http://www.cs.cornell.edu/courses/cs1110/2020sp

Lecture 18:
More Recursion!

CS 1110

Introduction to Computing Using Python

[E. Andersen, A. Bracy, D. Fan, D. Gries, L. Lee,
S. Marschner, C. Van Loan, W. White]

Recursion

Recursive Function:

A function that calls itself (directly or indirectly)

Recursive Definition:

A definition that is defined in terms of itself

From previous lecture: Factorial

Non-recursive definition:

$$n! = n \times n-1 \times \ldots \times 2 \times 1$$

$$= n\,(n-1 \times \ldots \times 2 \times 1)$$

Recursive definition:

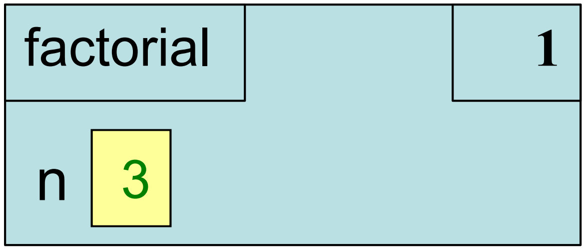
$n! = n\,(n-1)!$ for $n > 0$ **Recursive case**

$0! = 1$ **Base case**

Recursive Call Frames

```
  def factorial(n):
      """Returns: factorial of n.
      Precondition: n ≥ 0 an int"""
1     if n == 0:
2         return 1

3     return n*factorial(n-1)
```



factorial(3)

Recursive Call Frames

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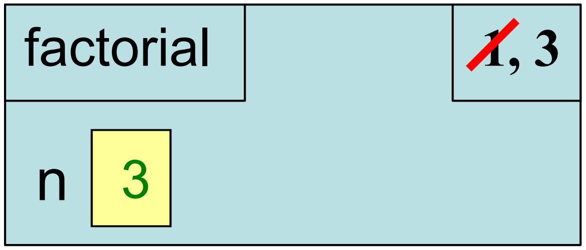

factorial(3)

Recursion

```
def factorial(n):
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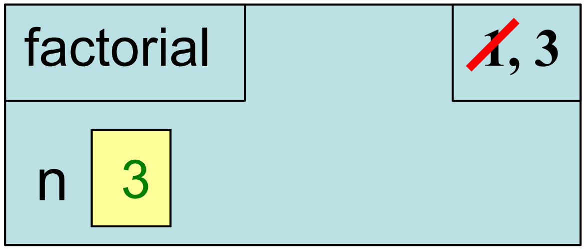


Now what?
Each call is a new frame.

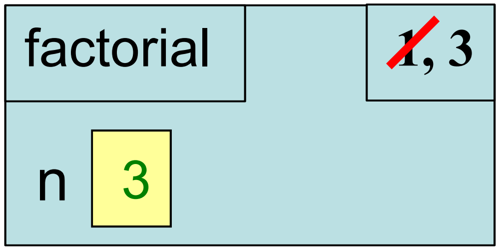
`factorial(3)`

What happens next? (Q)

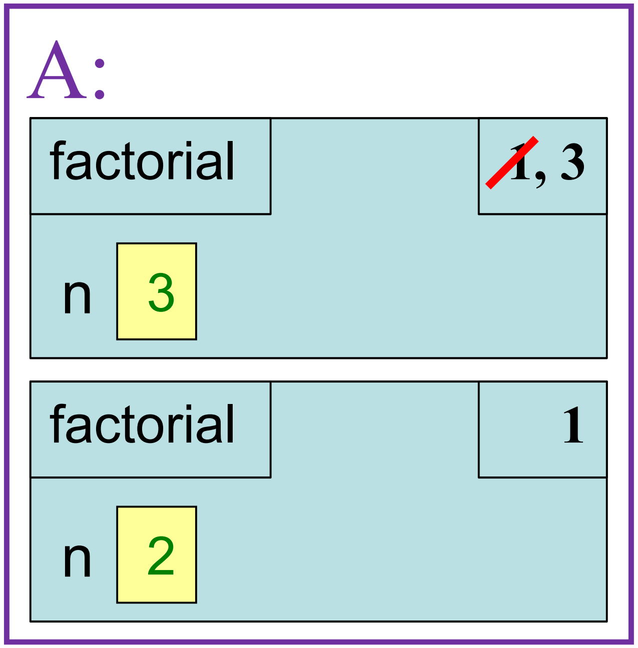
```
def factorial(n):
    """Returns: factorial of n.
    Pre: n ≥ 0 an int"""
    if n == 0:
        return 1

    return n*factorial(n-1)
```

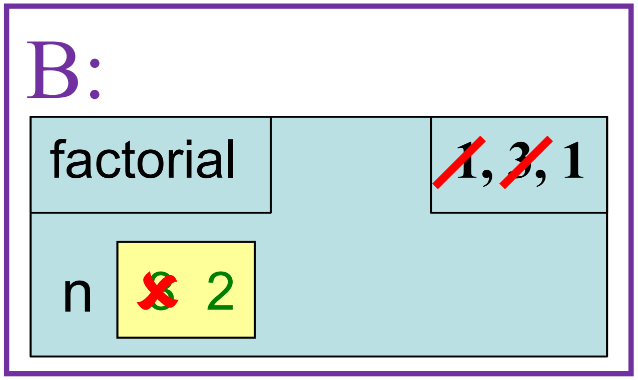
Call: factorial(3)



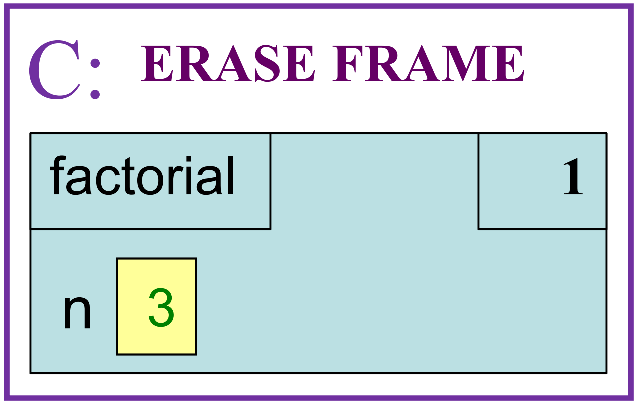
A:



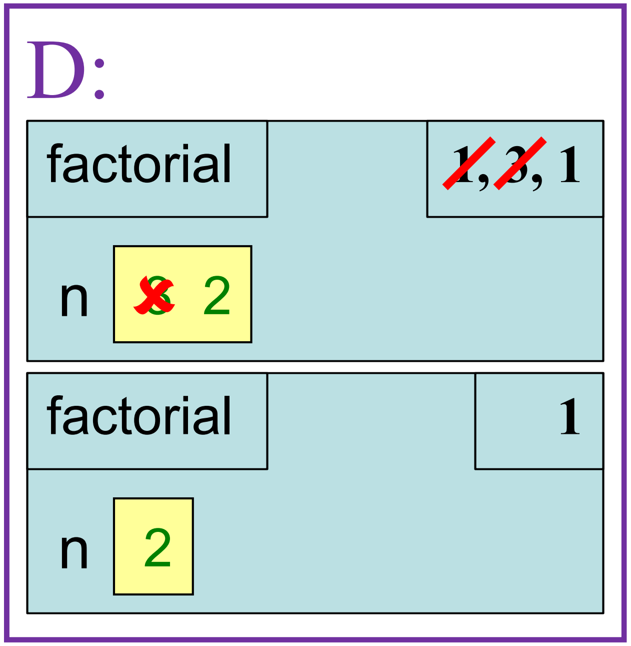
B:



C: **ERASE FRAME**



D:



Recursive Call Frames

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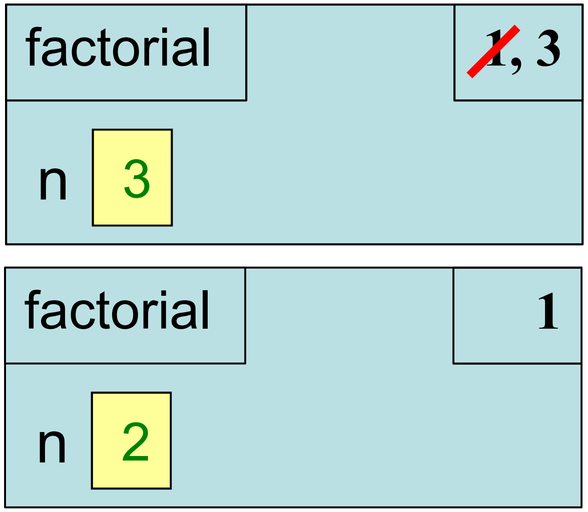
factorial(3)



Recursive Call Frames

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```

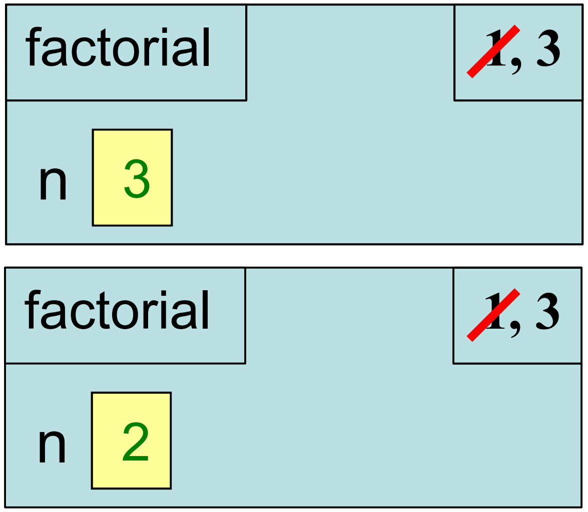

factorial(3)

Recursive Call Frames

