Lecture 25

# Searching & Sorting

# Announcements for This Lecture

## Prelim 2

- **Prelim, Tuesday at 7:30**
  - See webpage for rooms
  - Make-ups are all emailed!
  - Contact Amy if no email
- **Material up to Nov. 9**
  - Recursion + Loops + Classes
  - Review **Sun Nov. 19 at 6pm**
- **Graded after Break**
  - Need time for make-ups

## Assignments

- **A6** still not graded
  - Will be done by next week
  - Staff still working on it
- **A7** is due **Monday Dec. 4**
  - Extensions are possible
  - Work on it during lab

# Linear Search

```
def linear_search(v,b):
    """Returns: first occurrence of v in b (-1 if not found)
    Precond: b a list of number, v a number
    """
    # Loop variable
    i = 0
    while i < len(b) and b[i] != v:
        i = i + 1

    if i == len(b):  # not found
        return -1
    return i
```

How many entries do we have to look at?

# Linear Search

```
def linear_search(v,b):
    """Returns: first occurrence of v in b (-1 if not found)
    Precond: b a list of number, v a number
    """
    # Loop variable
    i = 0
    while i < len(b) and b[i] != v:
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    if i == len(b):  # not found
        return -1
    return i
```

How many entries do we have to look at?

All of them!

# Linear Search

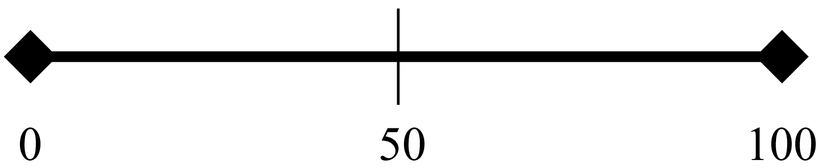
```
def linear_search(v,b):
    """Returns: last occurrence of v in b (-1 if not found)
    Precond: b a list of number, v a number
    """
    # Loop variable
    i = len(b)-1
    while i >= 0 and b[i] != v:
        i = i - 1

    # Equals -1 if not found
    return i
```

How many entries do we have to look at?

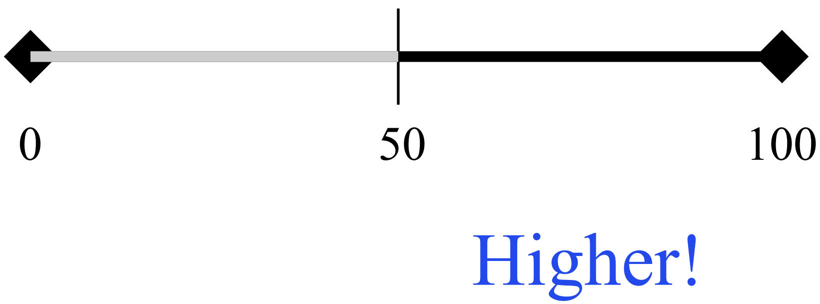
All of them!

# Is There a Better Way?




- Thinking of number 0..100
  - You get to guess number
  - I tell you higher or lower
  - Continue until get it right
- **Goal:** Keep # guesses low
  - Use my answers to help
- **Strategy?**
  - Start guess in the middle
  - Answer eliminates half
  - Go to middle of remaining

# Is There a Better Way?

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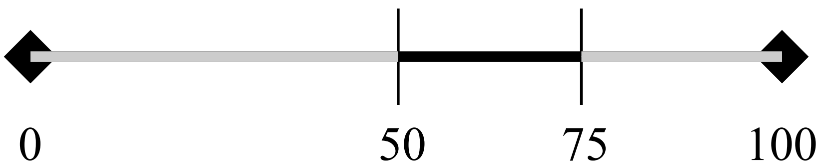


Lower!

