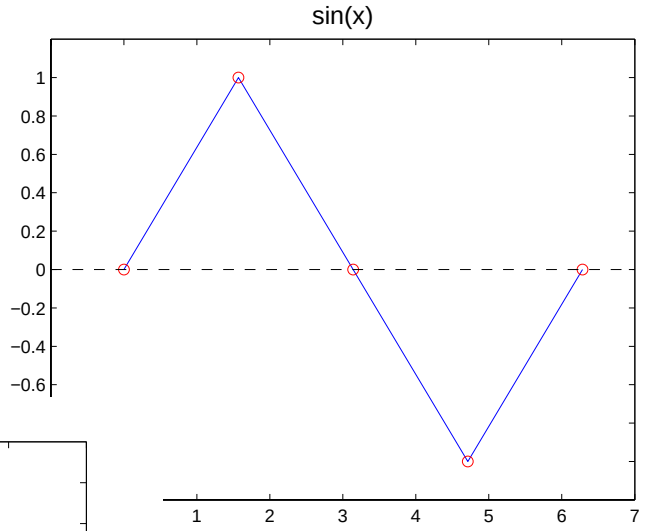
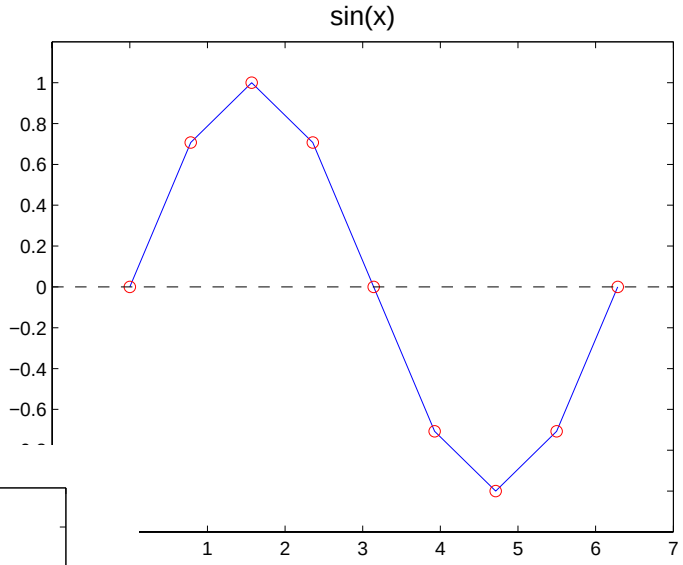
- Previous Lecture:
  - Review
  - Color as a 3-vector
  - Linear interpolation

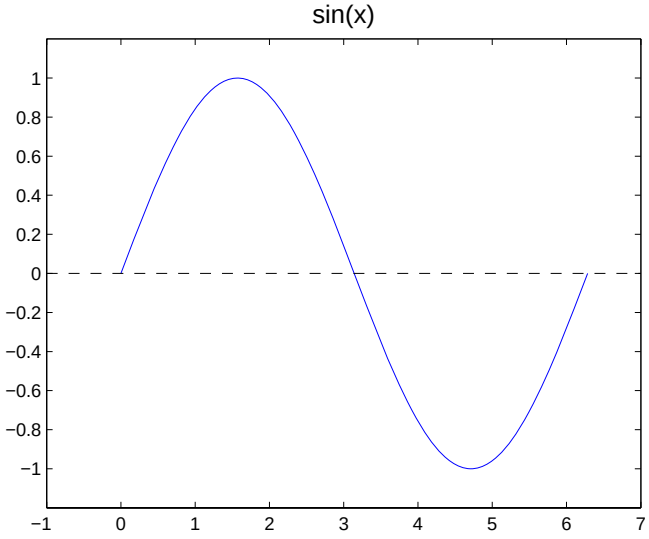
- Today's Lecture:
  - Finite/inexact arithmetic
  - Plotting continuous functions using vectors and vectorized code
  - Introduction to user-defined functions

- Announcement:
  - Prelim 1 tonight, Feb 25th at 7:30pm
    - Last names A-O in Statler Aud. main floor
    - Last names P-Z in Statler Aud. balcony

Discrete vs. continuous



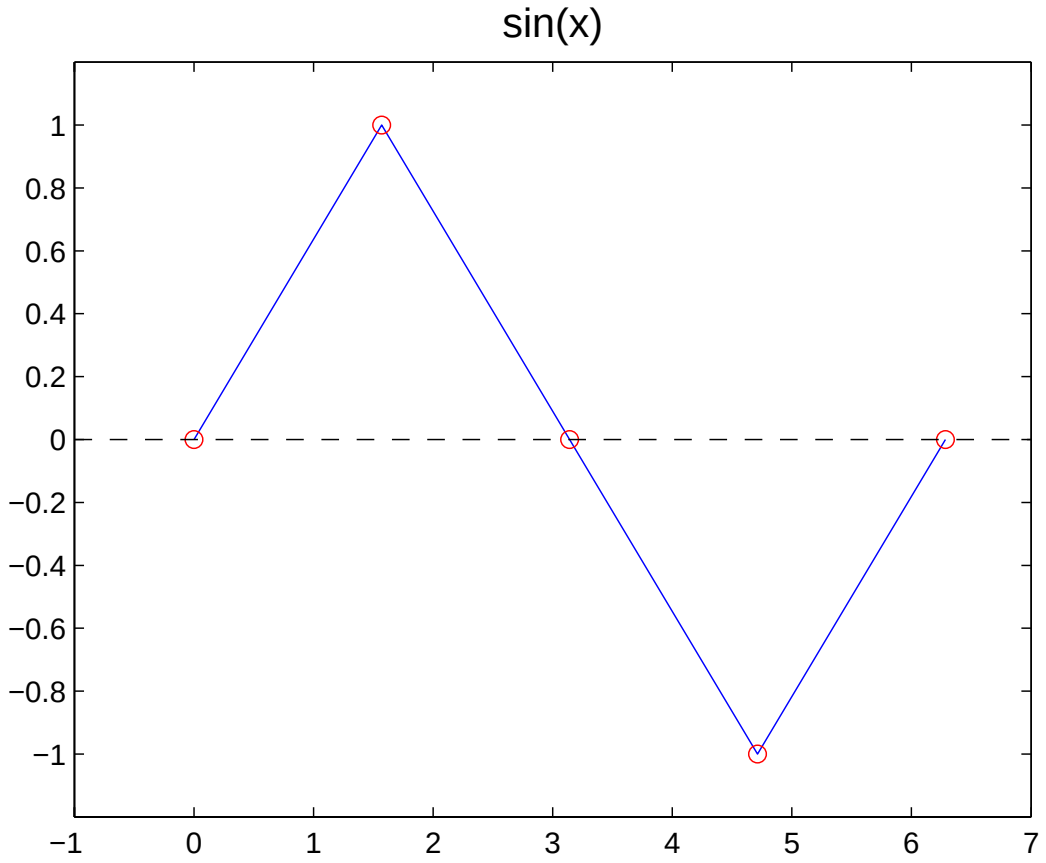




Plot made from discrete values, but it looks continuous since there're many points