```
  def factorial(n):
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```
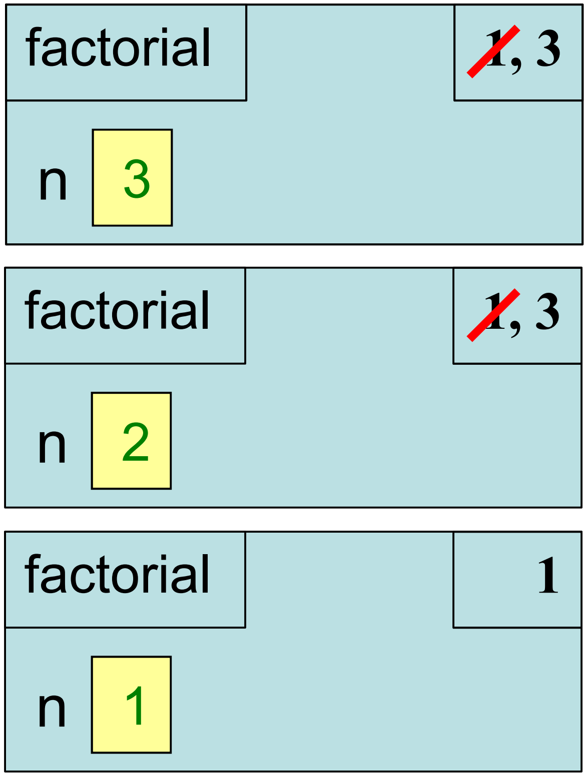

```
factorial(3)
```



