Module 30

Searching & Sorting

Linear Search

```
def linear_search(v,b):
    """Returns: first occurrence of v in b (-1 if not found)
    Precond: b a list of number, v a number
    """
    # Loop variable
    i = 0
    while i < len(b) and b[i] != v:
        i = i + 1

    if i == len(b):  # not found
        return -1
    return i
```

How many entries do we have to look at?

Linear Search

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def linear_search(v,b):
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    if i == len(b):  # not found
        return -1
    return i
```

How many entries do we have to look at?

All of them!

Linear Search

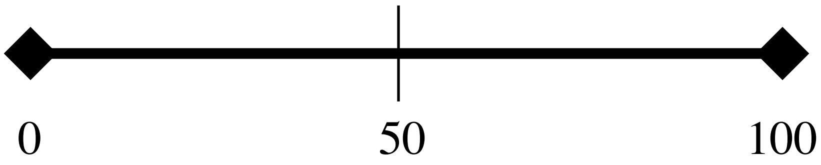
```
def linear_search(v,b):
    """Returns: first occurrence of v in b (-1 if not found)
    Precond: b a list of number, v a number
    """
    # Loop variable
    i = len(b)-1
    while i >= 0 and b[i] != v:
        i = i - 1

    # Equals -1 if not found
    return i
```

How many entries do we have to look at?

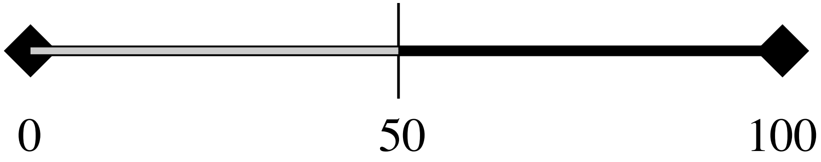
All of them!

Is There a Better Way?



- Thinking of number 0..100
  - You get to guess number
  - I tell you higher or lower
  - Continue until get it right
- **Goal:** Keep # guesses low
  - Use my answers to help
- **Strategy?**
  - Start guess in the middle
  - Answer eliminates half
  - Go to middle of remaining

Is There a Better Way?



Higher!

- Thinking of number 0..100
  - You get to guess number
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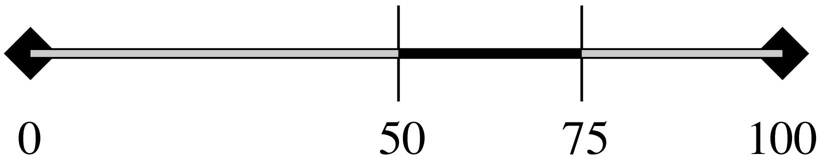

Lower!

- Thinking of number 0..100
  - You get to guess number
  - I tell you higher or lower
  - Continue until get it right
- **Goal:** Keep # guesses low
  - Use my answers to help
- **Strategy?**
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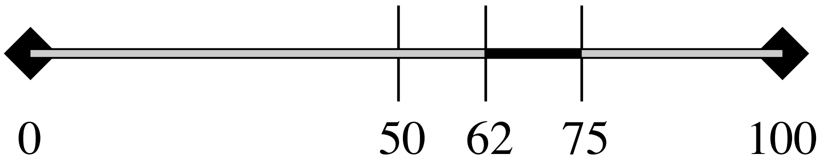
Is There a Better Way?



Higher!

- Thinking of number 0..100
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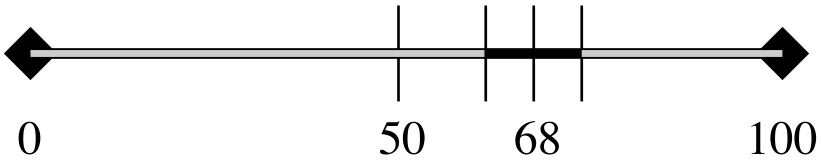
Is There a Better Way?



Correct!

- Thinking of number 0..100
  - You get to guess number
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- **Goal:** Keep # guesses low
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- **Strategy?**
  - Start guess in the middle
  - Answer eliminates half
  - Go to middle of remaining

Binary Search

