

Linear Search

```
def linear_search(v,b):
    """Returns: first occurrence of v in b (-1 if not found)
    Precond: b a list of number, v a number
    """
    # Loop variable
    i = 0
    while i < len(b) and b[i] != v:
        i = i + 1
    if i == len(b): # not found
        return -1
    return i
```

How many entries do we have to look at?

All of them!

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Binary Search

```
def binary_search(v,b):
    # Loop variable(s)
    i = 0, j = len(b)
    while i < j and b[i] != v:
        mid = (i+j)//2
        if b[mid] < v:
            j = mid
        elif b[mid] > v:
            i = mid
        else:
            return mid
    return -1 # not found
```

Requires that the data is sorted!

But few checks!

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The Sorting Challenge

- **Given:** A list of numbers
- **Goal:** Sort those numbers using only
 - Iteration (while-loops or for-loops)
 - Comparisons (< or >)
 - Assignment statements
- **Why?** For proper **analysis**.
 - Methods/functions come with hidden costs
 - Everything above has no hidden costs
 - Each comparison or assignment is “1 step”

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Horizontal Notation

- Want a pictorial way to visualize this sorting
 - Represent the list as long rectangle
 - We saw this idea in divide-and-conquer

- **Do not** show individual boxes
 - Just dividing lines between regions
 - Label dividing lines with indices
 - But index is either left or right of dividing line

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Visualizing Sorting

Start: b ?

Goal: b sorted

In-Progress: b sorted ?

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Insertion Sort

```
i = 0
while i < n:
    # Push b[i] down into its
    # sorted position in b[0..i]
    i = i+1
```

Remember the restrictions!

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Insertion Sort: Moving into Position

```

i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
    
```

0 i

2 4 4 6 6 7 | 5

0 i

2 4 4 6 6 5 | 7

0 i

2 4 4 6 5 6 | 7

0 i

2 4 4 5 6 6 | 7

swap shown in the
lecture about lists

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Insertion Sort: Performance

```

def push_down(b, i):
    """Push value at position i into
    sorted position in b[0..i-1]"""
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
    
```

- $b[0..i-1]$: i elements
- Worst case:
 - $i = 0$: 0 swaps
 - $i = 1$: 1 swap
 - $i = 2$: 2 swaps
- Pushdown is in a loop
 - Called for i in $0..n$
 - i swaps each time

Insertion sort is an n^2 algorithm

Total Swaps: $0 + 1 + 2 + 3 + \dots + (n-1) = (n-1)*n/2 = (n^2-n)/2$

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Algorithm "Complexity"

- **Given:** a list of length n and a problem to solve
- **Complexity:** rough number of steps to solve worst case
- Suppose we can compute 1000 operations a second:

Complexity	n=10	n=100	n=1000
$\log n$	0.003 s	0.006 s	0.01 s
n	0.01 s	0.1 s	1 s
$n \log n$	0.016 s	0.32 s	4.79 s
n^2	0.1 s	10 s	16.7 m
n^3	1 s	16.7 m	11.6 d
2^n	1 s	4×10^{19} y	3×10^{290} y

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A New Algorithm

Start: b

0 ? n

Goal: b

0 sorted n

In-Progress: b

0 sorted, $\leq b[i..]$ $\geq b[0..i-1]$ n

First segment always contains smaller values

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Selection Sort

0 i n

b [sorted, $\leq b[i..]$] [$\geq b[0..i-1]$]

i

2 4 4 6 6 | 8 9 9 7 8 9

i

2 4 4 6 6 | 7 9 9 8 8 9

i

2 4 4 6 6 7 | 9 9 8 8 9

```

i = 0
while i < n:
    # Find minimum in b[i..]
    # Move it to position i
    i = i+1
    
```

Remember the restrictions!

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What is the Problem?

- Both insertion, selection sort are **nested loops**
 - **Outer loop** over each element to sort
 - **Inner loop** to put next element in place
 - Each loop is n steps. $n \times n = n^2$
- To do better we must *eliminate* a loop
 - But how do we do that?
 - What is like a loop? **Recursion!**
 - Will see how to do this next lecture

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