factorial ~~1~~, 3

n 3

factorial ~~1~~, 3

n 2

factorial 1

n 1

Recursive Call Frames

```
def factorial(n):
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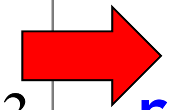
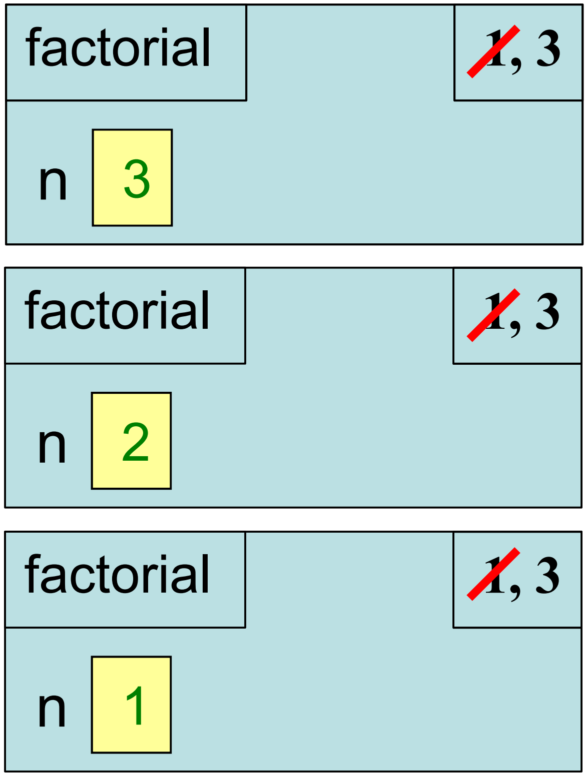


factorial(3)



factorial ~~1~~, 3

n 3

factorial ~~1~~, 3

n 2

factorial ~~1~~, 3

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Recursive Call Frames

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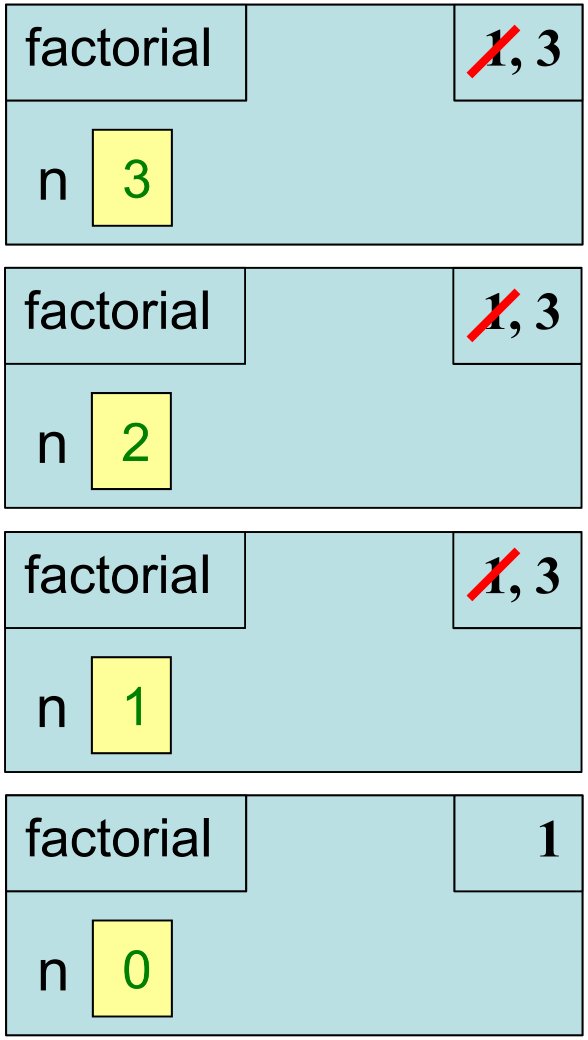
```
factorial(3)
```



Recursive Call Frames

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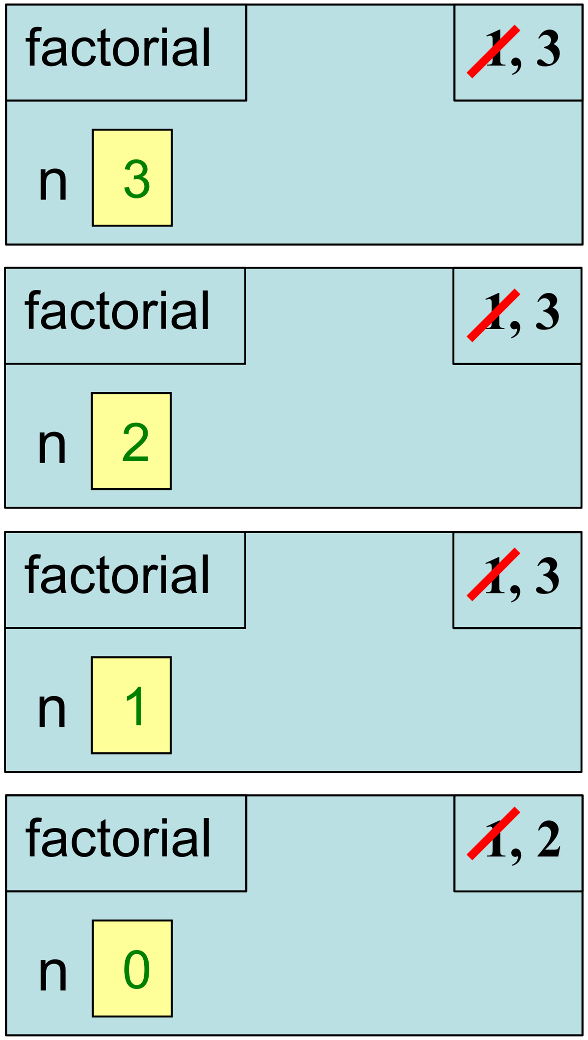
factorial(3)



factorial ~~1~~, 3
n 3

factorial ~~1~~, 3
n 2

factorial ~~1~~, 3
n 1

factorial ~~1~~, 2
n 0

Recursive Call Frames

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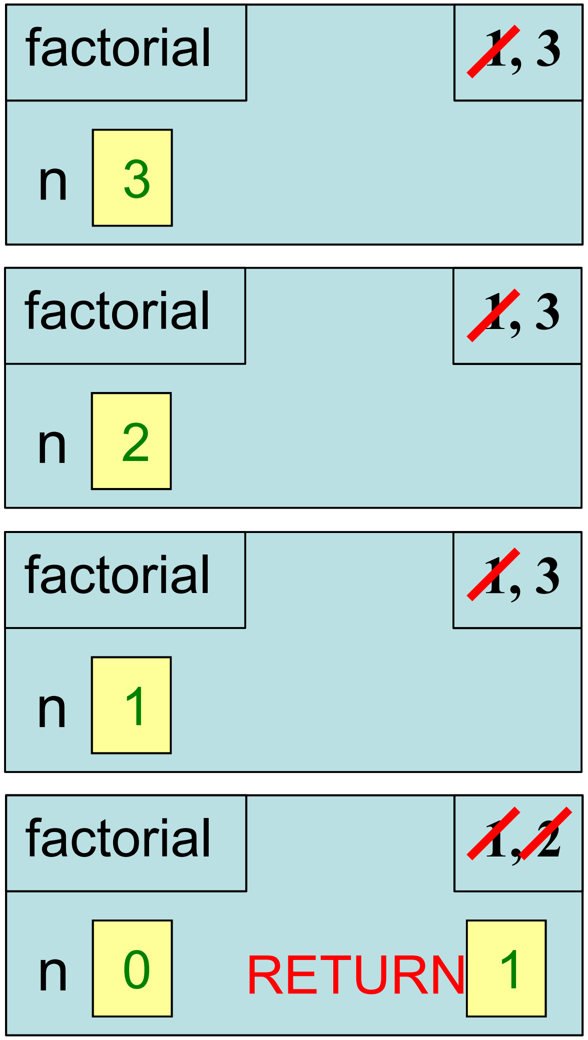
factorial(3)



Recursive Call Frames

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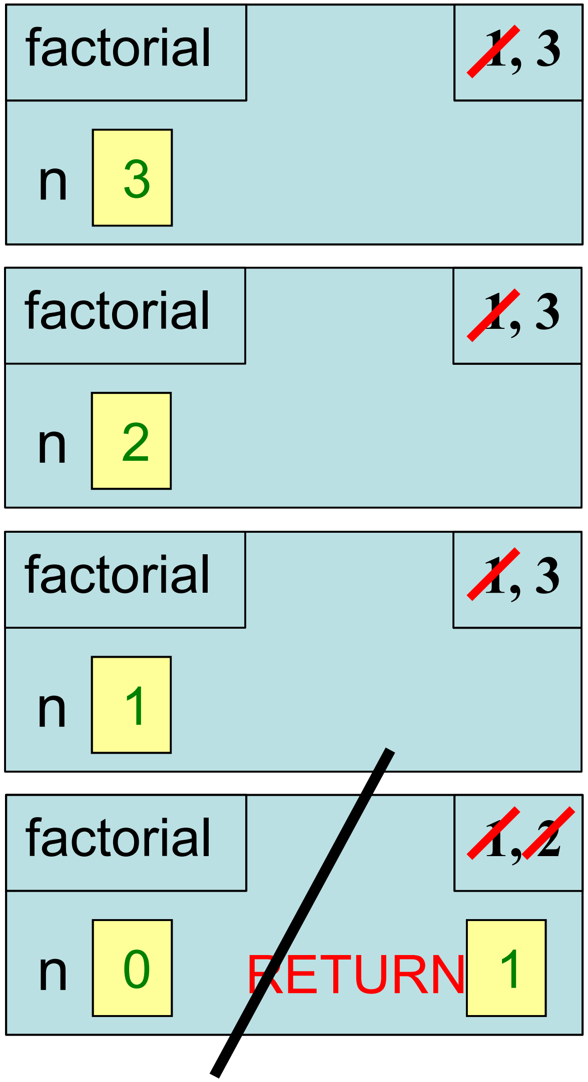

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def factorial(n):
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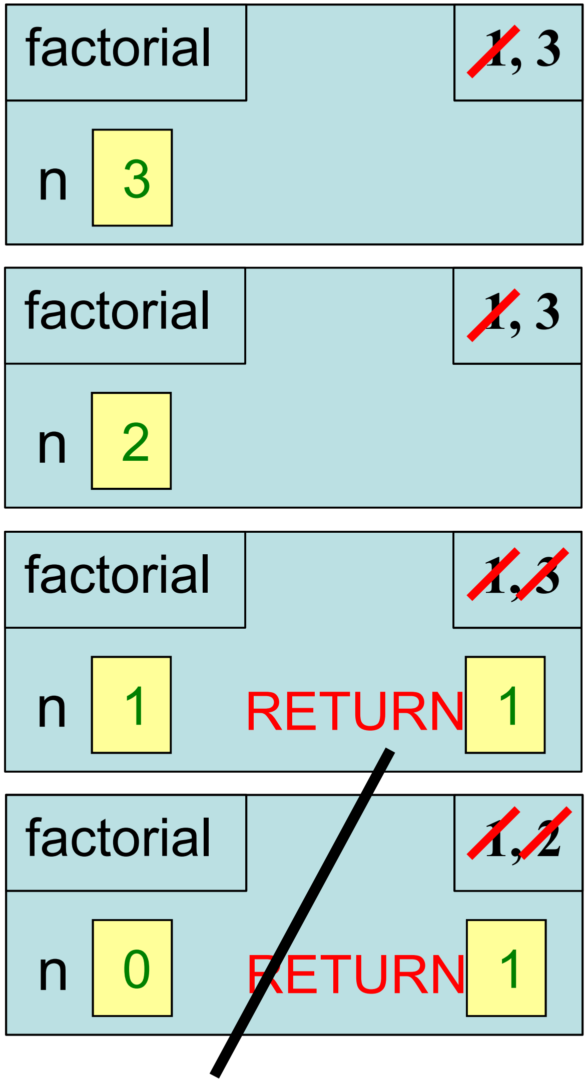

factorial ~~1~~, 3

n 3

factorial ~~1~~, 3

n 2

factorial ~~1~~, ~~3~~

n 1 RETURN 1

factorial ~~1~~, ~~2~~

n 0 RETURN 1

Recursive Call Frames