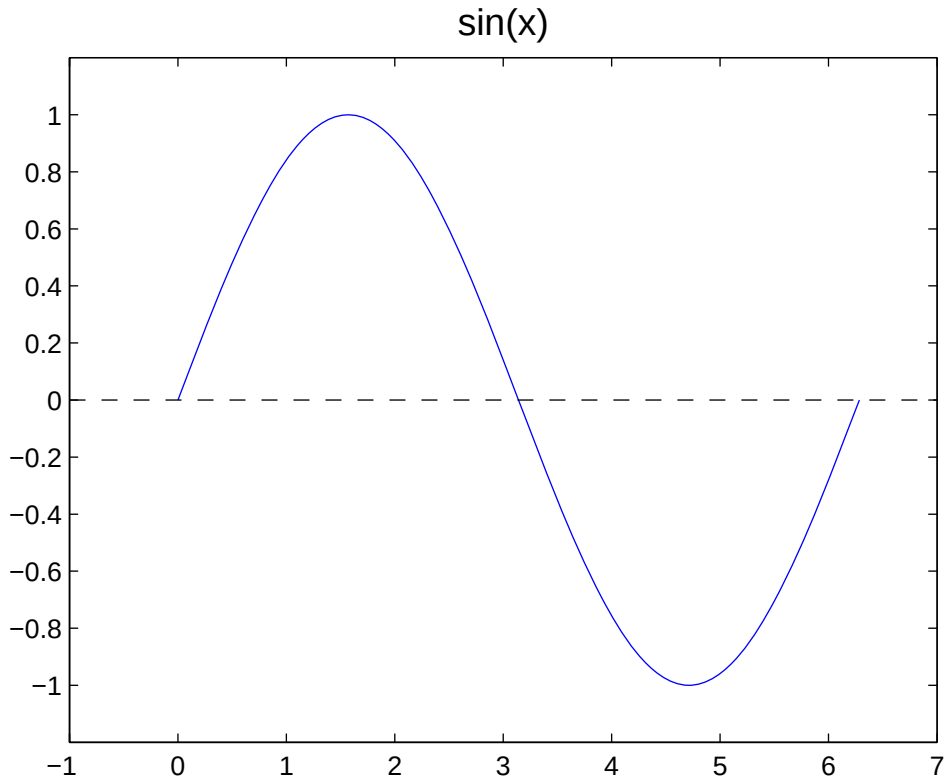
Plot a continuous function (from a table of values)

x	sin(x)
0.00	0.0
1.57	1.0
3.14	0.0
4.71	-1.0
6.28	0.0



Plot based on 5 points

Plot based on 200 discrete points, but it looks smooth

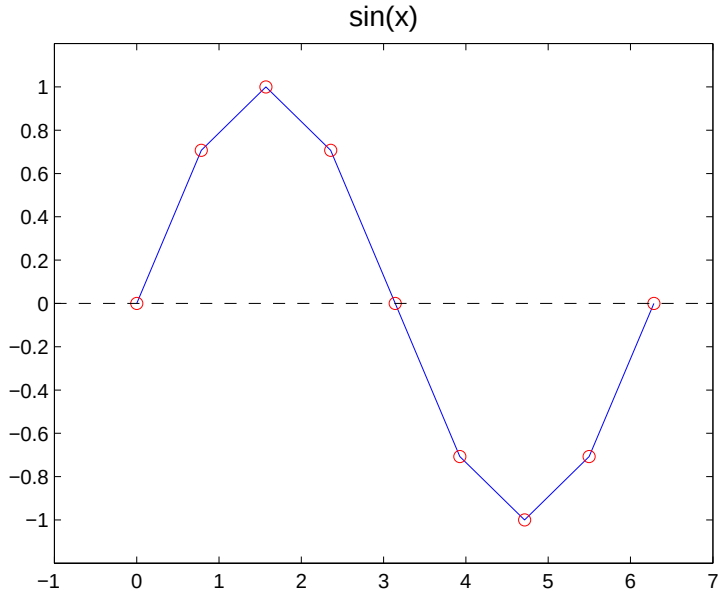


Generating tables and plots

x	sin(x)
0.000	0.000
0.784	0.707
1.571	1.000
2.357	0.707
3.142	0.000
3.927	-0.707
4.712	-1.000
5.498	-0.707
6.283	0.000

x, y are vectors. A vector is a 1-dimensional list of values

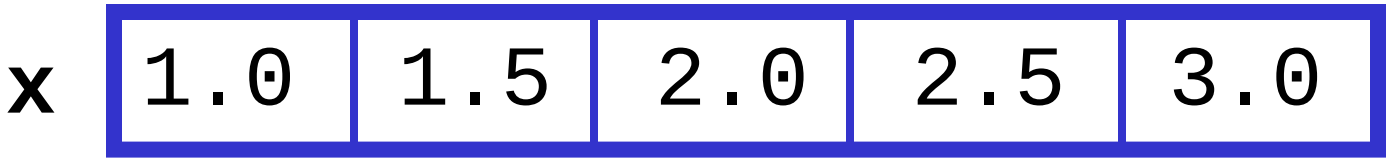
```
x= linspace(0,2*pi,9);
y= sin(x);
plot(x,y)
```