```
def binary_search(v,b):
    # Loop variable(s)
    i = 0, j = len(b)
    while i < j and b[i] != v:
        mid = (i+j)//2
        if b[mid] < v:
            j = mid
        elif b[mid] > v:
            i = mid
        else:
            return mid
    return -1 # not found
```

Requires that the data is sorted!

But few checks!

Observation About Sorting

- Sorting data can speed up searching
  - Sorting takes time, but do it once
  - Afterwards, can search many times
- Not just searching. Also speeds up
  - Duplicate elimination in data sets
  - Data compression
  - Physics computations in computer games
- Why it is a major area of computer science

The Sorting Challenge

- **Given:** A list of numbers
- **Goal:** Sort those numbers using only
  - Iteration (while-loops or for-loops)
  - Comparisons (< or >)
  - Assignment statements
- Why? For proper **analysis**.
  - Methods/functions come with hidden costs
  - Everything above has no hidden costs
  - Each comparison or assignment is “1 step”

This Requires Some Notation

- As the list is sorted…
  - Part of the list **will** be sorted
  - Part of the list will **not** be sorted
- Need a way to refer to portions of the list
  - Notation to refer to sorted/unsorted parts
- And have to do it **without** slicing!
  - Slicing makes a copy
  - Want to sort original list, not a copy

This Requires Some Notation

- As the list is sorted…
  - Part of the list **will** be sorted
  - Part of the list will **not** be sorted
- Need a w list
  - Notatio

But we will be less formal than in past years!

- And have to do it **without** slicing!
  - Slicing makes a copy
  - Want to sort original list, not a copy

Terminology: Range Notation

- m..n is a range containing n+1-m values
    - 2..5 contains 2, 3, 4, 5. Contains 5+1 – 2 = 4 values
    - 2..4 contains 2, 3, 4. Contains 4+1 – 2 = 3 values
    - 2..3 contains 2, 3. Contains 3+1 – 2 = 2 values
    - 2..2 contains 2. Contains 2+1 – 2 = 1 values
    - 2..1 contains ???

What does 2..1 contain?

A: nothing
B: 2,1
C: 1
D: 2
E: something else

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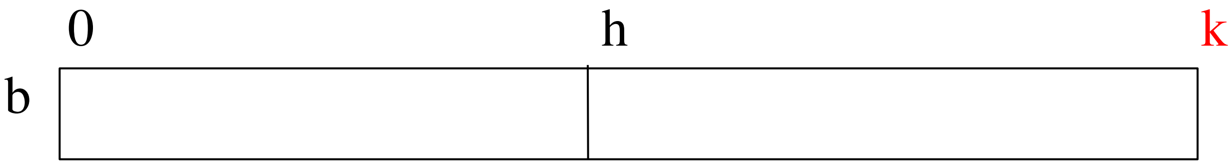
  - 2..2 contains 2. Contains 2+1 – 2 = 1 values
  - 2..1 contains ???

Not the same as `range(m,n)`

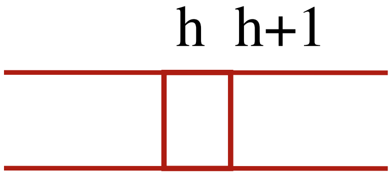
- The notation m..n, always implies that m <= n+1
  - So you can assume that even if we do not say it
  - If m = n+1, the range has 0 values

Horizontal Notation

- Want a pictoral way to visualize this sorting
  - Represent the list as long rectangle
  - We saw this idea in divide-and-conquer



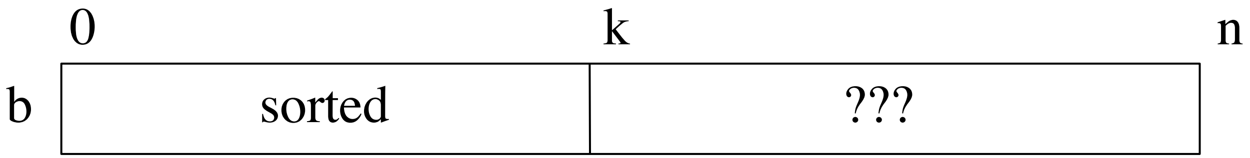
- Do **not** show individual boxes
  - Just dividing lines between regions
  - Label dividing lines with indices
  - But index is either left or right of dividing line



(h+1) – h = 1

Horizontal Notation

- Label regions with properties
  - **Example:** Sorted or ???

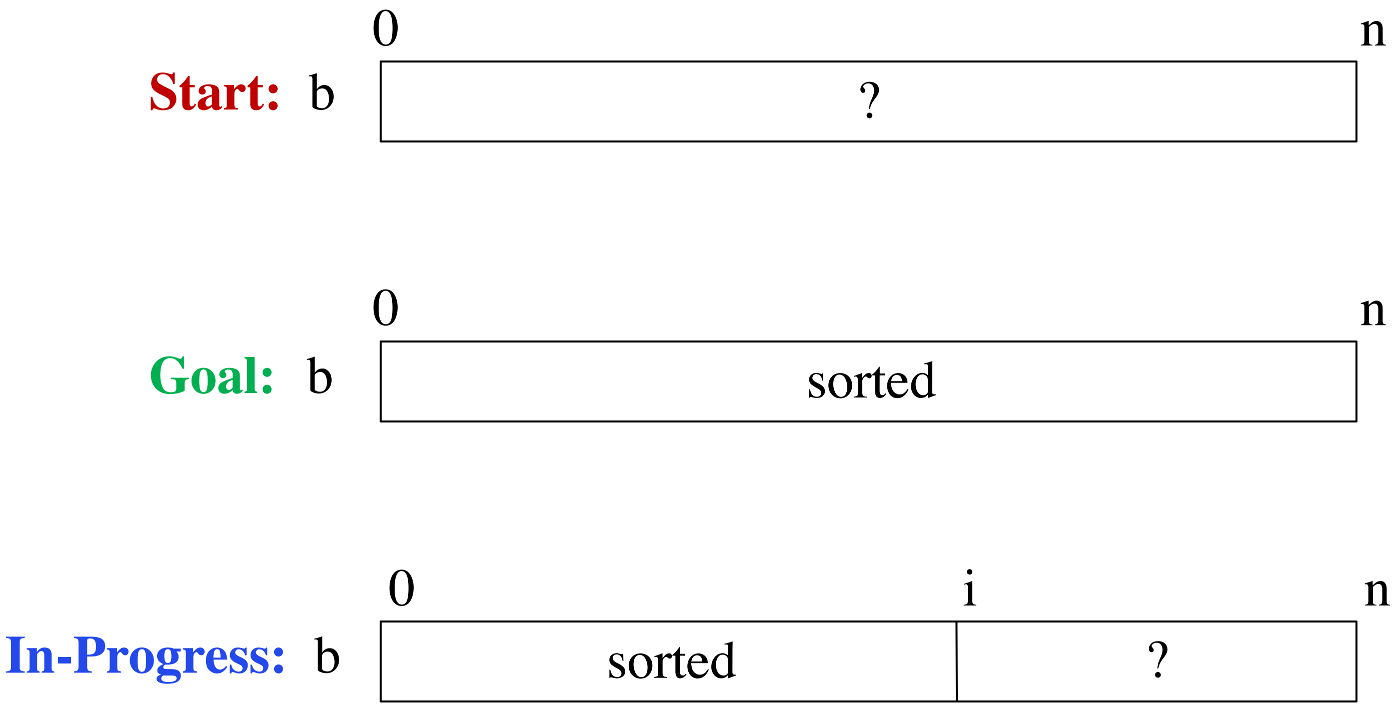


0 k n

b sorted ???

  - `b[0..k–1]` is sorted
  - `b[k..n-1]` **unknown** (might be sorted)
- Picture allows us to track progress

Visualizing Sorting

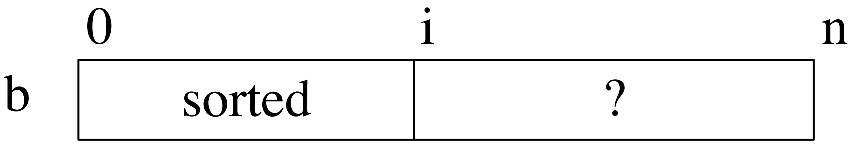


Start: b
0
n
?

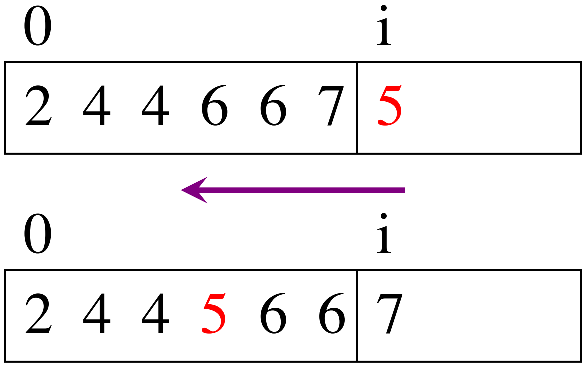
Goal: b
0
n
sorted

In-Progress: b
0
i
n
sorted
?

Insertion Sort