```
  def factorial(n):
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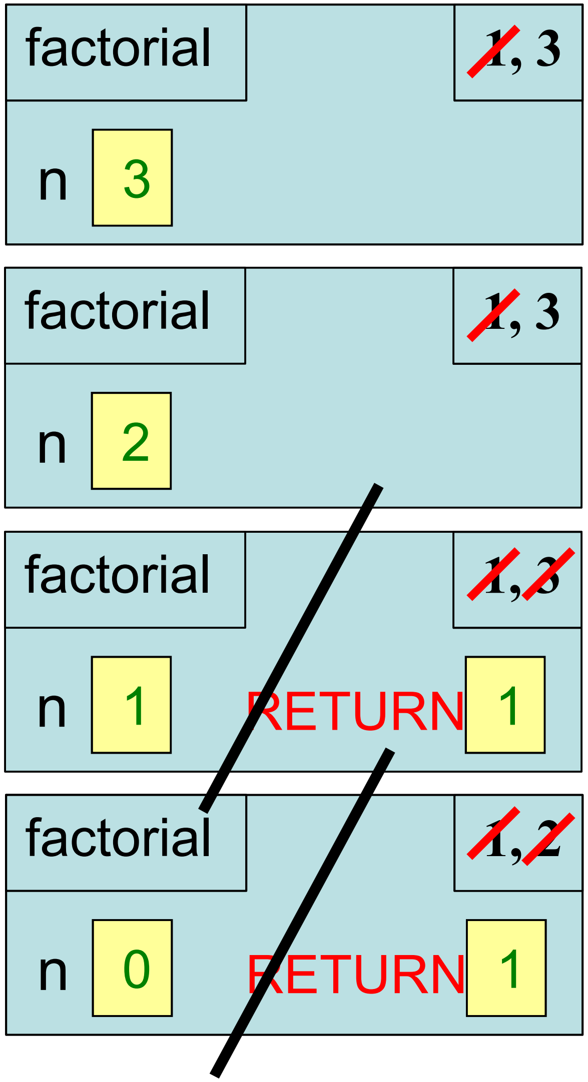

Recursive Call Frames

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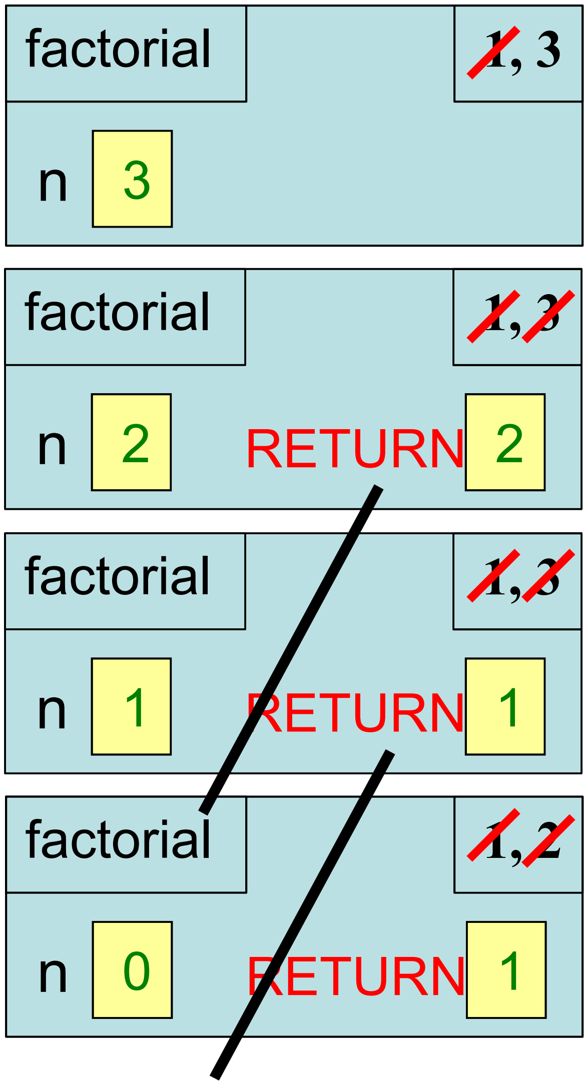
  factorial(3)
```



Recursive Call Frames

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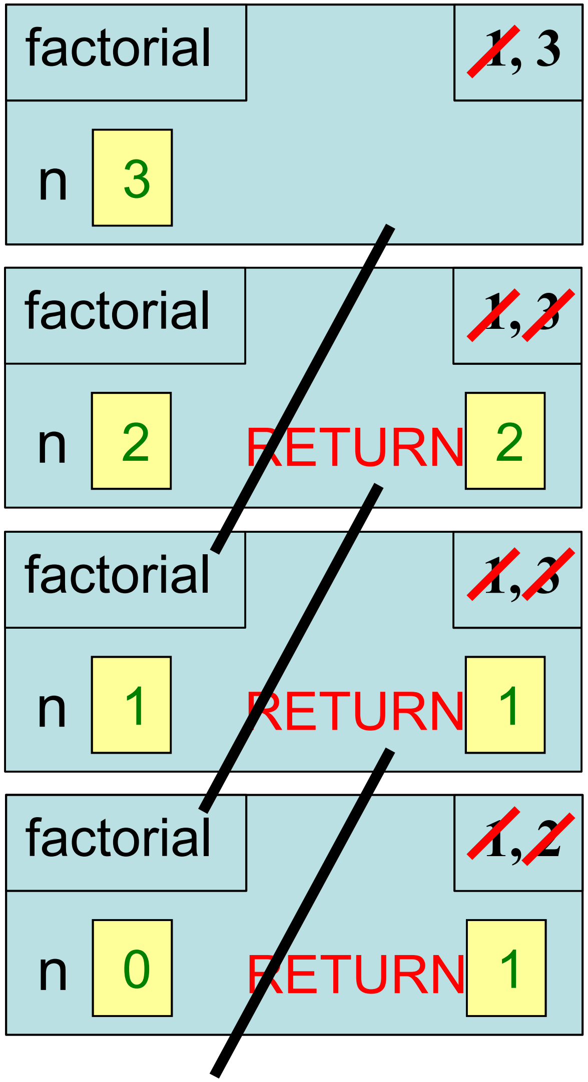

Recursive Call Frames

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```
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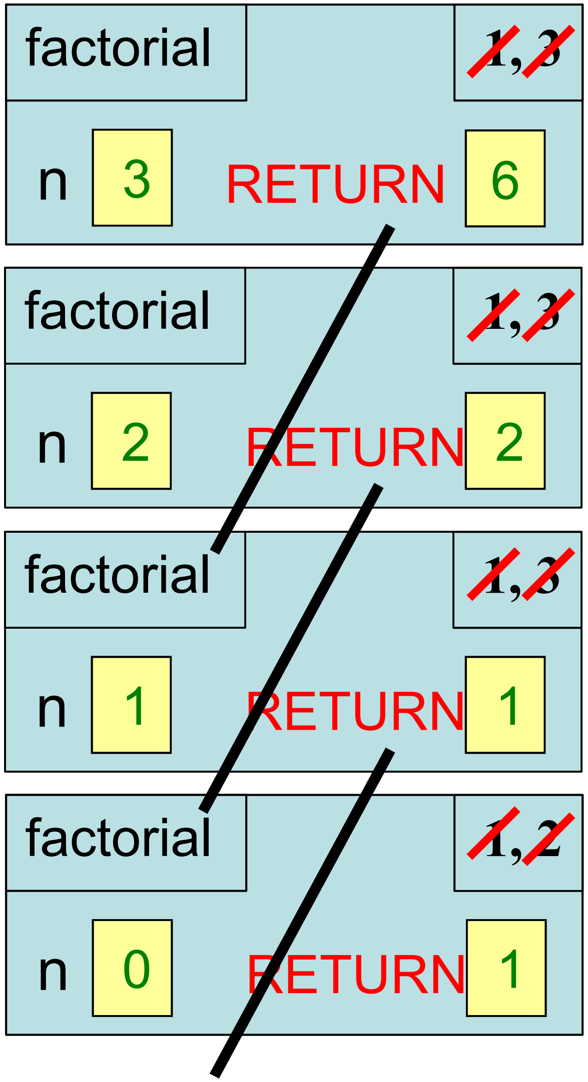
```