# Is There a Better Way?

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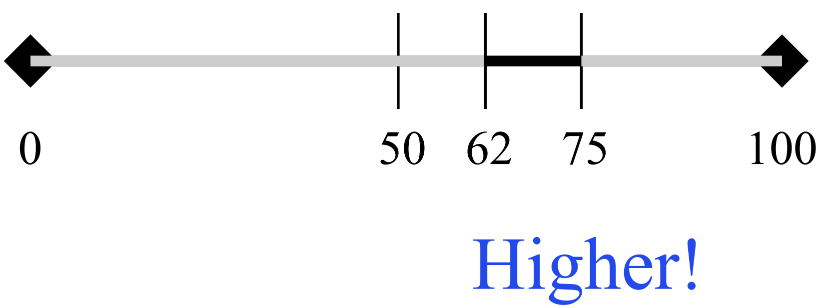

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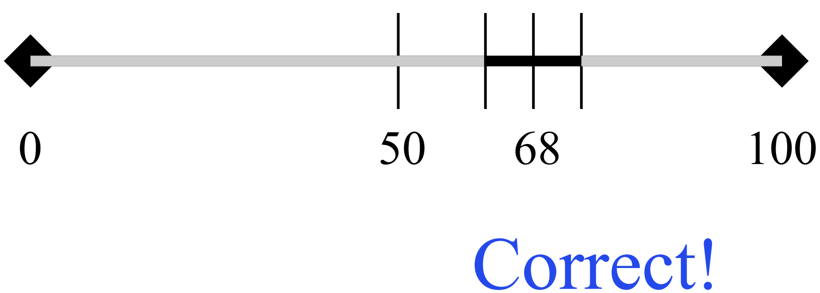


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- **Goal:** Keep # guesses low
  - Use my answers to help
- **Strategy?**
  - Start guess in the middle
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# Binary Search

```
def binary_search(v,b):
    # Loop variable(s)
    i = 0, j = len(b)
    while i < j and b[i] != v:
        mid = (i+j)//2
        if b[mid] < v:
            i = mid+1
        elif b[mid] > v:
            j = mid
        else:
            return mid
    return -1 # not found
```

Requires that the data is sorted!

But few checks!

# Observation About Sorting

- Sorting data can speed up searching
  - Sorting takes time, but do it once
  - Afterwards, can search many times
- Not just searching.  Also speeds up
  - Duplicate elimination in data sets
  - Data compression
  - Physics computations in computer games
- Why it is a major area of computer science

# The Sorting Challenge

- **Given:** A list of numbers
- **Goal:** Sort those numbers using only
  - Iteration (while-loops or for-loops)
  - Comparisons (< or >)
  - Assignment statements
- Why? For proper **analysis**.
  - Methods/functions come with hidden costs
  - Everything above has no hidden costs
  - Each comparison or assignment is "1 step"

# This Requires Some Notation

- As the list is sorted…
  - Part of the list **will** be sorted
  - Part of the list will **not** be sorted
- Need a way to refer to portions of the list
  - Notation to refer to sorted/unsorted parts
- And have to do it **without** slicing!
  - Slicing makes a **copy**
  - Want to sort original list, not a copy

# This Requires Some Notation

- As the list is sorted…
  - Part of the list **will** be sorted
  - Part of the list will **not** be sorted
- Need a w… list
  - Notatio…
- And have to do it **without** slicing!
  - Slicing makes a **copy**
  - Want to sort original list, not a copy

But we will be less formal than in previous years!

# Range Notation

- m..n is a range containing n+1-m values
  - 2..5 contains 2, 3, 4, 5. Contains 5+1 – 2 = 4 values
  - 2..4 contains 2, 3, 4. Contains 4+1 – 2 = 3 values
  - 2..3 contains 2, 3. Contains 3+1 – 2 = 2 values
  - 2..2 contains 2. Contains 2+1 – 2 = 1 values
  - 2..1 contains ???
- The notation m..n, always implies that m <= n+1
  - So you can assume that even if we do not say it
  - If m = n+1, the range has 0 values

# Range Notation

- m..n is a range containing n+1-m values
  - 2..5 contains 2, 3, 4, 5.
  - 2..4 contains 2, 3, 4.
  - 2..3 contains 2, 3.
  - 2..2 contains 2.
  - 2..1 contains ???