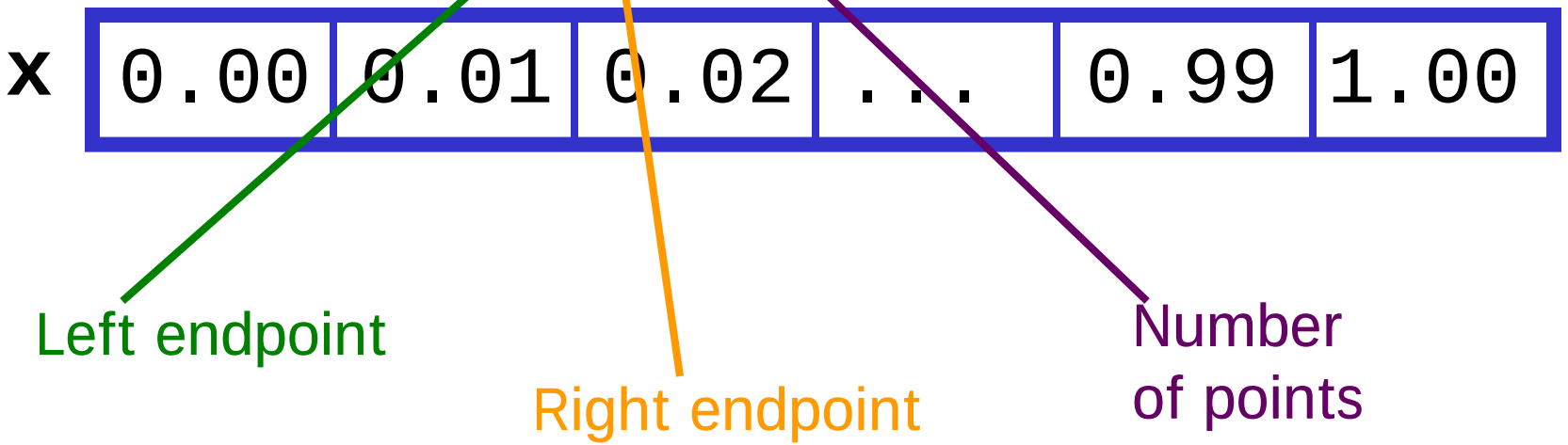
Note: x, y are shown in columns due to space limitation; they should be rows.

Built-in function `linspace`

```
x= linspace(1,3,5)
```



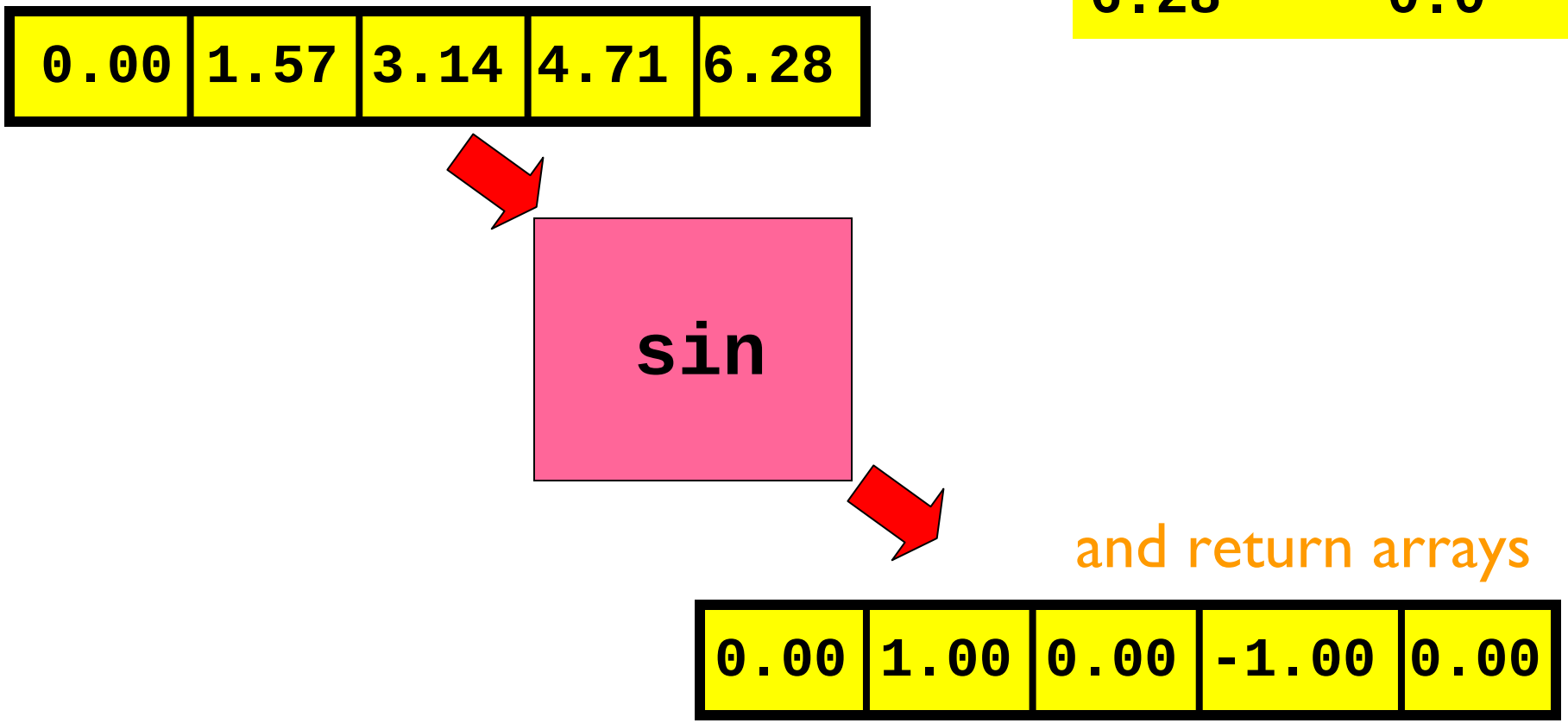

```
x= linspace(0,1,101)
```



How did we get all the sine values?

x	sin(x)
0.00	0.0
1.57	1.0
3.14	0.0
4.71	-1.0
6.28	0.0

Built-in functions accept arrays



Examples of functions that can work with arrays

```
x= linspace(0,1,200);
y= exp(x);
plot(x,y)
```

```
x= linspace(1,10,200);
y= log(x);
plot(x,y)
```

Does this assign to y the values
sin(0°), sin(1°), sin(2°), …, sin(90°)?

```
x = linspace(0,pi/2,90);

y = sin(x);
```

A: yes B: no

Can we plot this?

$$f(x) = \frac{\sin(5x)\exp(-x/2)}{1+x^2}$$

for
-2 <= x <= 3

Can we plot this?

$$f(x) = \frac{\sin(5x)\exp(-x/2)}{1+x^2}$$

for
-2 <= x <= 3

Yes!

Can we plot this?

See **plotComparison.m**

$$f(x) = \frac{\sin(5x)\exp(-x/2)}{1+x^2}$$

for
-2 <= x <= 3

Yes!

```
x = linspace(-2,3,200);
y = sin(5*x).*exp(-x/2)./(1 + x.^2);
plot(x,y)
```

Element-by-element arithmetic operations on arrays

Element-by-element arithmetic operations on arrays…
Also called "vectorized code"

```
x = linspace(-2,3,200);
y = sin(5*x).*exp(-x/2)./(1 + x.^2);
```

x and y are vectors

Contrast with scalar operations that we've used previously…

```
a = 2.1;
b = sin(5*a);
```

a and b are scalars

The operators are (mostly) the same; the operands may be scalars or vectors.

When an operand is a vector, you have "vectorized code."

