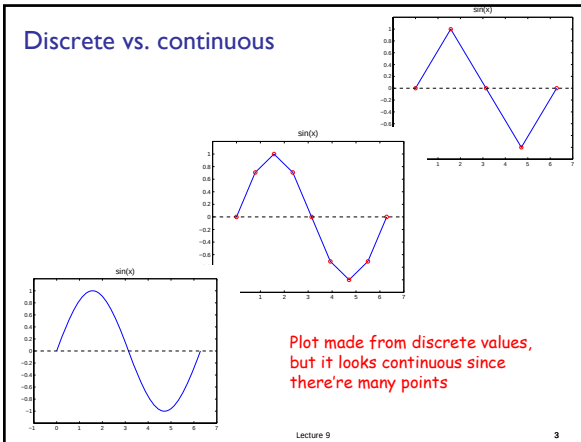
- Previous Lecture:
  - Review
  - Color as a 3-vector
  - Linear interpolation
- Today's Lecture:
  - Finite/inexact arithmetic
  - Plotting continuous functions using vectors and vectorized code
  - Introduction to user-defined functions
- Announcement:
  - Prelim 1 tonight, Feb 25th at 7:30pm
    - Last names A-O in Statler Aud. main floor
    - Last names P-Z in Statler Aud. balcony

Discrete vs. continuous



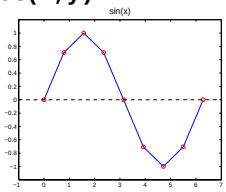
Plot made from discrete values, but it looks continuous since there're many points

Generating tables and plots

x	sin(x)
0.000	0.000
0.784	0.707
1.571	1.000
2.357	0.707
3.142	0.000
3.927	-0.707
4.712	-1.000
5.498	-0.707
6.283	0.000

x, y are vectors. A vector is a 1-dimensional list of values

```
x= linspace(0,2*pi,9);
y= sin(x);
plot(x,y)
```



Note: x, y are shown in columns due to space limitation; they should be rows.

Built-in function `linspace`

```
x= linspace(1,3,5)
```

x	1.0	1.5	2.0	2.5	3.0

```
x= linspace(0,1,101)
```

x	0.00	0.01	0.02	...	0.99	1.00

Left endpoint

Right endpoint

Number of points

How did we get all the sine values?

x	sin(x)
0.00	0.0
1.57	1.0
3.14	0.0
4.71	-1.0
6.28	0.0

Built-in functions accept arrays

0.00	1.57	3.14	4.71	6.28

sin

and return arrays

0.00	1.00	0.00	-1.00	0.00

Examples of functions that can work with arrays

```
x= linspace(0,1,200);
y= exp(x);
plot(x,y)
```

```
x= linspace(1,10,200);
y= log(x);
plot(x,y)
```

Does this assign to y the values
sin(0°), sin(1°), sin(2°), …, sin(90°)?

```
x = linspace(0,pi/2,90);

y = sin(x);
```

A: yes B: no

Can we plot this?

See `plotComparison.m`

$$f(x) = \frac{\sin(5x)\exp(-x/2)}{1+x^2}$$ for -2 <= x <= 3

Yes!

```
x = linspace(-2,3,200);
y = sin(5*x).*exp(-x/2)./(1 + x.^2);
plot(x,y)
```

Element-by-element arithmetic operations on arrays

Element-by-element arithmetic operations on arrays…
Also called "vectorized code"

```
x = linspace(-2,3,200);
y = sin(5*x).*exp(-x/2)./(1 + x.^2);
```

x and y are vectors

Contrast with scalar operations that we've used previously…

```
a = 2.1;
b = sin(5*a);
```

a and b are scalars

The operators are (mostly) the same; the operands may be scalars or vectors.

When an operand is a vector, you have "vectorized code."

Vectorized code
—a Matlab-specific feature

See Sec 4.1 for list of vectorized arithmetic operations

- Code that performs element-by-element arithmetic/relational/logical operations on array operands in one step
- Scalar operation: x + y
  where x, y are scalar variables
- Vectorized code: x + y
  where x and/or y are vectors. If x and y are both vectors, they must be of the same shape and length

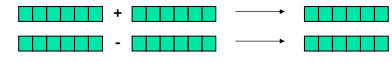
Vectorized multiplication

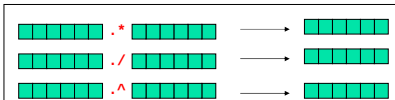
	a	2	1	.5	8
×	b	1	2	0	1
=	c	2	2	0	8

Matlab code: `c= a .* b`

Vectorized code
element-by-element arithmetic operations on arrays

See full list of ops in §4.1





A dot (.) is necessary in front of these math operators

Reciprocate

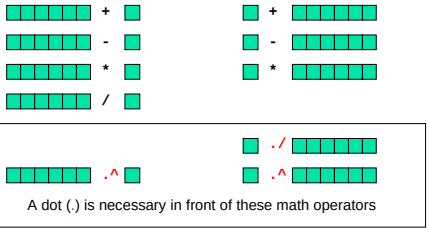
	x	1
/	y	2 1 .5 8
=	z	.5 1 2 .125

Matlab code: `z= x ./ y`

Vectorized code

See full list of ops in §4.1

element-by-element arithmetic operations between an array and a scalar



The dot in □.*□ , □.*□ , □./□ not necessary but OK

Computer Arithmetic—floating point arithmetic

Suppose you have a calculator with a window like this:

