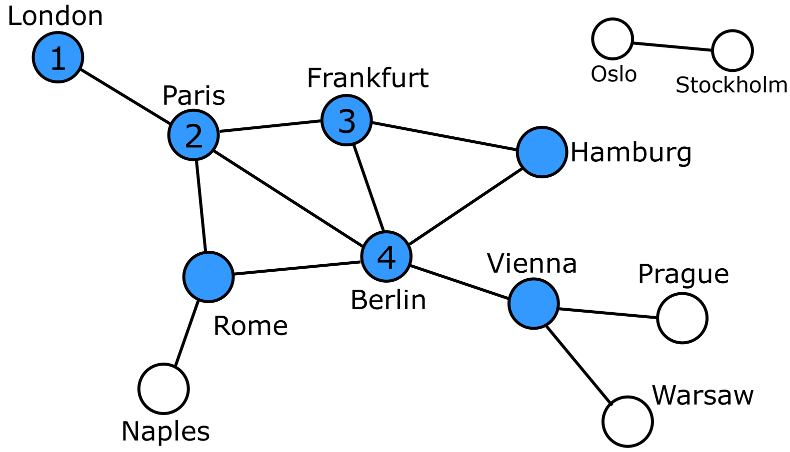
Current node: Frankfurt
Todo list: [Berlin, Rome, Hamburg]

Graph traversal (queue)



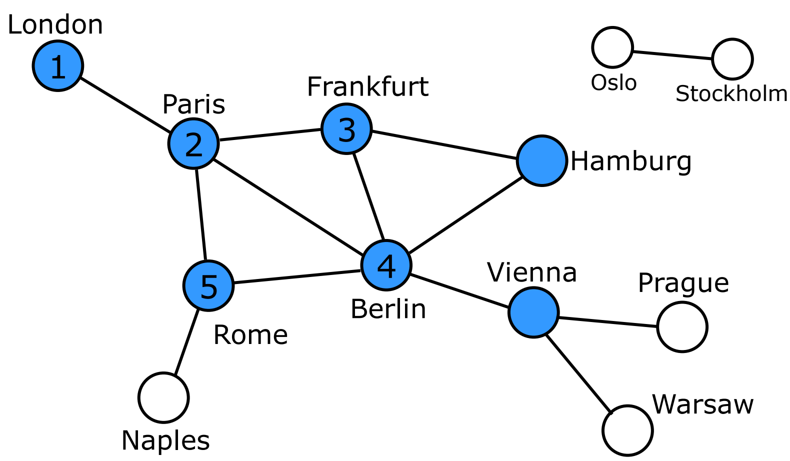
Current node: Berlin
Todo list: [Rome, Hamburg]

Graph traversal (queue)



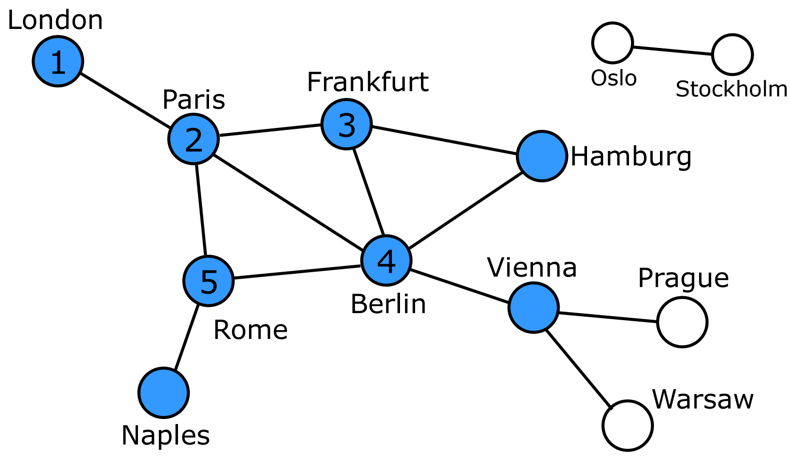
Current node: Berlin
Todo list: [Rome, Hamburg, Vienna]

Graph traversal (queue)