Vectorized addition

	x	2	1	.5	8
+	y	1	2	0	1
=	z	3	3	.5	9

Matlab code: `z= x + y`

Vectorized subtraction

	x	2	1	.5	8
-	y	1	2	0	1
=	z	1	-1	.5	7

Matlab code: `z= x - y`

Vectorized code

—a Matlab-specific feature

See Sec 4.1 for list of vectorized arithmetic operations

- Code that performs element-by-element arithmetic/relational/logical operations on array operands in one step

- Scalar operation: x + y

  where x, y are scalar variables

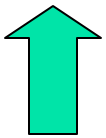
- Vectorized code: x + y

  where x and/or y are vectors. If x and y are both vectors, they must be of the same shape and length

Vectorized multiplication

	a	2	1	.5	8
×	**b**	1	2	0	1
=	**c**	2	2	0	8

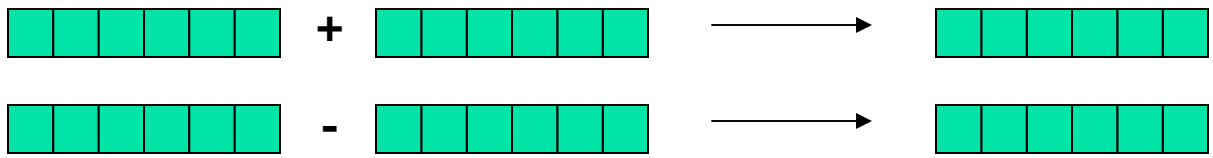
Matlab code: **c= a .* b**

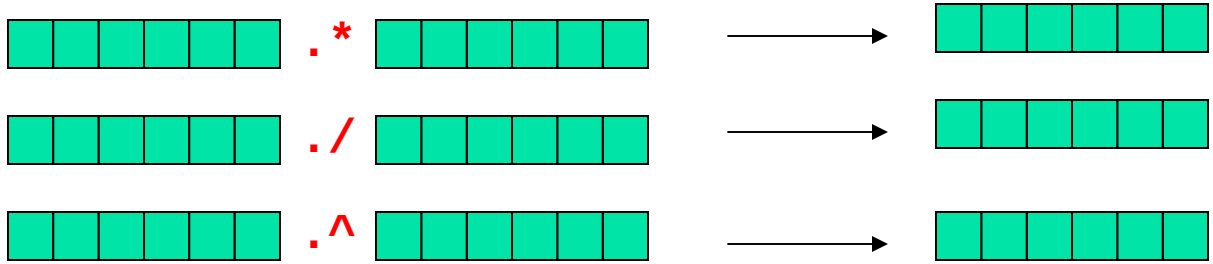


Vectorized code

element-by-element arithmetic operations on arrays

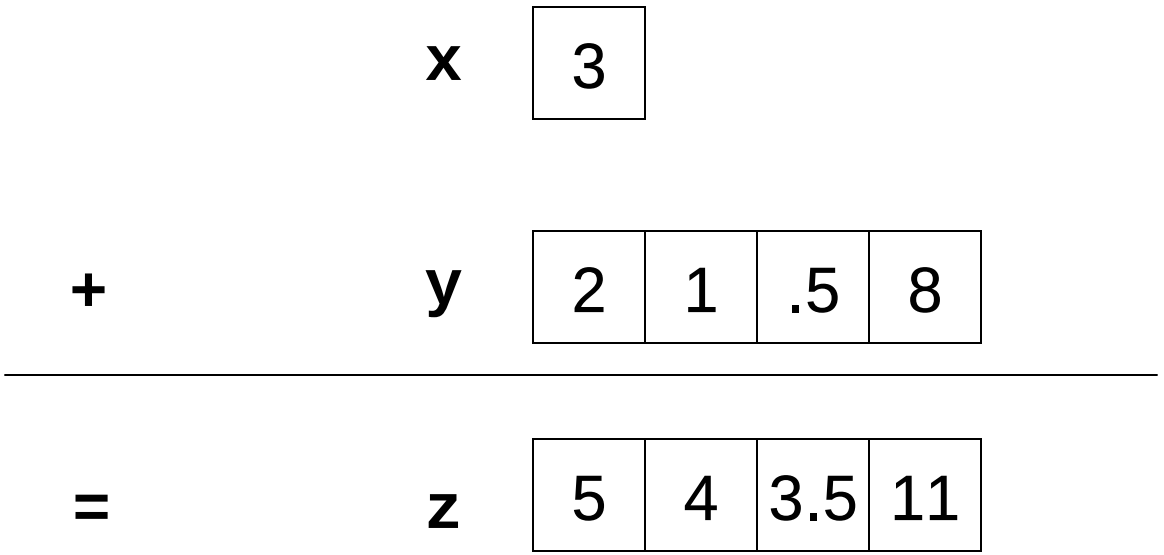
See full list of ops in §4.1





A dot (.) is necessary in front of these math operators

Shift

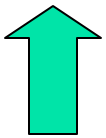


Matlab code: `z= x + y`

Reciprocate

	x	1			
/	y	2	1	.5	8