+	2	4	1	-	3

representing 2.41×10^{-3}

Floating point addition

+	2	4	1	-	3
+	1	0	0	-	4

Result:

+	2	5	1	-	3

Floating point addition

+	2	4	1	-	3
+	1	0	0	-	6

Result:

+	2	4	1	-	3

Not enough room to represent .002411

The loop DOES terminate given the limitations of floating point arithmetic!

```
k = 0;
while 1 + 1/2^k > 1
   k = k+1;
end
disp(k)
```

1 + 1/2^53 is calculated to be just 1, so "53" is printed.

Patriot missile failure

www.namsa.nato.int/gallery/systems

In 1991, a Patriot Missile failed, resulting in 28 deaths and about 100 injured. The cause?

Computer arithmetic is *inexact*

- There is error in computer arithmetic—floating point arithmetic—due to limitation in "hardware." Computer memory is finite.
- What is $1 + 10^{-16}$?
  - 1.0000000000000001 in real arithmetic
  - 1 in floating point arithmetic (IEEE)
- Read Sec 4.3

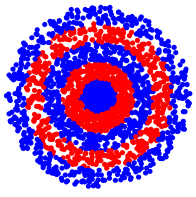
Built-in functions

- We've used many Matlab built-in functions, e.g., `rand`, `abs`, `floor`, `rem`
- Example: `abs(x-.5)`
- Observations:
  - `abs` is set up to be able to work with any valid data
  - `abs` *doesn't prompt us for input; it expects that we provide data* that it'll then work on

User-defined functions

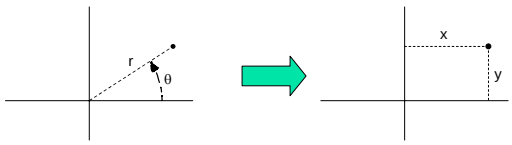
- We can write our own functions to perform a specific task
  - Example: generate a random floating point number in a specified interval
  - Example: convert polar coordinates to x-y (Cartesian) coordinates

Draw a bulls eye figure with randomly placed dots



- What are the main tasks?
- Accommodate variable number of rings—loop
- For each ring
  - Need many dots
  - For each dot
    - Generate random position
    - Choose color
    - Draw it

Convert from polar to Cartesian coordinates



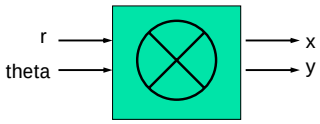
Polar coordinates Cartesian coordinates

```
c= input('How many concentric rings? ');
d= input('How many dots? ');

% Put dots btwn circles with radii rRing and (rRing-1)
for rRing= 1:c
  % Draw d dots
  for count= 1:d

    % Generate random dot location (polar coord.)
    theta= _______
    r= _______

    % Convert from polar to Cartesian
    x= _______
    y= _______

    % Use plot to draw dot
  end
end
```

A common task! Create a function **polar2xy** to do this. **polar2xy** likely will be useful in other problems as well.


```
function [x, y] = polar2xy(r,theta)
% Convert polar coordinates (r,theta) to
% Cartesian coordinates (x,y).
% theta is in degrees.

rads= theta*pi/180;  % radian
x= r*cos(rads);
y= r*sin(rads);
```

Think of **polar2xy** as a factory




```
function [x, y] = polar2xy(r,theta)
% Convert polar coordinates (r,theta) to
% Cartesian coordinates (x,y).
% theta is in degrees.

rads= theta*pi/180;  % radian
x= r*cos(rads);
y= r*sin(rads);
```

A function file **polar2xy.m**

```
r= input('Enter radius: ');
theta= input('Enter angle in degrees: ');

rads= theta*pi/180;  % radian
x= r*cos(rads);
y= r*sin(rads);
```

(Part of) a script file

```
function [x, y] = polar2xy(r,theta)
```

Output parameter list enclosed in [ ]

Function name (This file's name is **polar2xy.m**)

Input parameter list enclosed in ( )

Function header is the "contract" for how the function will be used (called)

You have this function:

```
function  [x, y] = polar2xy(r, theta)
% Convert polar coordinates (r, theta) to
% Cartesian coordinates (x,y).  Theta in degrees.
…
```

Code to call the above function:

```
% Convert polar (r1,t1) to Cartesian (x1,y1)
r1= 1;   t1= 30;
[x1, y1]= polar2xy(r1, t1);
plot(x1, y1, 'b*')
…
```


General form of a user-defined function

```
function [out1, out2, …]= functionName (in1, in2, …)
% 1-line comment to describe the function
% Additional description of function

   Executable code that at some point assigns
   values to output parameters out1, out2, …
```

- *in1*, *in2*, … are defined when the function begins execution. Variables *in1*, *in2*, … are called function *parameters* and they hold the function *arguments* used when the function is invoked (called).
- *out1*, *out2*, … are not defined until the executable code in the function assigns values to them.