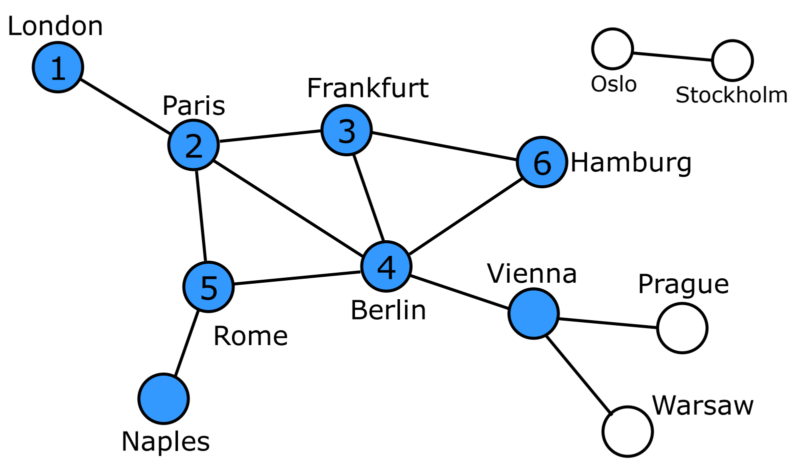
Current node: Rome
Todo list: [Hamburg, Vienna]

Graph traversal (queue)



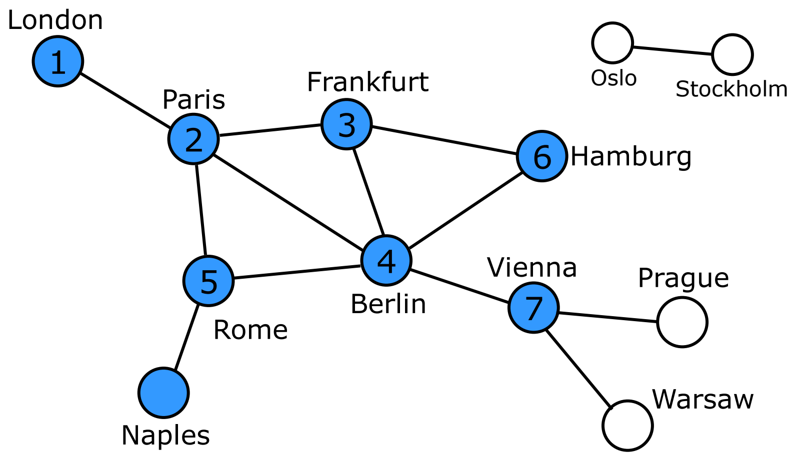
Current node: Rome
Todo list: [Hamburg, Vienna, Naples]

Graph traversal (queue)



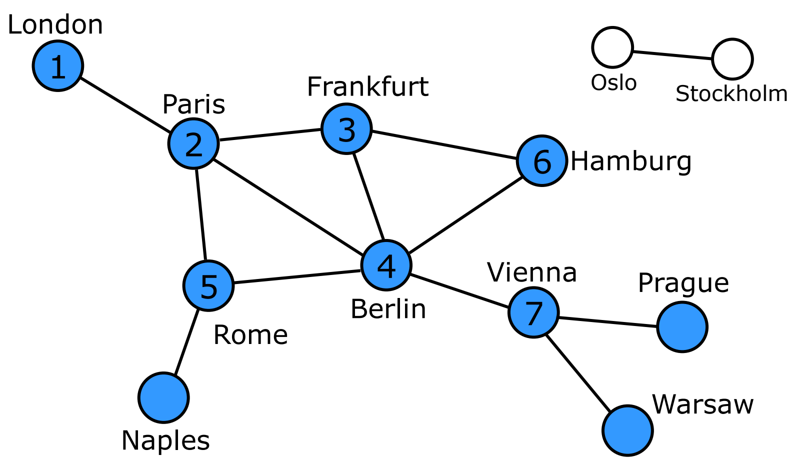
Current node: Hamburg
Todo list: [Vienna, Naples]

Graph traversal (queue)