```
i = 0
while i < n:
    # Push b[i] down into its
    # sorted position in b[0..i]
    i = i+1
```

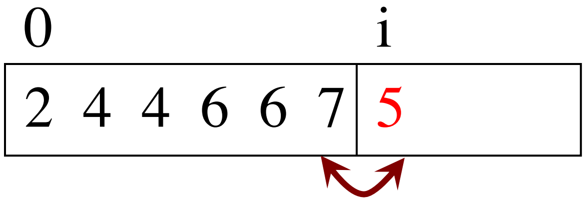


Remember the restrictions!

Insertion Sort: Moving into Position

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```



swap shown in the lecture about lists

Insertion Sort: Moving into Position

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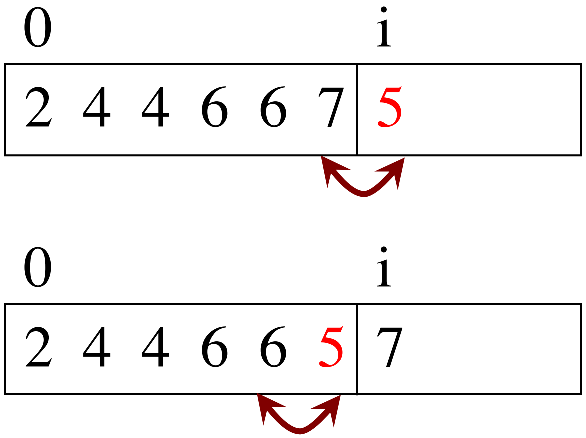

Insertion Sort: Moving into Position