Recursive Call Frames

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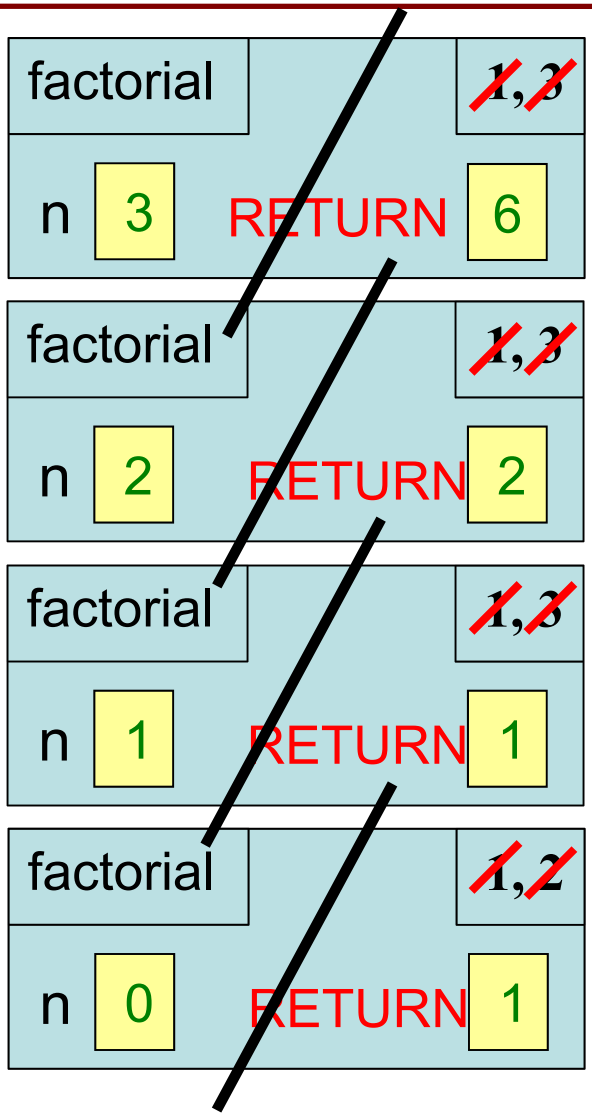


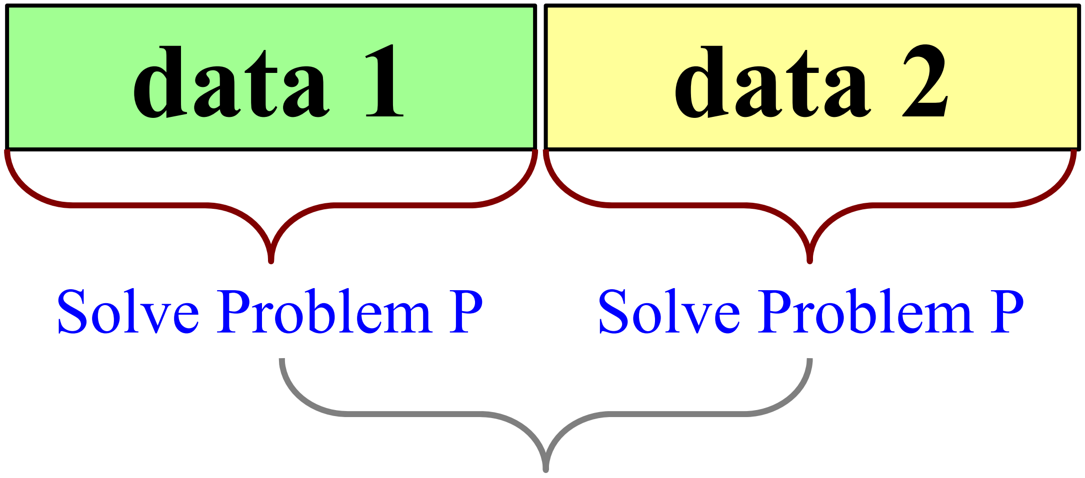
[Start next video:
ways to divide (and conquer)]

Divide and Conquer

Goal: Solve problem P on a piece of data

data

Idea: Split data into two parts and solve problem



data 1 **data 2**

Solve Problem P Solve Problem P

Combine Answer!

Example: Reversing a String

```
def reverse(s):
    """Returns: reverse of s

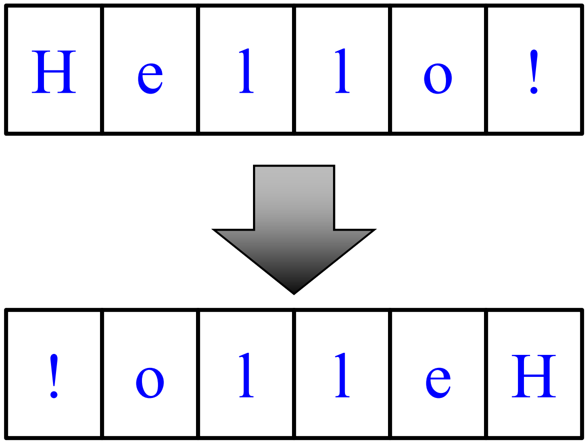
    Precondition: s a string"""
    # 1. Handle base case



    # 2. Break into two parts



    # 3. Combine the result
```



H	e	l	l	o	!
!	o	l	l	e	H

Example: Reversing a String

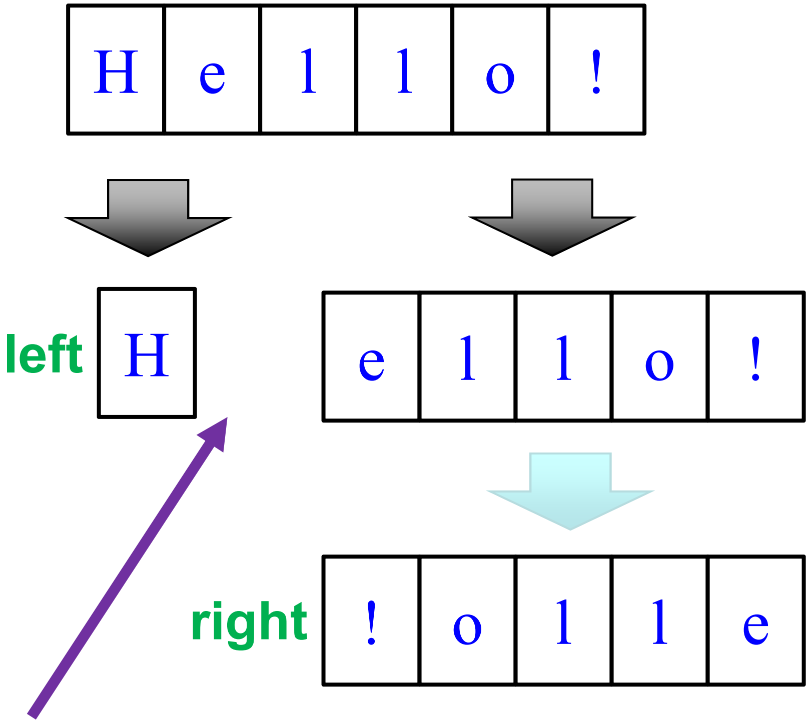
```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case



    # 2. Break into two parts
    left  = reverse(s[0])
    right = reverse(s[1:])


    # 3. Combine the result
```



If this is how we break it up….

How do we combine it?

How to Combine? (Q)