=	z	.5	1	2	.125

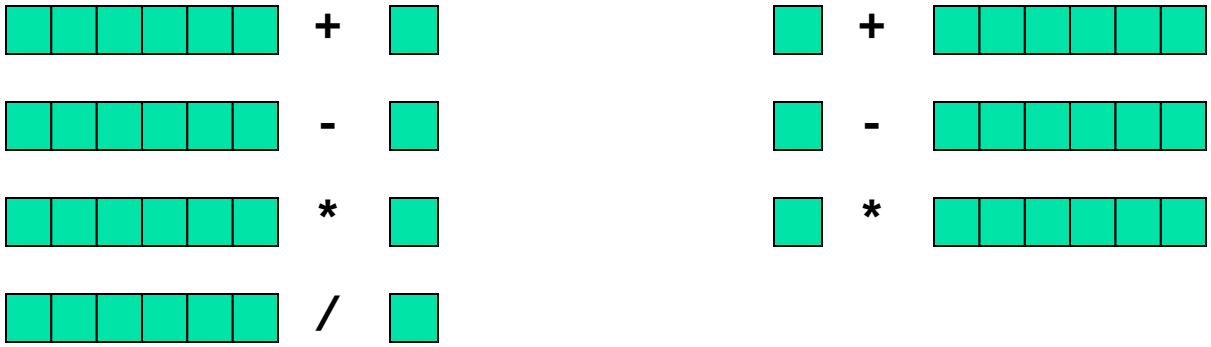
Matlab code: `z= x ./ y`



See full list of ops in §4.1

Vectorized code

element-by-element arithmetic operations between an array and a scalar

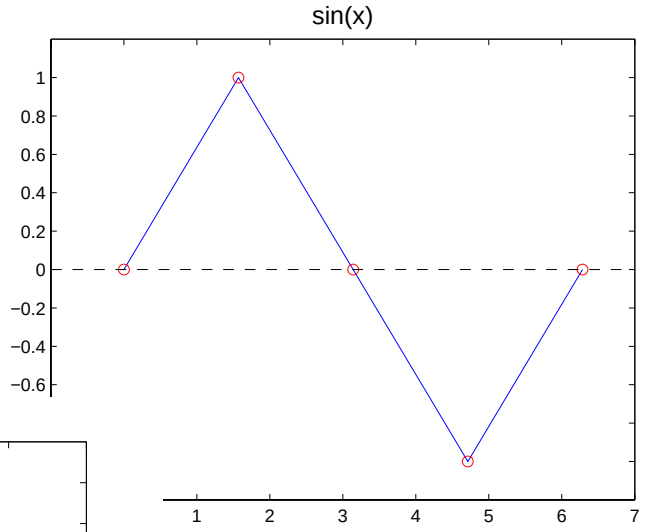


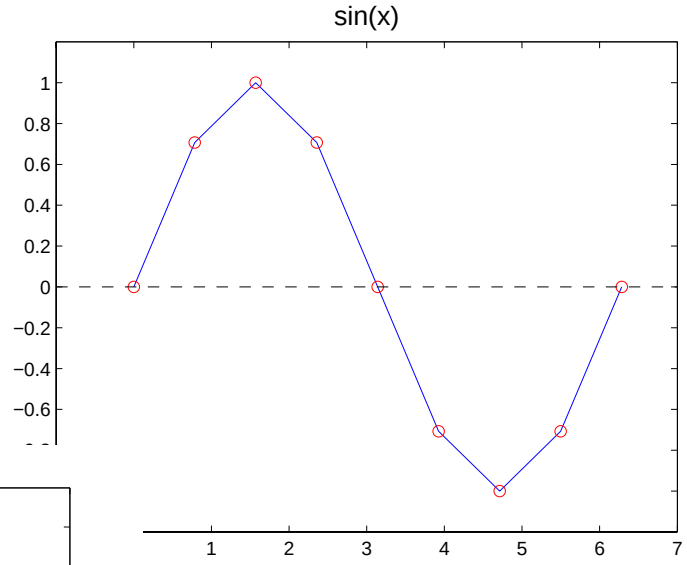
A dot (.) is necessary in front of these math operators

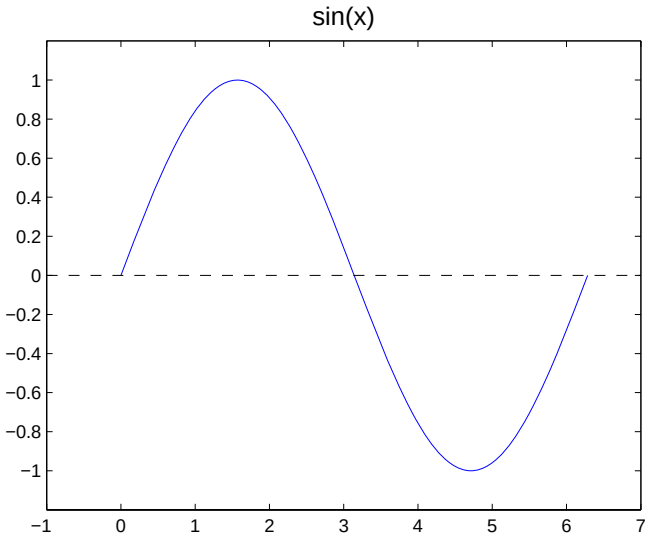
The dot in .* , .* , ./ not necessary but OK

Discrete vs. continuous

Plots are made from discrete values, but when there're many points the plot looks continuous







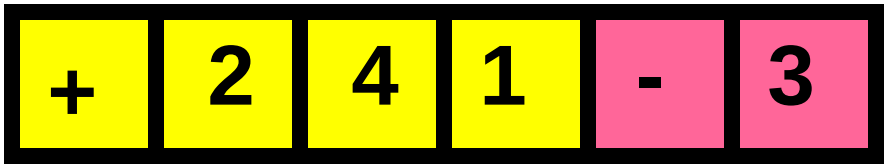
There're similar considerations with computer arithmetic

Does this script print anything?

```
k = 0;
while 1 + 1/2^k > 1
    k = k+1;
end
disp(k)
```

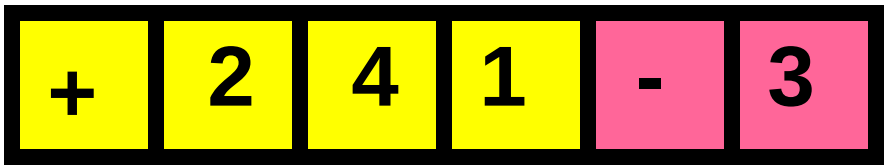
Computer Arithmetic—floating point arithmetic

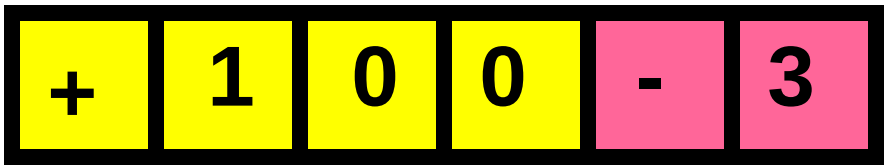
Suppose you have a calculator with a window like this:



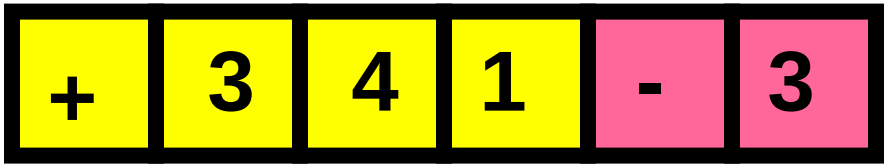
representing 2.41×10^{-3}

Floating point addition

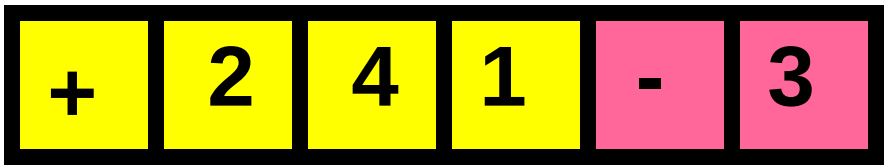


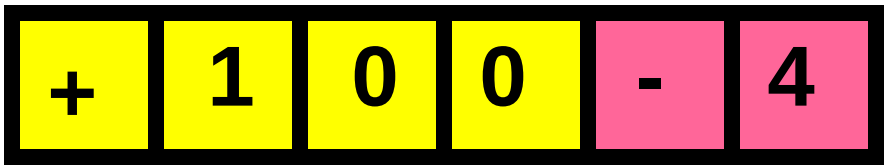


Result:

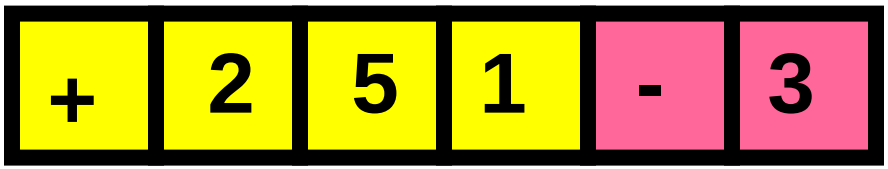


Floating point addition

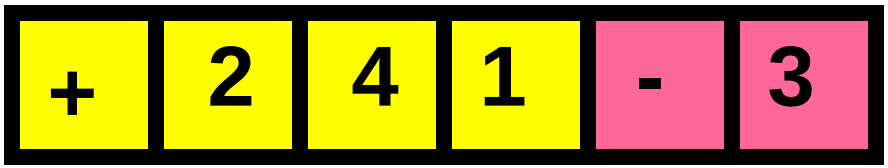


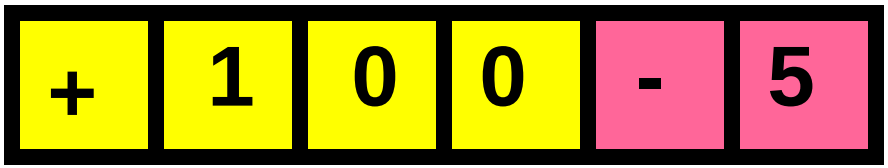


Result:

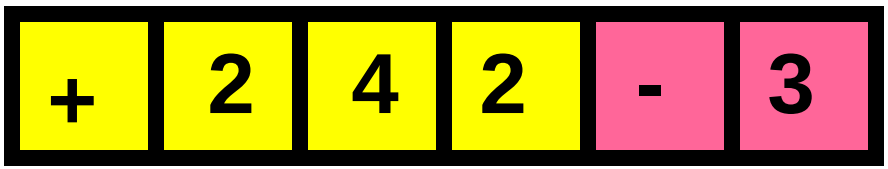


Floating point addition

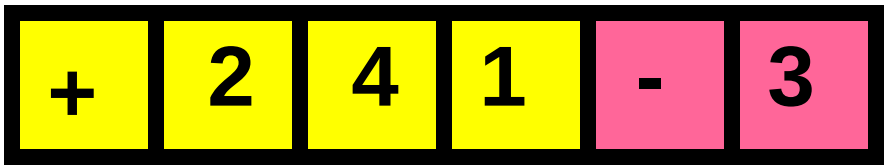


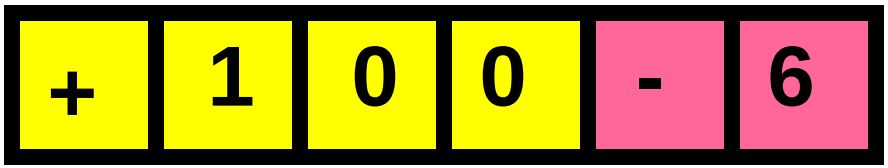


Result:

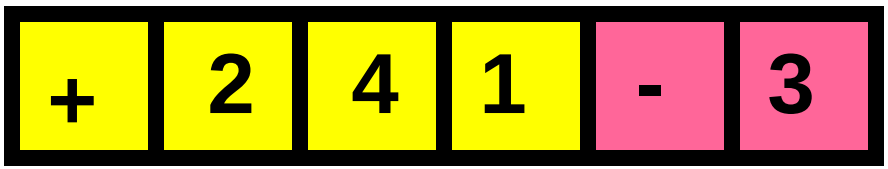


Floating point addition

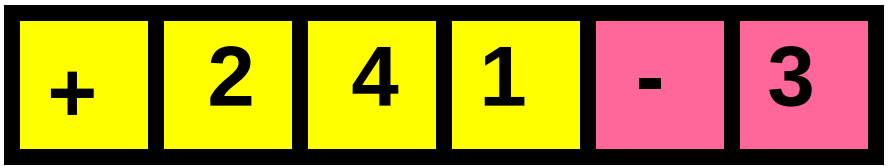


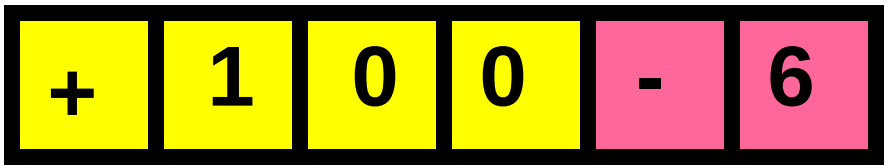


Result:

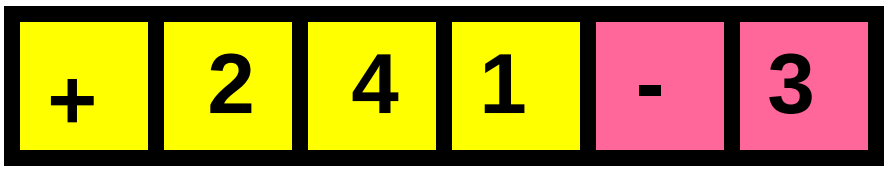


Floating point addition





Result:



Not enough room to represent .002411

The loop DOES terminate given the limitations of floating point arithmetic!

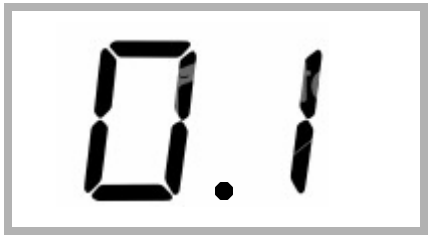
```
k = 0;
while 1 + 1/2^k > 1
    k = k+1;
end
disp(k)
```

1 + 1/2^53 is calculated to be just 1, so "53" is printed.