```
i = 0
while i < n:
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    i = i+1

def push_down(b, i):
    j = i
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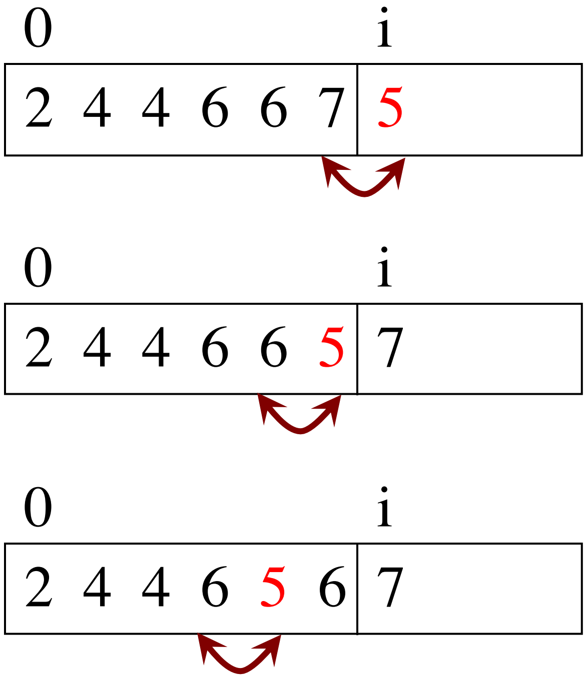