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case



    # 2. Break into two parts
    left  = reverse(s[0])
    right = reverse(s[1:])

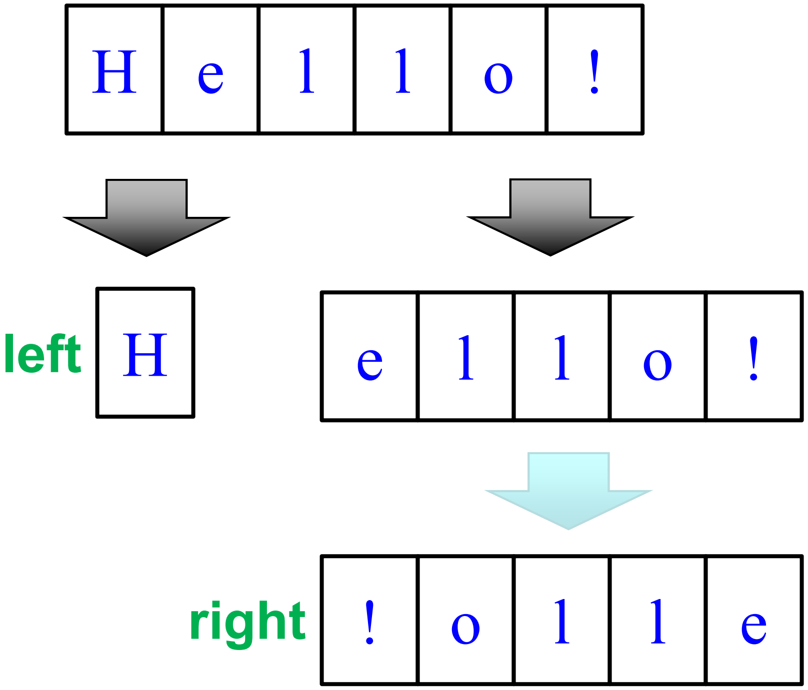
    # 3. Combine the result
    return
```

A: left + right

B: right + left

C: left

D: right



Example: Reversing a String

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def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case



    # 2. Break into two parts
    left   = reverse(s[0])
    right = reverse(s[1:])

    # 3. Combine the result
     return right+left
```
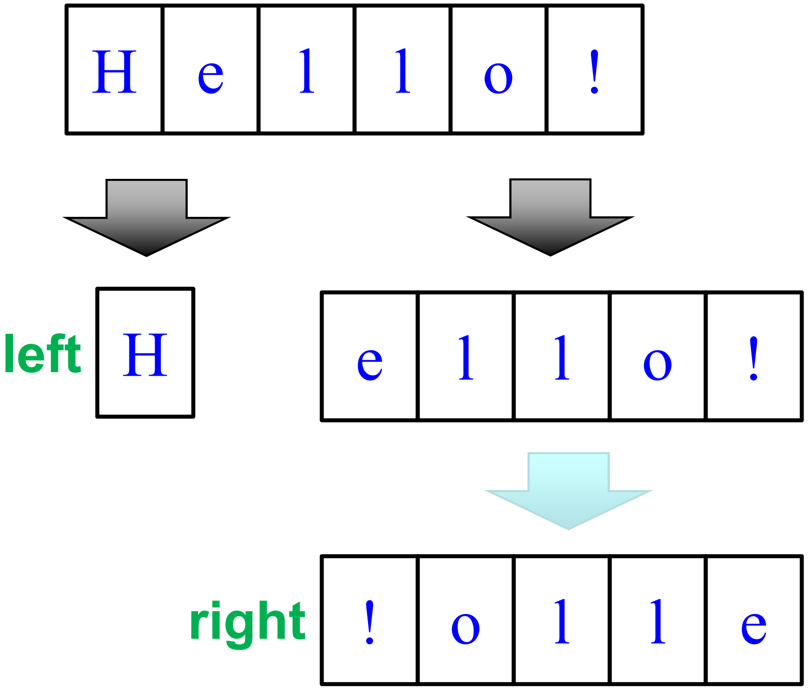



What is the Base Case? (Q)

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case
```

H	e	l	l	o	!

```
A: if s == "":
       return s
```

```
B: if len(s) <= 2:
       return s
```

```
C: if len(s) <= 1:
       return s
```

```
    # 2. Break into two parts
    left  = reverse(s[0])
    right = reverse(s[1:])

    # 3. Combine the result
     return right+left
```

D: Either A or C would work

E: A, B, and C would all work

Example: Reversing a String

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case
    if len(s) <= 1:
        return s

    # 2. Break into two parts
    left  = ~~reverse(s[0])~~    s[0]
    right = reverse(s[1:])

    # 3. Combine the result
    return right+left
```

Base Case

Recursive Case

Alternate Implementation (Q)

```
def reverse(s):
    """Returns: reverse of s
    Precondition: s a string"""
    # 1. Handle base case
    if len(s) <= 1:
        return s

    # 2. Break into two parts
    half   = len(s)//2
    left   = reverse(s[:half])
    right = reverse(s[half:])

    # 3. Combine the result
    return right+left
```

Does this work?

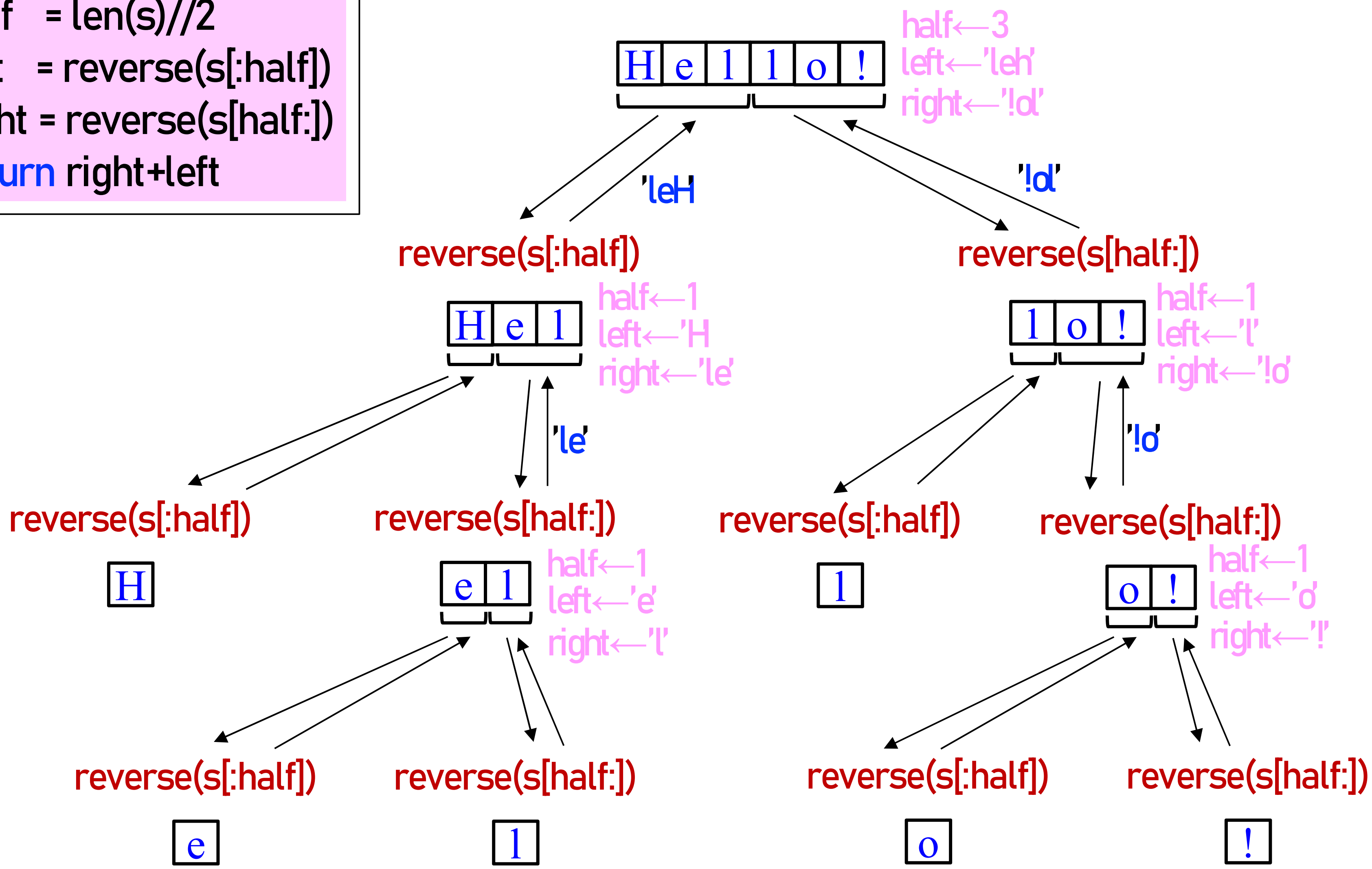
A: **YES**

B: **NO**

```
def reverse(s):
    if len(s) <= 1:
        return s
    half   = len(s)//2
    left   = reverse(s[:half])
    right = reverse(s[half:])
    return right+left
```

Execute the function call reverse('Hello!')