Patriot missile failure

www.namsa.nato.int/gallery/systems

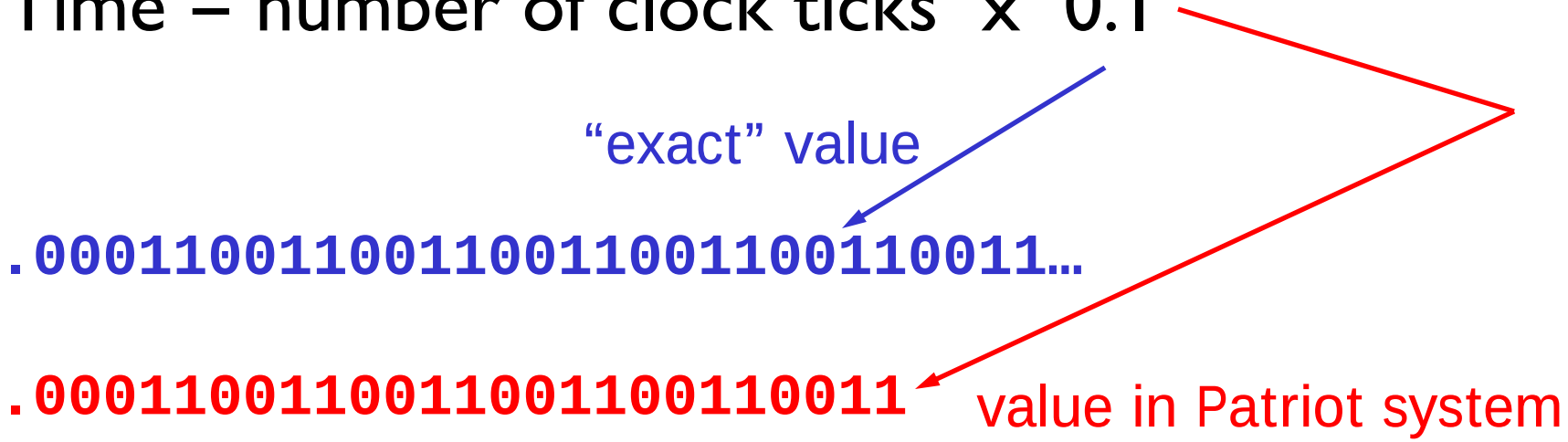
In 1991, a Patriot Missile failed, resulting in 28 deaths and about 100 injured. The cause?



Inexact representation of time/number

- System clock represented time in tenths of a second: a clock tick every 1/10 of a second

- Time = number of clock ticks x 0.1



Error of .000000095 every clock tick

Resulting error

… after 100 hours

.000000095 x (100x60x60)

0.34 second

At a velocity of 1700 m/s, missed target by more than 500 meters!

Computer arithmetic is *inexact*

- There is error in computer arithmetic—floating point arithmetic—due to limitation in "hardware." Computer memory is finite.

- What is $1 + 10^{-16}$?
  - 1.0000000000000001 in real arithmetic
  - 1 in floating point arithmetic (IEEE)

- Read Sec 4.3

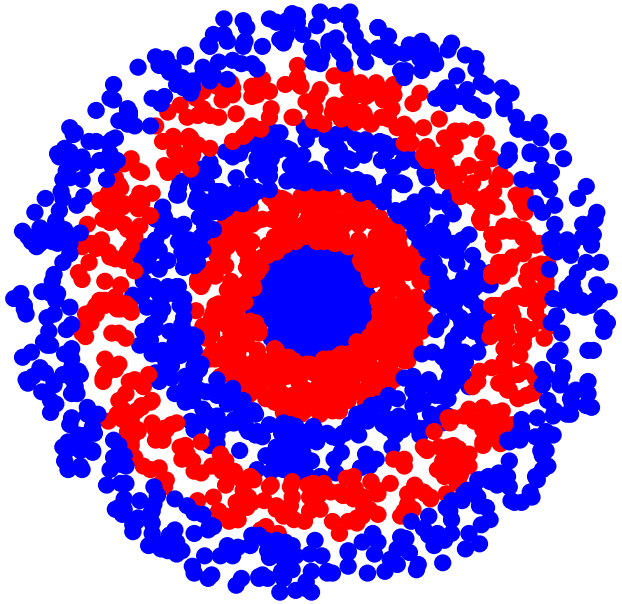
Built-in functions

- We've used many Matlab built-in functions, e.g., **`rand`**, **`abs`**, **`floor`**, **`rem`**
- Example: **`abs(x-.5)`**
- Observations:
  - **`abs`** is set up to be able to work with any valid data
  - **`abs`** *doesn't prompt us for input; it expects that we provide data* that it'll then work on

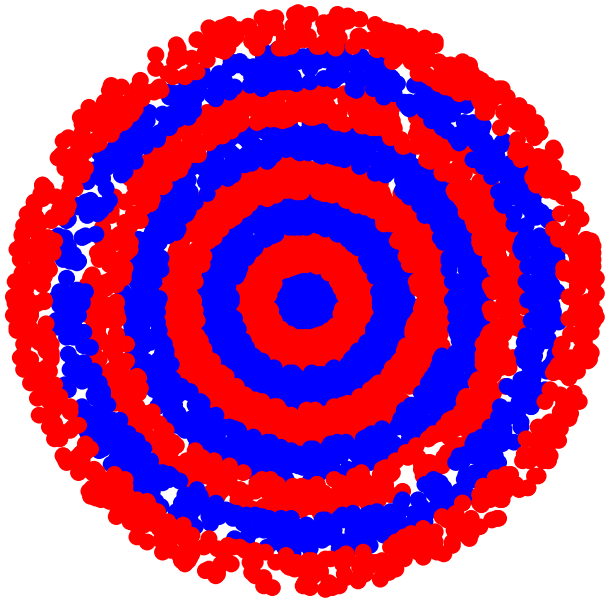
User-defined functions

- We can write our own functions to perform a specific task
  - Example: generate a random floating point number in a specified interval
  - Example: convert polar coordinates to x-y (Cartesian) coordinates

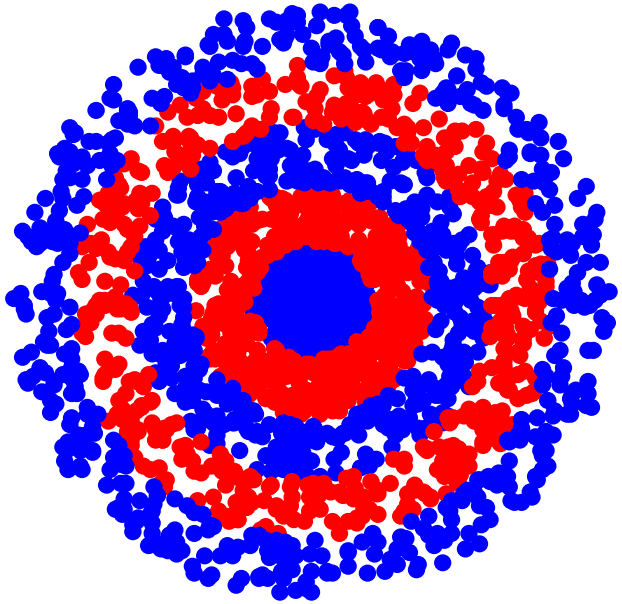
Draw a bulls eye figure with randomly placed dots



