Lecture 26

Advanced Sorting
Announcements for This Lecture

Assignment & Lab

• A6 is not graded yet
  ▪ Done early next week
  ▪ Survey still open today

• A7 due Mon, Dec. 5
  • Extensions are possible
  • Contact your lab instructor

• Lab Today: Office Hours
  • Get help on A7 Planetoids
  • Anyone can go to any lab

Optional Videos

• ALL all are now posted
  ▪ Lesson 30 for today
  ▪ Lesson 28 is next week
Recall Our Problem

- Both insertion, selection sort are **nested loops**
  - **Outer loop** over each element to sort
  - **Inner loop** to put next element in place
  - Each loop is $n$ steps. $n \times n = n^2$
- To do better we must **eliminate** a loop
  - But how do we do that?
  - What is like a loop? **Recursion!**
  - First need an **intermediate** algorithm
The Partition Algorithm

- Given a list segment \( b[h..k] \) with some value \( x \) in \( b[h] \):

  \[
  \begin{array}{c|c}
  \text{h} & \text{k} \\
  \hline
  \text{Start:} & b \begin{array}{c|c|c|c|c|c|c|c|c}
  \hline
  x & ? \\
  \end{array} \\
  \end{array}
  \]

- Swap elements of \( b[h..k] \) to get this answer

  \[
  \begin{array}{c|c|c|c|c}
  \text{h} & \text{i} & \text{i+1} & \text{k} \\
  \hline
  \text{Goal:} & b \begin{array}{c|c|c|c|c|c|c|c|c}
  \hline
  \leq x & x & \geq x \\
  \end{array} \\
  \end{array}
  \]

  \[
  \begin{array}{c|c|c|c|c|c|c|c|c}
  \hline
  \text{change:} & b \begin{array}{c|c|c|c|c|c|c|c|c}
  \hline
  3 & 5 & 4 & 1 & 6 & 2 & 3 & 8 & 1 \\
  \end{array} \\
  \end{array}
  \]

  \[
  \begin{array}{c|c|c|c|c|c|c|c|c}
  \hline
  \text{into} & b \begin{array}{c|c|c|c|c|c|c|c|c}
  \hline
  1 & 2 & 1 & 3 & 5 & 4 & 6 & 3 & 8 \\
  \end{array} \\
  \end{array}
  \]

  or

  \[
  \begin{array}{c|c|c|c|c|c|c|c|c}
  \hline
  \text{or} & b \begin{array}{c|c|c|c|c|c|c|c|c}
  \hline
  1 & 2 & 3 & 1 & 3 & 4 & 5 & 6 & 8 \\
  \end{array} \\
  \end{array}
  \]

- \( x \) is called the **pivot value**
  - \( x \) is not a program variable
  - \( x \) denotes value initially in \( b[h] \)
Designing the Partition Algorithm

- Given a list \( b[h..k] \) with some value \( x \) in \( b[h] \):

  \[
  \begin{array}{c|c|c}
  h & b & k \\
  \hline
  \text{Start:} & b & x \\
  \text{Goal:} & b & \leq x & x & \geq x \\
  \text{In-Progress:} & b & \leq x & x & ? & \geq x
  \end{array}
  \]

- Swap elements of \( b[h..k] \) to get this answer

Indices \( b, h \) important!
Might partition only part
def partition(b, h, k):
    """Partition list b[h..k] around a pivot x = b[h]"""
    i = h; j = k+1; x = b[h]
    while i < j:
        if b[i+1] >= x:
            # Move to end of block.
            swap(b,i+1,j-1)
            j = j - 1
        else:  # b[i+1] < x
            swap(b,i,i+1)
            i = i + 1
    return i

partition(b,h,k), not partition(b[h:k+1])
Remember, slicing always copies the list!
We want to partition the original list
def partition(b, h, k):
    """Partition list b[h..k] around a pivot x = b[h]"""
    i = h; j = k+1; x = b[h]

    while i < j-1:
        if b[i+1] >= x:
            # Move to end of block.
            swap(b, i+1, j-1)
            j = j - 1
        else:
            # b[i+1] < x
            swap(b, i, i+1)
            i = i + 1

    return i
def partition(b, h, k):
    '''Partition list b[h..k] around a pivot x = b[h]'''
    i = h; j = k+1; x = b[h]

    while i < j-1:
        if b[i+1] >= x:
            # Move to end of block.
            swap(b, i+1, j-1)
            j = j - 1
        else:
            # b[i+1] < x
            swap(b, i, i+1)
            i = i + 1

    return i
def partition(b, h, k):
    
    '''Partition list \[b[h..k]\] around a pivot \(x = b[h]\)'''
    i = h; j = k+1; x = b[h]
    
    while i < j-1:
        if b[i+1] >= x:
            # Move to end of block.
            swap(b,i+1,j-1)
            j = j - 1
        else:  # b[i+1] < x
            swap(b,i,i+1)
            i = i + 1
    
    return i

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def partition(b, h, k):
    """Partition list b[h..k] around a pivot x = b[h]""
    i = h; j = k + 1; x = b[h]

    while i < j - 1:
        if b[i + 1] >= x:
            # Move to end of block.
            swap(b, i + 1, j - 1)
            j = j - 1
        else:
            # b[i+1] < x
            swap(b, i, i + 1)
            i = i + 1

    return i
Why is this Useful?