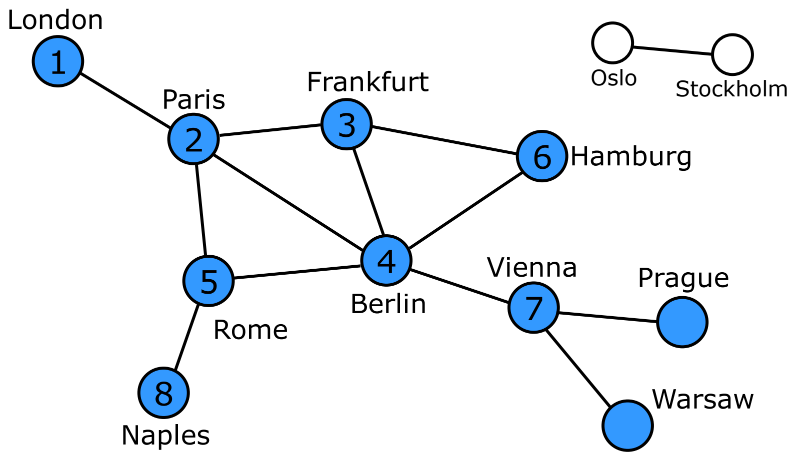
Current node: Vienna
Todo list: [Naples]

Graph traversal (queue)



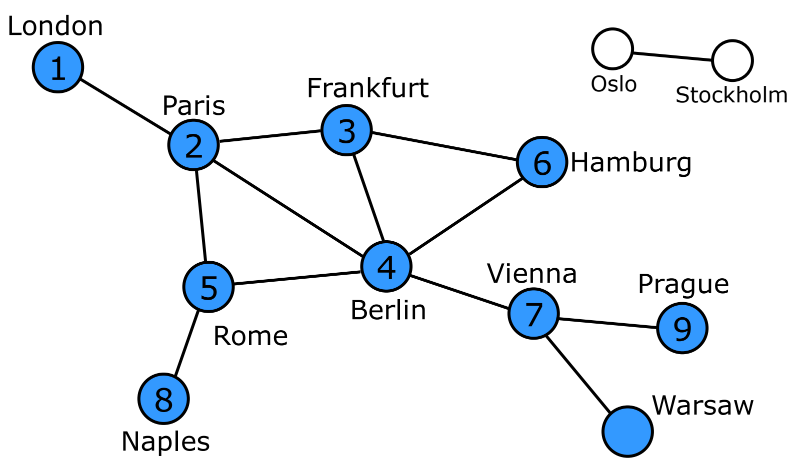
Current node: Vienna
Todo list: [Naples, Prague, Warsaw]

Graph traversal (queue)



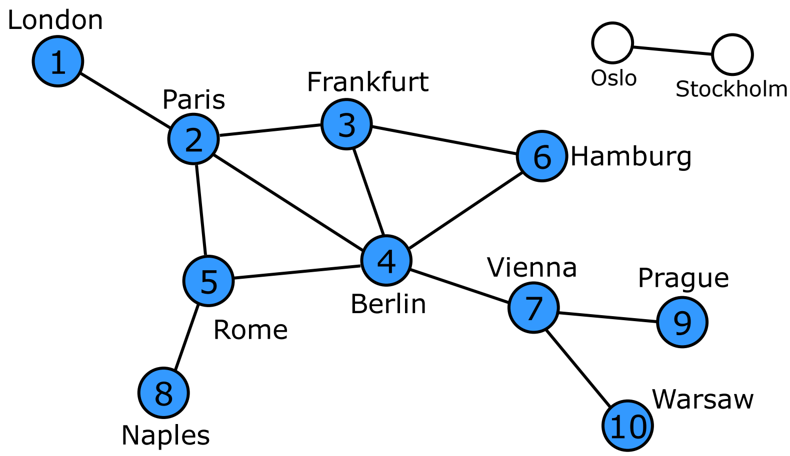
Current node: Naples
Todo list: [Prague, Warsaw]

Graph traversal (queue)



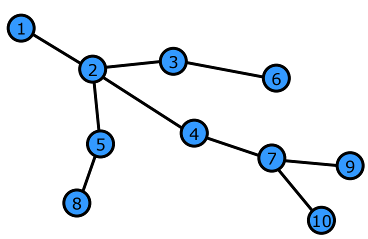
Current node: Prague
Todo list: [Warsaw]

Graph traversal (queue)



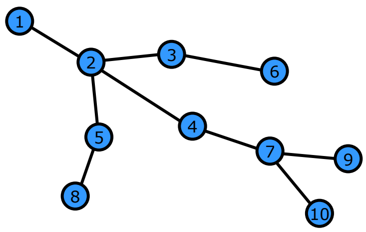
Current node: Warsaw
Todo list: [ ]