- Dots are randomly placed within concentric rings
- User decides how many rings, how many dots



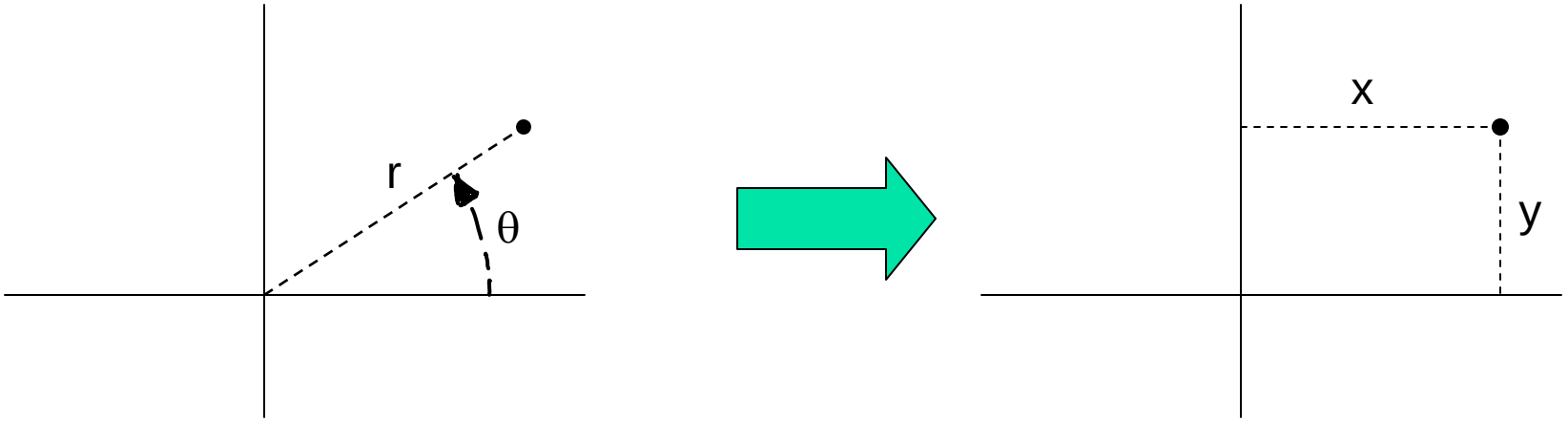
Draw a bulls eye figure with randomly placed dots



- What are the main tasks?
- Accommodate variable number of rings—loop

- For each ring
  - Need many dots
  - For each dot
    - Generate random position
    - Choose color
    - Draw it

Convert from polar to Cartesian coordinates



Polar coordinates

Cartesian coordinates

```
c= input('How many concentric rings? ');
d= input('How many dots? ');

% Put dots btwn circles with radii rRing and (rRing-1)
for rRing= 1:c
  % Draw d dots
  for count= 1:d

    % Generate random dot location (polar coord.)
    theta= _______
    r= _______

    % Convert from polar to Cartesian
    x= _______
    y= _______

    % Use plot to draw dot
  end
end
```

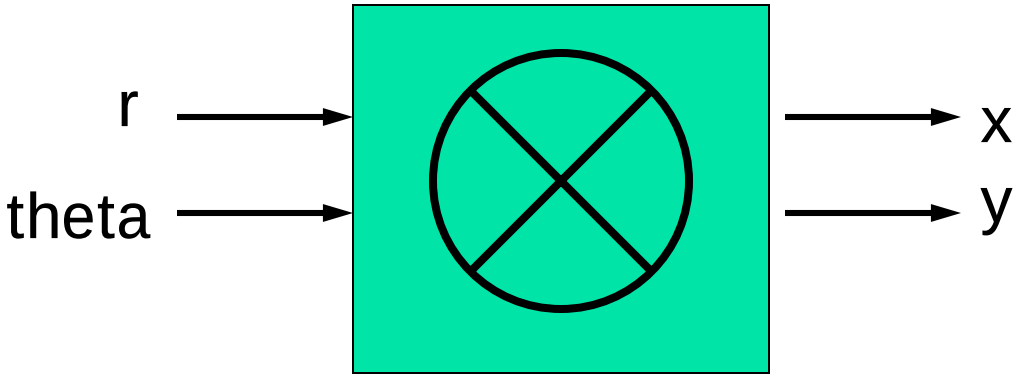
A common task! Create a function **polar2xy** to do this. **polar2xy** likely will be useful in other problems as well.

```
function [x, y] = polar2xy(r,theta)
% Convert polar coordinates (r,theta) to
% Cartesian coordinates (x,y).
% theta is in degrees.

rads= theta*pi/180;  % radian
x= r*cos(rads);
y= r*sin(rads);
```

A function file **polar2xy.m**

```
function [x, y] = polar2xy(r,theta)
% Convert polar coordinates (r,theta) to
% Cartesian coordinates (x,y).
% theta is in degrees.

rads= theta*pi/180;  % radian
x= r*cos(rads);
y= r*sin(rads);
```

Think of **`polar2xy`** as a factory



```
function [x, y] = polar2xy(r,theta)
% Convert polar coordinates (r,theta) to
% Cartesian coordinates (x,y).
% theta is in degrees.

rads= theta*pi/180;  % radian
x= r*cos(rads);
y= r*sin(rads);
```

A function file **polar2xy.m**

```
r= input('Enter radius: ');
theta= input('Enter angle in degrees: ');

rads= theta*pi/180;  % radian
x= r*cos(rads);
y= r*sin(rads);
```

(Part of) a script file

```
function [x, y] = polar2xy(r,theta)
% Convert polar coordinates (r,theta) to
% Cartesian coordinates (x,y).
% theta is in degrees.

rads= theta*pi/180;  % radian
x= r*cos(rads);
y= r*sin(rads);
```

A function file **polar2xy.m**

```
r= input('Enter radius: ');
theta= input('Enter angle in degrees: ');

rads= theta*pi/180;  % radian
x= r*cos(rads);
y= r*sin(rads);
```

(Part of) a script file

```
% Put dots btw circles with radii rRing and (rRing-1)
for rRing= 1:c
  % Draw d dots
  for count= 1:d

    % Generate random dot location
    theta= ______;
    r= ______;

    % Convert from polar to Cartesian
    rads= theta*pi/180;
    x= r*cos(rads);
    y= r*sin(rads);

    % Draw the dot



  end
end
```

```
[x,y] = polar2xy(r,theta);
```