• Will use this algorithm to replace inner loop
  ▪ The inner loop cost us n swaps every time

• Can this reduce the number of swaps?
  ▪ Worst case is $k-h$ swaps
  ▪ This is $n$ if partitioning the whole list
  ▪ But less if only partitioning part

• **Idea:** Break up list and partition only part?
  ▪ This is **Divide-and-Conquer**!
Sorting with Partitions

• Given a list segment \( b[h..k] \) with some value \( x \) in \( b[h] \):

\[
\begin{array}{c|c|c}
\text{h} & x & \text{k} \\
\hline
\text{Start:} & b & ? \\
\end{array}
\]

• Swap elements of \( b[h..k] \) to get this answer

\[
\begin{array}{c|c|c|c}
\text{h} & \text{i} & \text{i+1} & \text{k} \\
\hline
\text{Goal:} & b & \leq x & x & \geq x \\
\end{array}
\]

Partition Recursively

Recursive partitions = sorting
- Called \textbf{QuickSort} (why???)
- Popular, fast sorting technique
Sorting with Partitions

- Given a list segment $b[h..k]$ with some value $x$ in $b[h]$:

  - Start: $b$ with $x$?

- Swap elements of $b[h..k]$ to get this answer

  - Goal: $b$ with $y$? $x$ $>= x$

Partition Recursively

Recursive partitions = sorting
- Called **QuickSort** (why???)
- Popular, fast sorting technique
Sorting with Partitions

• Given a list segment \( b[h..k] \) with some value \( x \) in \( b[h] \):

Start: \( b \)

\[
\begin{array}{c|c|c|c|c}
\hline
h & x & ? & \vline & k \\
\hline
\end{array}
\]

• Swap elements of \( b[h..k] \) to get this answer

Goal: \( b \)

\[
\begin{array}{c|c|c|c|c|c}
\hline
\leq y & y & \geq y & x & \geq x & k \\
\hline
\end{array}
\]

Partition Recursively

Recursive partitions = sorting

- Called QuickSort (why???)
- Popular, fast sorting technique
def quick_sort(b, h, k):
    """Sort the array fragment b[h..k]"""
    if b[h..k] has fewer than 2 elements:
        return
    j = partition(b, h, k)
    # b[h..j-1] <= b[j] <= b[j+1..k]
    # Sort b[h..j-1] and b[j+1..k]
    quick_sort(b, h, j-1)
    quick_sort(b, j+1, k)

- **Worst Case:**
  - array already sorted
  - Or almost sorted
  - $n^2$ in that case
- **Average Case:**
  - array is scrambled
  - $n \log n$ in that case
  - Best sorting time!

<table>
<thead>
<tr>
<th>h</th>
<th>i</th>
<th>i+1</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&lt;= x</td>
<td>x</td>
<td>&gt;= x</td>
<td></td>
</tr>
</tbody>
</table>
So Does that Solve It?

- Worst case still seems bad! Still $n^2$
  - But only happens in small number of cases
  - Just happens that case is common (already sorted)
- Can greatly reduce issue with randomization
  - Swap start with random element in list
  - Now pivot is random and already sorted unlikely

<table>
<thead>
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<td>?</td>
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Start: b

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So Does that Solve It?

- Worst case still seems bad! Still $n^2$
  - But only happens in small number of cases
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Makes it “good enough” for most applications
Can We Do Better?

- Recursion seems to be the solution
  - Partitioned the list into two halves
  - Recursively sorted each half
- How about a traditional divide-and-conquer?
  - *Divide* the list into two halves
  - *Recursively sort* the two halves
  - *Combine* the two sort halves
- How do we do the last step?
Combining Two Sorted Lists

1  4  5  6  11  12

2  3  7  8  9  10
Combining Two Sorted Lists

Pick from list with the least
Combining Two Sorted Lists

Pick from list with the least
Combining Two Sorted Lists

Pick from list with the least

List 1:
1
2
3

List 2:
3
7
8
9
10

List 3:
4
5
6
11
12
Combining Two Sorted Lists

Pick from list with the least

1 2 3 4
7 8 9 10
4 5 6 11 12
Combining Two Sorted Lists

Pick from list with the least
Combining Two Sorted Lists

Pick from list with the least
Combining Two Sorted Lists

Pick from list with the least
Combining Two Sorted Lists

Pick from list with the least
Combining Two Sorted Lists

Pick from list with the least
Combining Two Sorted Lists

Pick from list with the least
Combining Two Sorted Lists

Finish off remaining list
Combining Two Sorted Lists

Finish off remaining list

1
2
3
4
5
6
7
8
9
10
11
12
Combining Two Sorted Lists

Does this look familiar?
def merge_sort(b, h, k):
    """Sort the array fragment b[h..k]"""
    if b[h..k] has fewer than 2 elements:
        return
    # Divide and recurse
    mid = (h+k)//2
    merge_sort (b, h, m)
    merge_sort (b, m+1, k)
    # Combine
    merge(b,h,mid,k) # Merge halves into b

• Seems simpler than qsort
  ▪ Straight-forward d&c
  ▪ Merge easy to implement
• What is the catch?
  ▪ Merge requires a copy
  ▪ We did not allow copies
  ▪ Copying takes n steps
  ▪ But so does merge/partition
• n log n ALWAYS
def merge_sort(b, h, k):
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    merge(b,h,mid,k) # Merge halves into b

• Seems simpler than qsort
  ▪ Straight-forward d&c
  ▪ Merge easy to implement

• What is the catch?
  ▪ Merge requires a copy
  ▪ We did not allow copies
  ▪ Copying takes O(n) time
  ▪ But so does merge/partition

• O(n log n) ALWAYS

Proof beyond scope of course
What Does Python Use?

- The `sort()` method is **Timsort**
  - Invented by Tim Peters in 2002
  - Combination of insertion sort and merge sort
- Why a combination of the two?
  - Merge sort requires copies of the data
  - Copying pays off for large lists, but not small lists
  - Insertion sort is not that slow on small lists
  - Balancing two properly still gives $n \log n$
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Most of time spent here