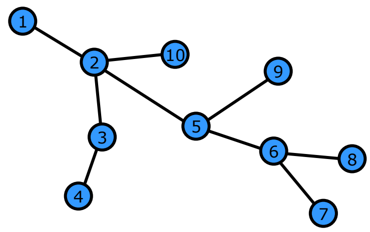
Breadth-first search (BFS)



- We visit all the vertices at the same level (same distance to the root) before moving on to the next level

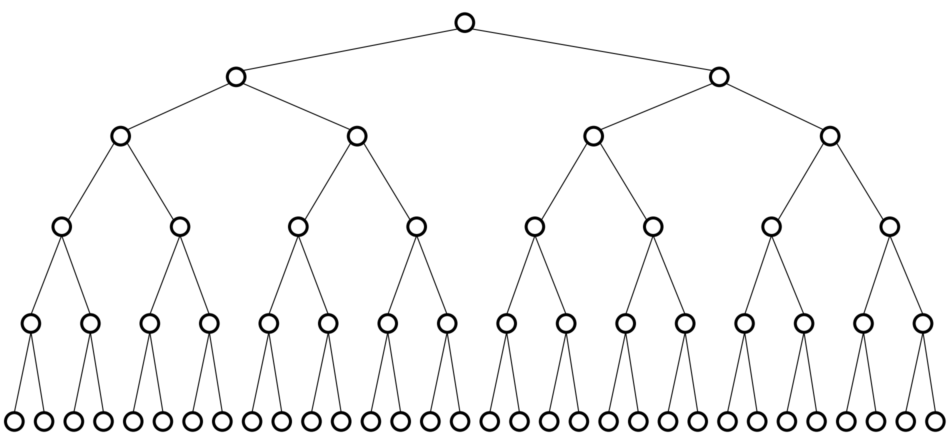
BFS vs. DFS





Breadth-first (queue)   Depth-first (stack)

BFS vs. DFS



(tree = graph with no cycles)

Basic algorithms

BREADTH-FIRST SEARCH (Graph G)

- While there is an uncolored node **r**
  - Choose a new color
  - Create an empty queue **Q**
  - Let **r** be the root node, color it, and add it to **Q**
  - While **Q** is not empty
    - Dequeue a node **v** from **Q**
    - For each of **v**'s neighbors **u**
      - If **u** is not colored, color it and add it to **Q**

Basic algorithms

DEPTH-FIRST SEARCH (Graph G)

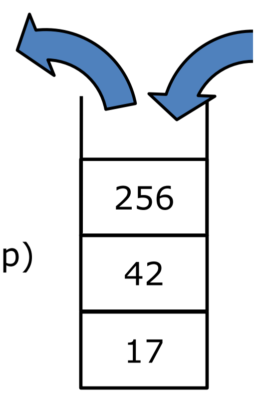
- While there is an uncolored node **r**
  - Choose a new color
  - Create an empty stack **S**
  - Let **r** be the root node, color it, and push it on **S**
  - While **S** is not empty
    - Pop a node **v** from **S**
    - For each of **v**'s neighbors **u**
      - If **u** is not colored, color it and push it onto **S**

Queues and Stacks

- Examples of Abstract Data Types (ADTs)
- ADTs fulfill a contract:
  - The contract tells you what the ADT can do, and what the behavior is
  - For instance, with a stack:
    - We can push and pop
    - If we push X onto S and then pop S, we get back X, and S is as before
- Doesn't tell you *how* it fulfills the contract

Implementing DFS

- How can we implement a stack?
  - Needs to support several operations:
  - Push (add an element to the top)
  - Pop (remove the element from the top)
  - IsEmpty



Implementing a stack

- IsEmpty

  function e = IsEmpty(S)
      e = (length(S) == 0);

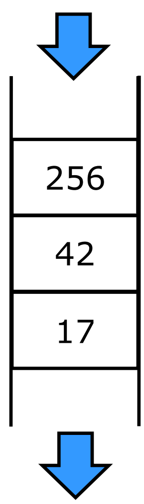
- Push (add an element to the top)

  function S = push(S, x)
      S = [S x]

- Pop (remove an element from the top)

  function [S, x] = pop(S)
      n = length(S); x = S(n); S = S(1:n-1);
      % but what happens if n = 0?

Implementing BFS



- How can we implement a queue?
  - Needs to support several operations:
  - Enqueue (add an element to back)
  - Dequeue (remove an element from front)
  - IsEmpty
- Not quite as easy as a stack…