Insertion Sort: Moving into Position

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while i < n:
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def push_down(b, i):
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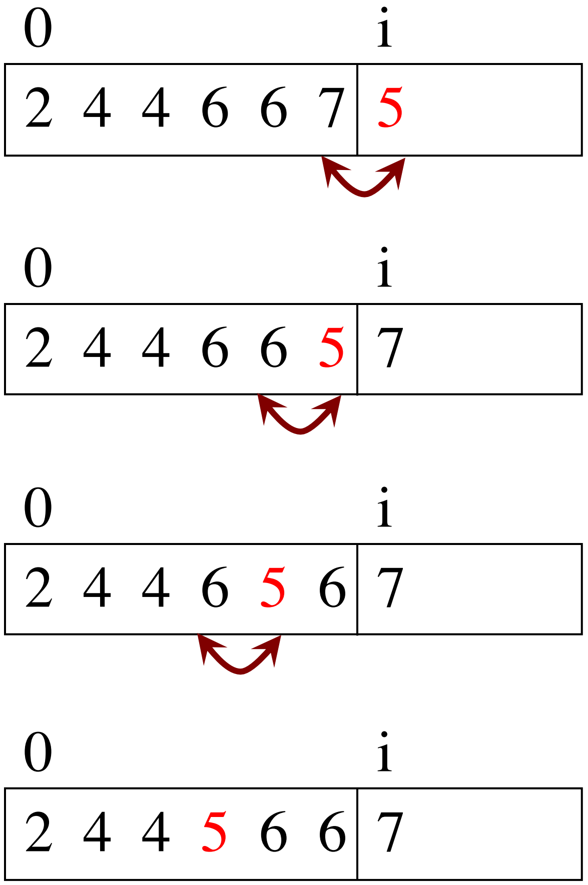
```

swap shown in the lecture about lists



The Importance of Helper Functions

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

VS

Can you understand all this code below?

```
i = 0
while i < n:
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            temp = b[j]
            b[j] = b[j-1]
            b[j-1] = temp
        j = j -1
    i = i +1
```

Measuring Performance

- Performance is a tricky thing to measure
  - Different computers run at different speeds
  - Memory also has a major effect as well
- Need an independent way to measure
  - Measure in terms of “basic steps”
  - **Example:** Searching counted # of checks
- For sorting, we measure in terms of **swaps**
  - Three assignment statements
  - Present in all sorting algorithms

Insertion Sort: Performance

```
def push_down(b, i):
    """Push value at position i into
    sorted position in b[0..i-1]"""
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

- b[0..i-1]: i elements
- Worst case:
  - i = 0: 0 swaps
  - i = 1: 1 swap
  - i = 2: 2 swaps
- Pushdown is in a loop
  - Called for i in 0..n
  - i swaps each time