Co lues

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Co ues

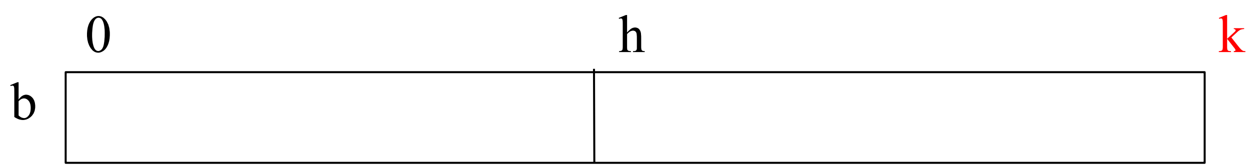
Contains 2+1 – 2 = 1 values

Not the same as range(m,n)

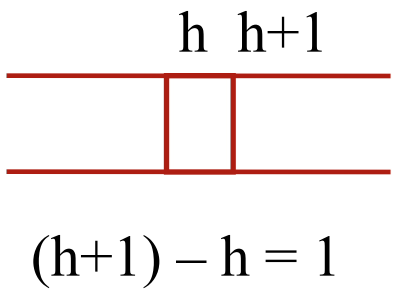
- The notation m..n, always implies that m <= n+1
  - So you can assume that even if we do not say it
  - If m = n+1, the range has 0 values

# Horizontal Notation

- Want a pictoral way to visualize this sorting
  - Represent the list as long rectangle
  - We saw this idea in divide-and-conquer

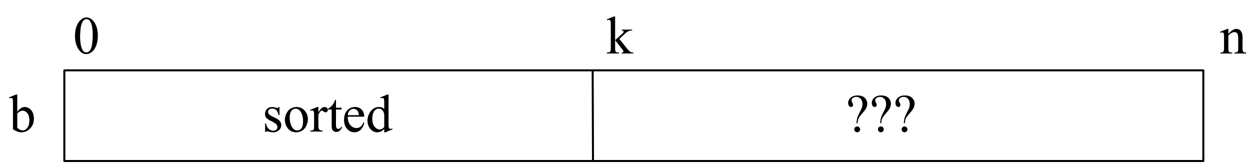



- Do **not** show individual boxes
  - Just dividing lines between regions
  - Label dividing lines with indices
  - But index is either left or right of dividing line