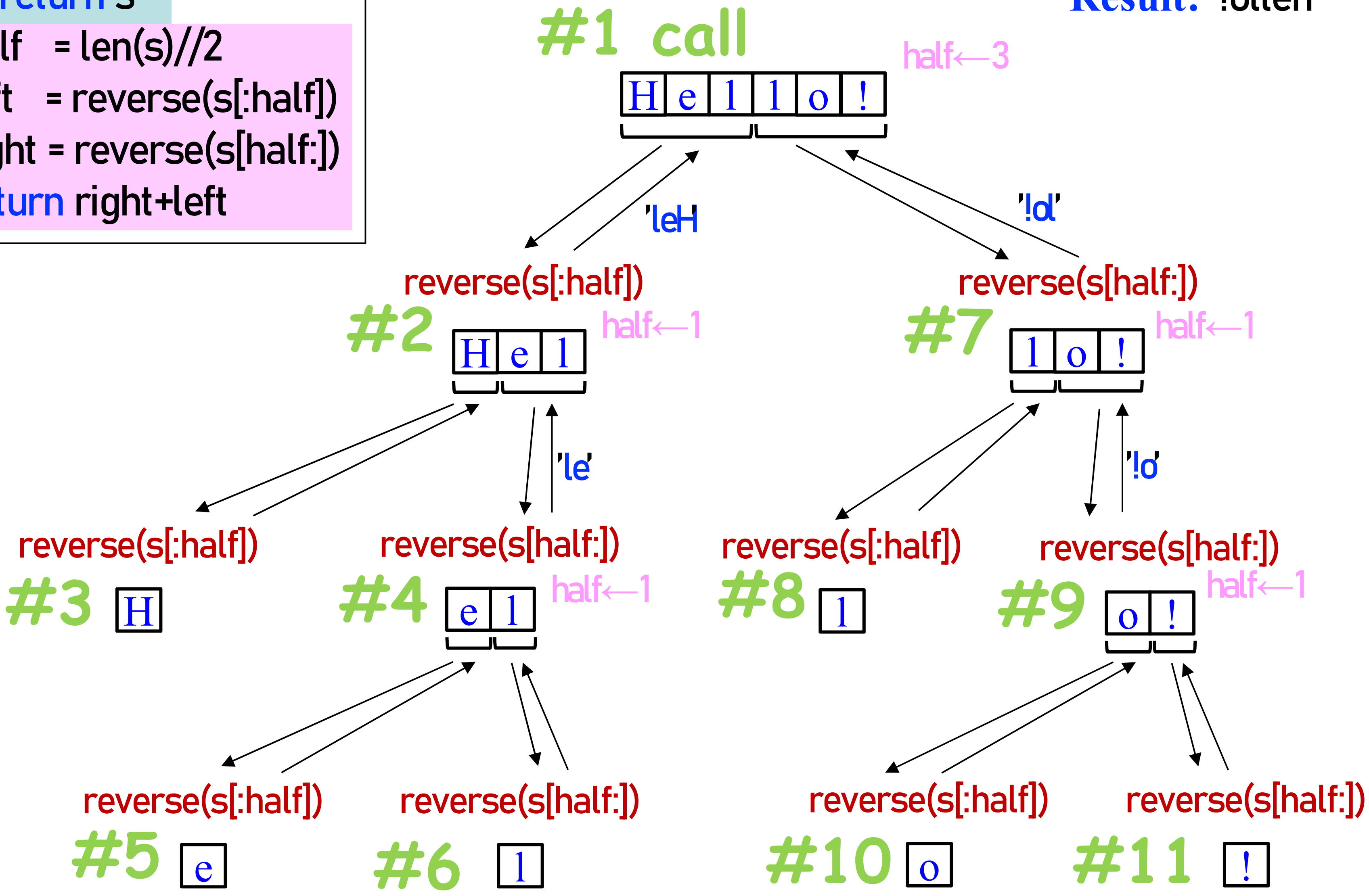
Result: '!olleh'



```
def reverse(s):
    if len(s) <= 1:
        return s
    half   = len(s)//2
    left   = reverse(s[:half])
    right = reverse(s[half:])
    return right+left
```

Execute the function call reverse('Hello!')

Result: '!olleh'



Example: Palindromes

- **Example:**

AMANAPLANACANALPANAMA

MOM

- Dictionary definition: “a word that reads (spells) the same backward as forward”

- Can we define recursively?

Example: Palindromes

- String with ≥ 2 characters is a palindrome if:
    - its first and last characters are equal, and
    - the rest of the characters form a palindrome
- **Example:**

have to be the same

AMANAPLANACANALPANAMA

has to be a palindrome

- **Implement:** `def ispalindrome(s):`

```
    """Returns: True if s is a palindrome"""
```

Example: Palindromes

String with ≥ 2 characters is a palindrome if:

- its first and last characters are equal, and
- the rest of the characters form a palindrome

Recursive Definition

```
def ispalindrome(s):
    """Returns: True if s is a palindrome"""
    if len(s) < 2:
        return True

    ends = s[0] == s[-1]
    middle = ispalindrome(s[1:-1])
    return ends and middle
```