Total Swaps: $0 + 1 + 2 + 3 + \ldots (n-1) = (n-1)*n/2 = (n^2-n)/2$

Insertion Sort: Performance

```
def push_down(b, i):
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- b[0..i-1]: i elements
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  - i = 0: 0 swaps
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Insertion sort is an n^2 algorithm

Total Swaps: $0 + 1 + 2 + 3 + \ldots (n-1) = (n-1)*n/2 = (n^2-n)/2$

Algorithm “Complexity”

- **Given**: a list of length n and a problem to solve
- **Complexity**: *rough* number of steps to solve worst case
- Suppose we can compute 1000 operations a second:

Complexity	n=10	n=100	n=1000
log n	0.003 s	0.006 s	0.01 s
n	0.01 s	0.1 s	1 s
n log n	0.016 s	0.32 s	4.79 s
n^2	0.1 s	10 s	16.7 m
n^3	1 s	16.7 m	11.6 d
2^n	1 s	4×10^{19} y	3×10^{290} y

Algorithm “Complexity”

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Binary Search

Linear Search

Insertion Sort

Algorithm “Complexity”

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Complexity	n=10	n=100	n=1000
log n			0.01 s
n			1 s
n log n			4.79 s
n^2			16.7 m
n^3	1 s	16.7 m	11.6 d
2^n	1 s	4×10^{19} y	3×10^{290} y

Major Topic in 2110:

Beyond scope of this course

Insertion Sort is Not Great

- Typically n^2 is okay, but not great
  - Will perform horribly on large data
  - Very bad when performance critical (games)
- We would like to do better than this
  - Can we get n swaps (**no**)?
  - How about $n \log n$ (**maybe**)
- This will require a new algorithm
  - Let’s return to horizontal notation

A New Algorthm

0 n

Start: b | ? |

0 n

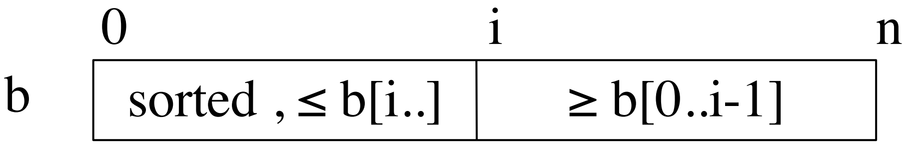
Goal: b | sorted |

0 i n

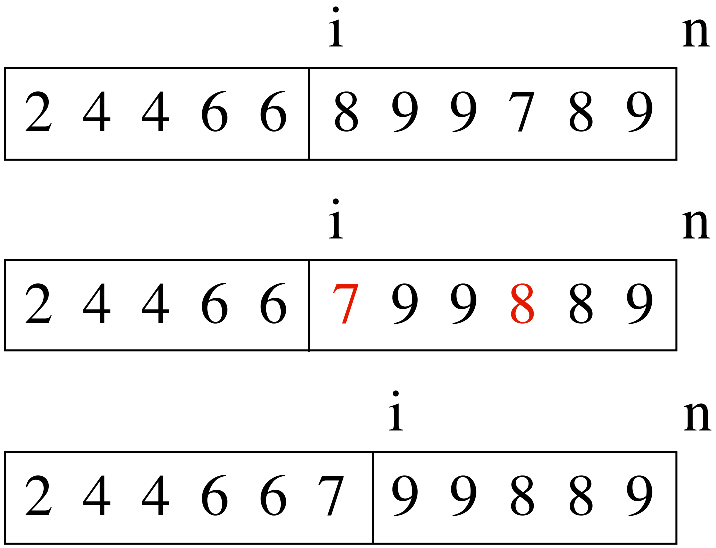
In-Progress: b | sorted, ≤ b[i..] | ≥ b[0..i-1] |

First segment always contains smaller values

Selection Sort



```
i = 0
while i < n:
    # Find minimum in b[i..]
    # Move it to position i
    i = i+1
```

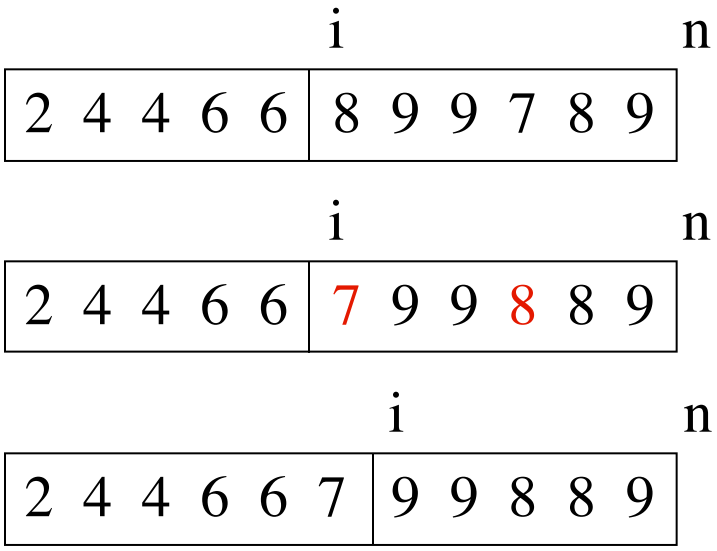


Remember the restrictions!

Selection Sort

How fast is this?

```
i = 0
while i < n:
    j = index of min of b[i..n-1]
    swap(b,i,j)
    i = i+1
```

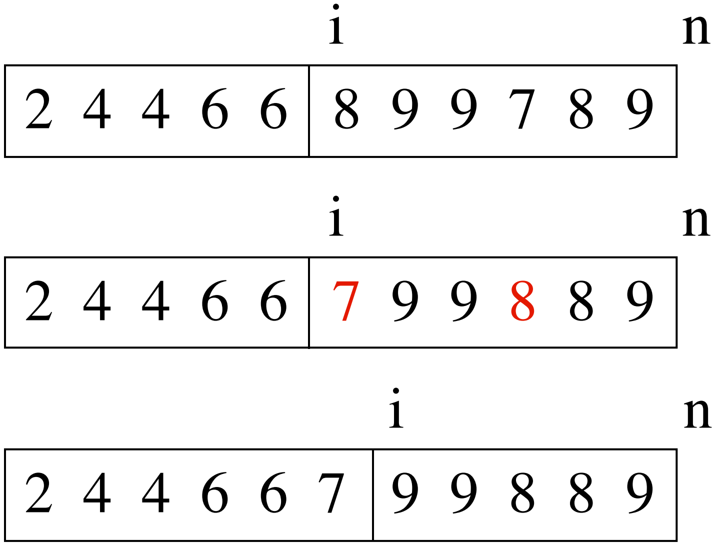


Selection Sort

This is also n^2!