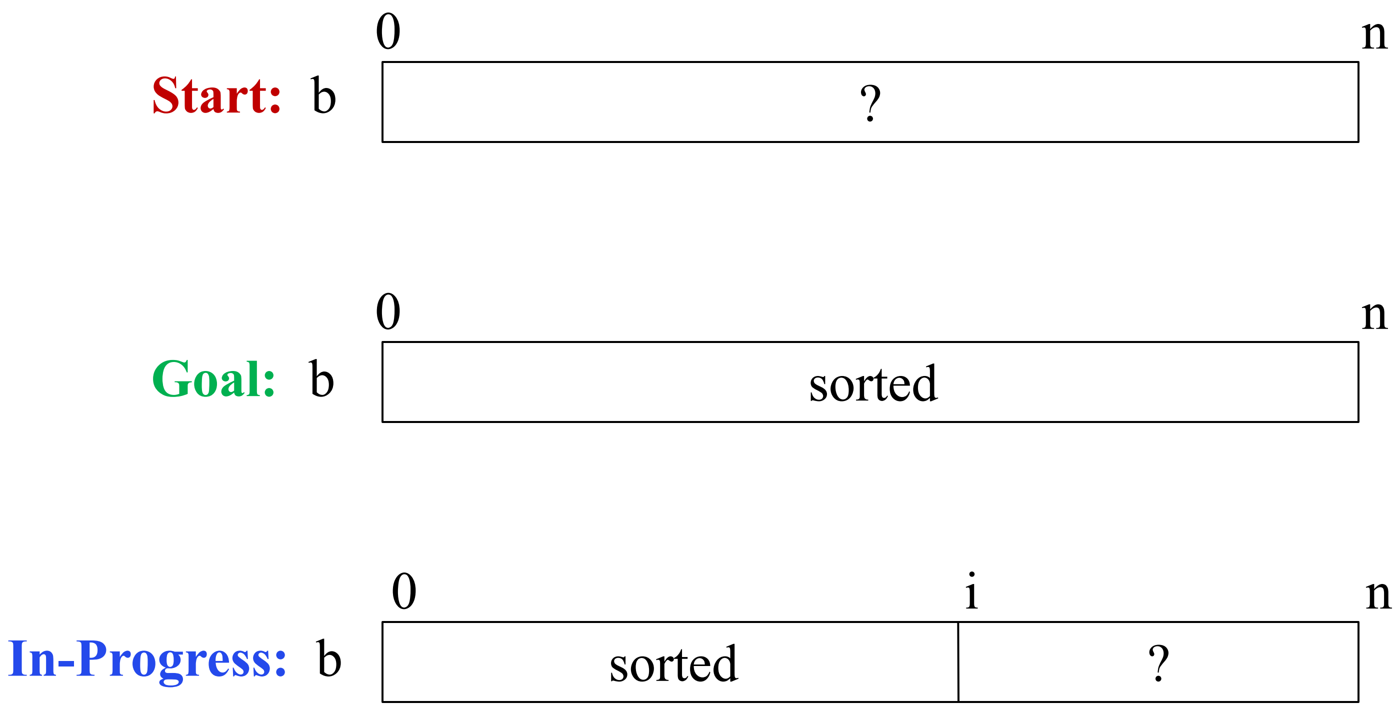
# Horizontal Notation

- Label regions with properties
  - **Example:** Sorted or ???




  - `b[0..k-1]` is sorted
  - `b[k..n-1]` **unknown** (might be sorted)
- Picture allows us to track progress

# Visualizing Sorting



# Insertion Sort

| | 0 | i | n |
|---|---|---|---|
| b | sorted | ? | |

```
i = 0
while i < n:
    # Push b[i] down into its
    # sorted position in b[0..i]
    i = i+1
```

| 0 | | | | | | i |
|---|---|---|---|---|---|---|
| 2 | 4 | 4 | 6 | 6 | 7 | 5 |

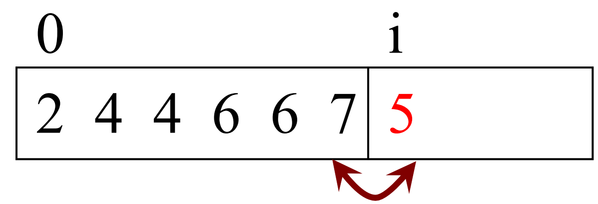
| 0 | | | | | | i |
|---|---|---|---|---|---|---|
| 2 | 4 | 4 | 5 | 6 | 6 | 7 |

**Remember the restrictions!**

# Insertion Sort: Moving into Position

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```




swap shown in the lecture about lists

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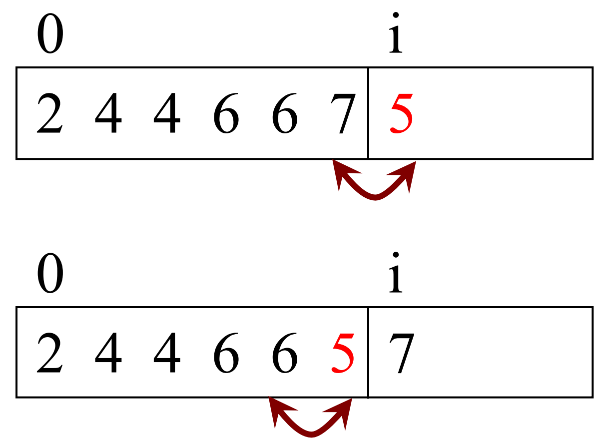