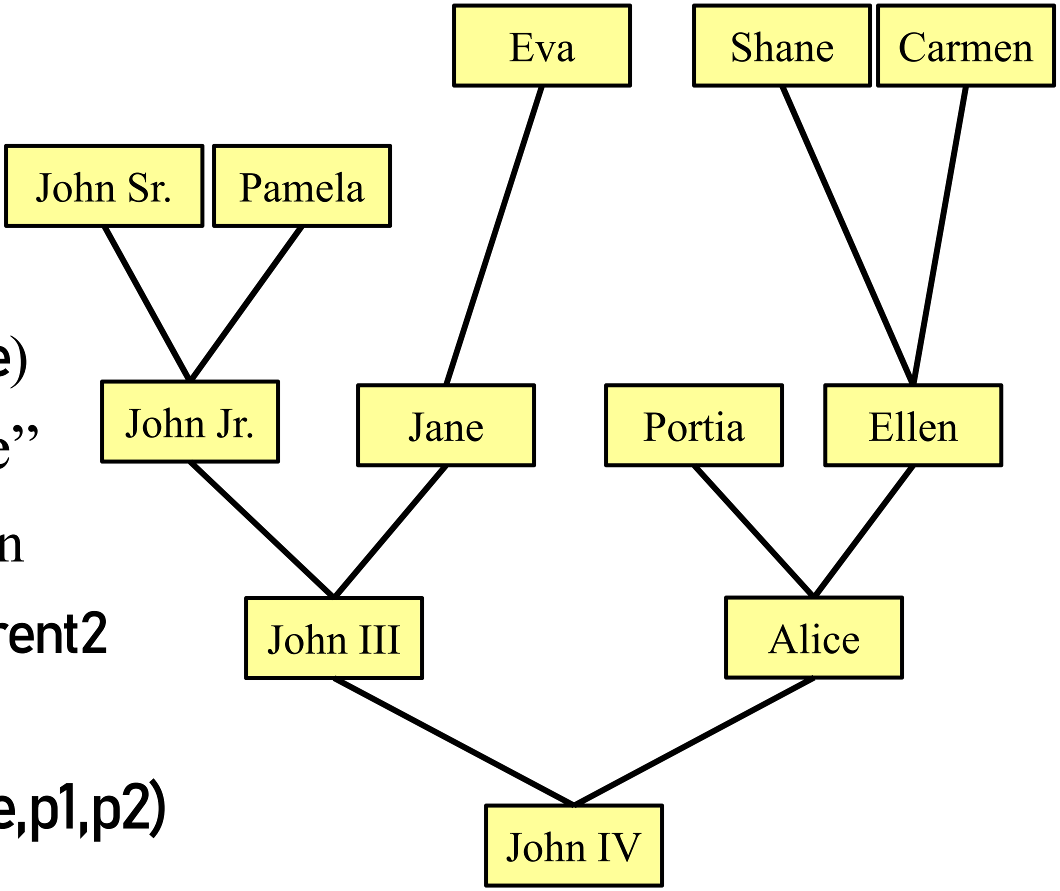
Base case

Recursive case

*[Start next video:
recursion and objects]*

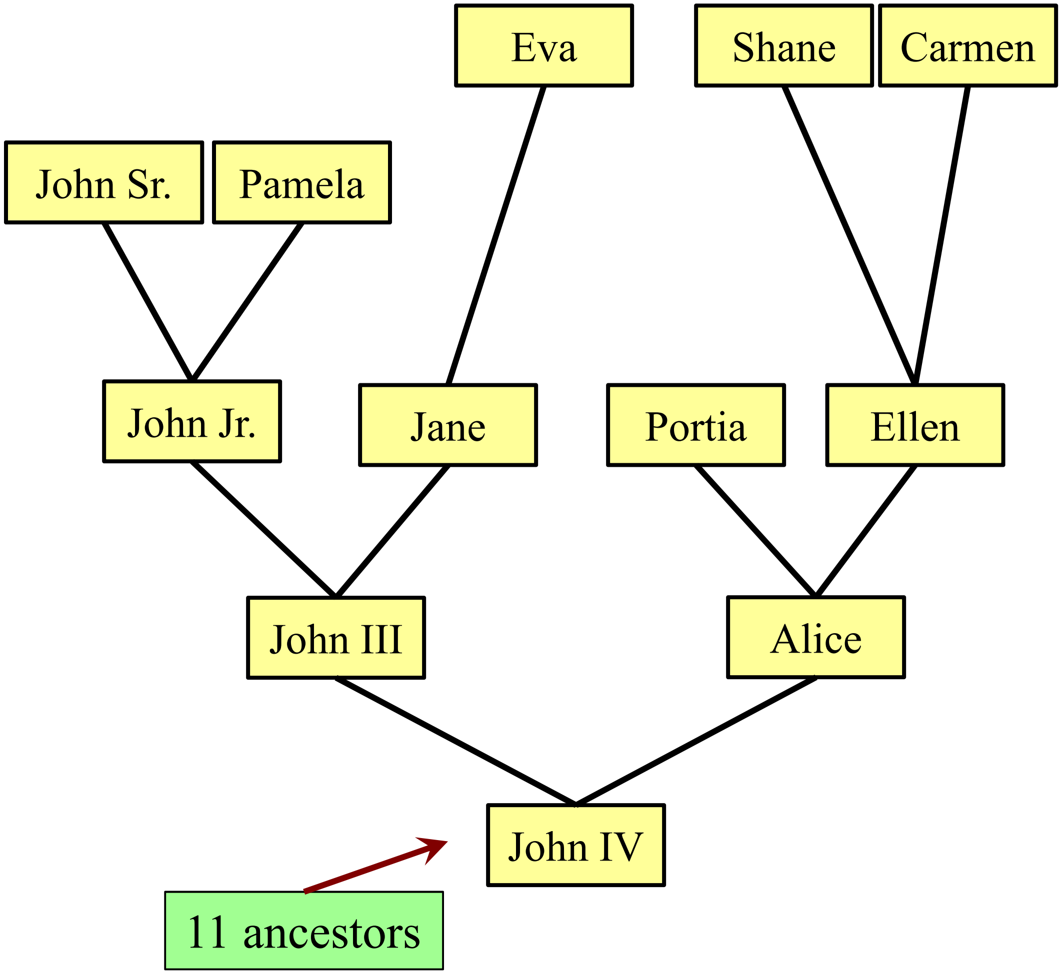
Recursion and Objects

- Class Person
  - Objects have 3 attributes
  - **name**: String
  - **parent1**: Person (or **None**)
  - **parent2**: Person (or **None**)
- Represents the "family tree"
  - Goes as far back as known
  - Attributes **parent1** and **parent2** are **None** if not known
- **Constructor**: `Person(name,p1,p2)`



Recursion and Objects

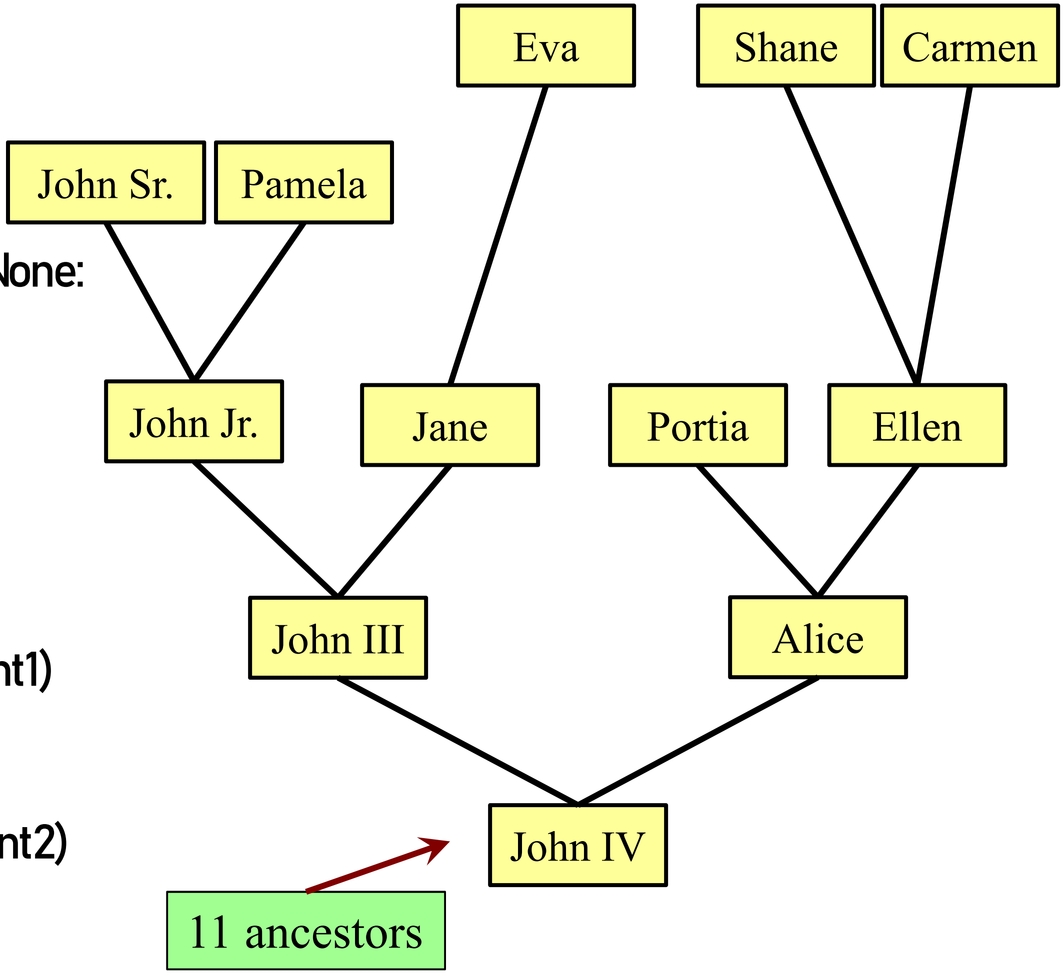
```
def num_ancestors(p):
    """Returns: num of known ancestors
    Pre: p is a Person"""
    # 1. Handle base case.
    # No parents
    # (no ancestors)


    # 2. Break into two parts
    # Has parent1 or parent2
    # Count ancestors of each one
    # (plus parent1, parent2 themselves)



    # 3. Combine the result
```



11 ancestors

Recursion and Objects

```
def num_ancestors(p):
    """Returns: num of known ancestors
    Pre: p is a Person"""
    # 1. Handle base case.
    if p.parent1 == None and p.parent2 == None:
        return 0


    # 2. Break into two parts
    parent1s = 0
    if p.parent1 != None:
        parent1s = 1+num_ancestors(p.parent1)
    parent2s = 0
    if p.parent2 != None:
        parent2s = 1+num_ancestors(p.parent2)

    # 3. Combine the result
    return parent1s+parent2s
```



Recursion and Objects

```
def num_ancestors(p):
    """Returns: num of known ancestors
    Pre: p is a Person"""
    # 1. Handle base case.
    if p.parent1 == None and p.parent2 == None:
        return 0

    # 2. Break into two parts
    parent1s = 0
    if p.parent1 != None:
       parent1s = 1+num_ancestors(p.parent1s)
    parent2s = 0
    if p.parent2 != None:
       parent2s = 1+num_ancestors(p.parent2s)

    # 3. Combine the result
    return parent1s+parent2s
```

We don’t actually need this.
It is handled by the conditionals in #2.

Challenge: All Ancestors

```
def all_ancestors(p):
    """Returns: list of all ancestors of p"""
    # 1. Handle base case.
    # 2. Break into parts.
    # 3. Combine answer.
```