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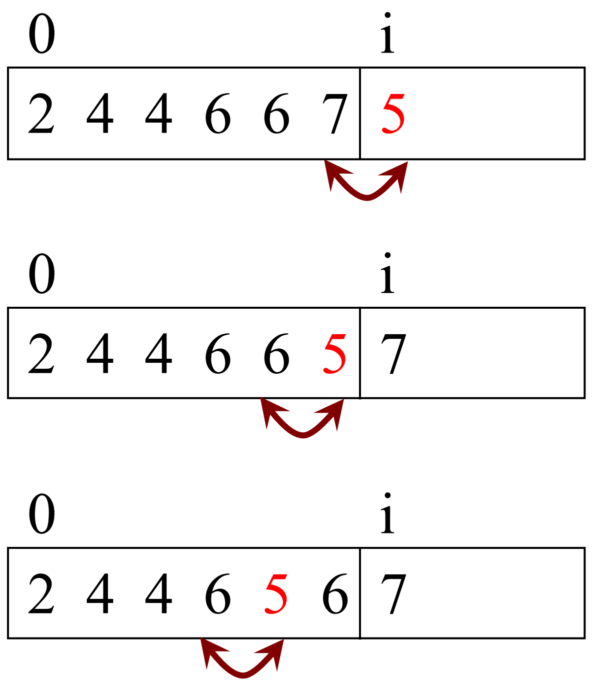

# Insertion Sort: Moving into Position

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```

swap shown in the lecture about lists

0 i
2 4 4 6 6 7 | 5

0 i
2 4 4 6 6 5 | 7

0 i
2 4 4 6 5 6 | 7

0 i
2 4 4 5 6 6 | 7

# The Importance of Helper Functions

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

VS

Can you understand all this code below?

```
i = 0
while i < n:
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            temp = b[j]
            b[j] = b[j-1]
            b[j-1] = temp
        j = j -1
    i = i +1
```

# Measuring Performance

- Performance is a tricky thing to measure
  - Different computers run at different speeds
  - Memory also has a major effect as well
- Need an independent way to measure
  - Measure in terms of “basic steps”
  - **Example:** Searching counted # of checks
- For sorting, we measure in terms of **swaps**
  - Three assignment statements
  - Present in all sorting algorithms

# Insertion Sort: Performance

```
def push_down(b, i):
    """Push value at position i into
    sorted position in b[0..i-1]"""
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

- b[0..i-1]: i elements
- Worst case:
  - i = 0: 0 swaps
  - i = 1: 1 swap
  - i = 2: 2 swaps
- Pushdown is in a loop
  - Called for i in 0..n
  - i swaps each time

**Total Swaps:** $0 + 1 + 2 + 3 + \ldots (n-1) = (n-1)*n/2 = (n^2-n)/2$

# Insertion Sort: Performance

```
def push_down(b, i):
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```

- b[0..i-1]: i elements
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- Pushdown is in a loop
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Insertion sort is an $n^2$ algorithm

**Total Swaps:** $0 + 1 + 2 + 3 + \ldots (n-1) = (n-1)*n/2 = (n^2-n)/2$

# Algorithm “Complexity”

- **Given**: a list of length n and a problem to solve
- **Complexity**: *rough* number of steps to solve worst case
- Suppose we can compute 1000 operations a second:

| Complexity | n=10 | n=100 | n=1000 |
|---|---|---|---|
| log n | 0.003 s | 0.006 s | 0.01 s |
| n | 0.01 s | 0.1 s | 1 s |
| n log n | 0.016 s | 0.32 s | 4.79 s |
| $n^2$ | 0.1 s | 10 s | 16.7 m |
| $n^3$ | 1 s | 16.7 m | 11.6 d |
| $2^n$ | 1 s | $4x10^{19}$ y | $3x10^{290}$ y |

# Algorithm "Complexity"

- **Given**: a list of length n and a problem to solve
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| Complexity | [illegible] | n=100 | n=1000 |
|---|---|---|---|
| log n | [illegible] | 0.006 s | 0.01 s |
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Binary Search

Linear Search

Insertion Sort

# Algorithm "Complexity"

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| Complexity | n=10 | n=100 | n=1000 |
|---|---|---|---|
| log n | | | 0.01 s |
| n | | | 1 s |
| n log n | | | 4.79 s |
| $n^2$ | | | 16.7 m |
| $n^3$ | 1 s | 16.7 m | 11.6 d |
| $2^n$ | 1 s | $4 \times 10^{19}$ y | $3 \times 10^{290}$ y |

**Major Topic in 2110:**
Beyond scope of this course

# Insertion Sort is Not Great

- Typically $n^2$ is okay, but not great
  - Will perform horribly on large data
  - Very bad when performance critical (games)
- We would like to do better than this
  - Can we get n swaps (**no**)?
  - How about n log n (**maybe**)
- This will require a new algorithm
  - Let's return to horizontal notation

# A New Algorithm

**Start:** b

| 0 | n |
|---|---|
| ? | |

**Goal:** b

| 0 | n |
|---|---|
| sorted | |

**In-Progress:** b

| 0 | i | n |
|---|---|---|
| sorted, $\leq$ b[i..] | $\geq$ b[0..i-1] | |

First segment always contains smaller values

# Selection Sort

| | 0 | i | n |
|---|---|---|---|
| b | sorted , ≤ b[i..] | ≥ b[0..i-1] | |

```
i = 0
while i < n:
    # Find minimum in b[i..]
    # Move it to position i
    i = i+1
```

| | | | | | i | | | | | | n |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 4 | 6 | 6 | 8 | 9 | 9 | 7 | 8 | 9 | |

| | | | | | i | | | | | | n |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 4 | 6 | 6 | 7 | 9 | 9 | 8 | 8 | 9 | |

| | | | | | | i | | | | | n |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 4 | 6 | 6 | 7 | 9 | 9 | 8 | 8 | 9 | |

**Remember the restrictions!**

# Selection Sort

How fast is this?

```
i = 0
while i < n:
    j = index of min of b[i..n-1]
    swap(b,i,j)
    i = i+1
```

| | i | n |
|---|---|---|
| 2 4 4 6 6 | 8 9 9 7 8 9 | |

| | i | n |
|---|---|---|
| 2 4 4 6 6 | 7 9 9 8 8 9 | |

| | i | n |
|---|---|---|
| 2 4 4 6 6 7 | 9 9 8 8 9 | |

# Selection Sort

**This is also $n^2$!**

```
i = 0
while i < n:
    j = index of min of b[i..n-1]
    swap(b,i,j)
    i = i+1
```

This is n steps

| | | | | | i | | | | | | n |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 4 | 6 | 6 | 8 | 9 | 9 | 7 | 8 | 9 | |

| | | | | | i | | | | | | n |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 4 | 6 | 6 | 7 | 9 | 9 | 8 | 8 | 9 | |

| | | | | | | i | | | | | n |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 4 | 6 | 6 | 7 | 9 | 9 | 8 | 8 | 9 | |

# What is the Problem?

- Both insertion, selection sort are **nested loops**
  - **Outer loop** over each element to sort
  - **Inner loop** to put next element in place
  - Each loop is n steps. $n \times n = n^2$
- To do better we must *eliminate* a loop
  - But how do we do that?
  - What is like a loop? **Recursion!**
  - Will see how to do this next lecture