```
i = 0
while i < n:
    j = index of min of b[i..n-1]
    swap(b,i,j)
    i = i+1
```

This is n steps

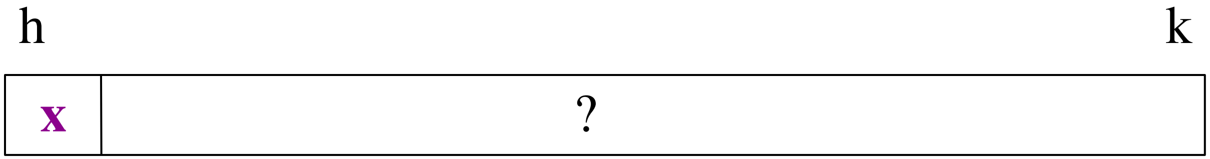


What is the Problem

- Both insertion, selection sort are nested loops
  - Outer loop over each element to sort
  - Inner loop to put next element in place
  - Each loop is n steps. $n \times n = n^2$
- To do better we must *eliminate* a loop
  - But with what? Recursion!
- But to do this we have to back up a bit
  - Need to introduce an intermediate algorithm

The Problem Statement

- Given a list b[h..k] with some value x in b[h]:

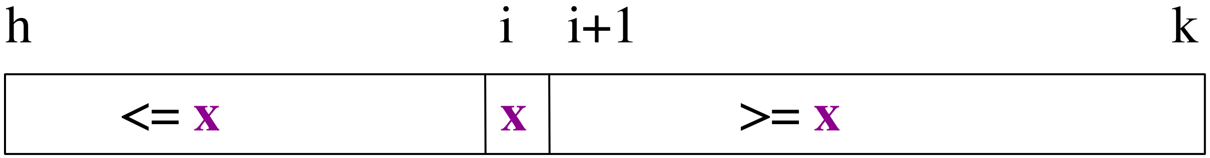


Start: b

h k

x ?

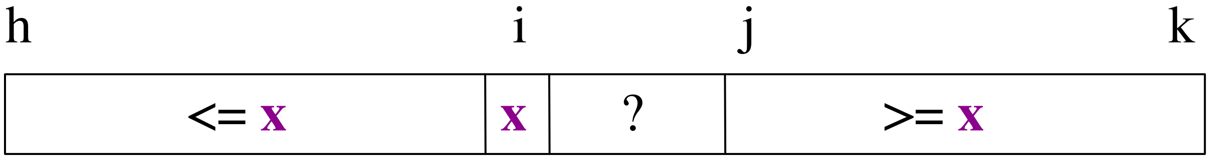
- Swap elements of b[h..k] to get this answer



Goal: b

h i i+1 k

<= x x >= x



In-Progress: b

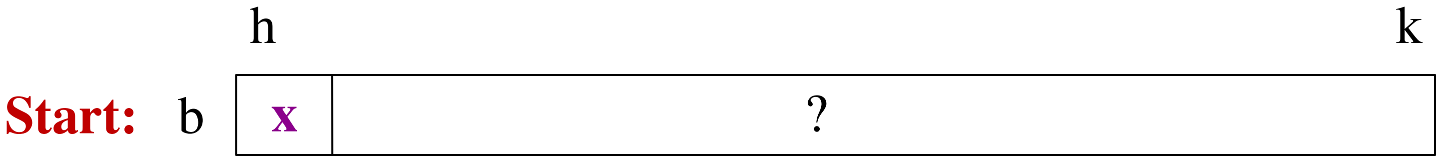
h i j k

<= x x ? >= x

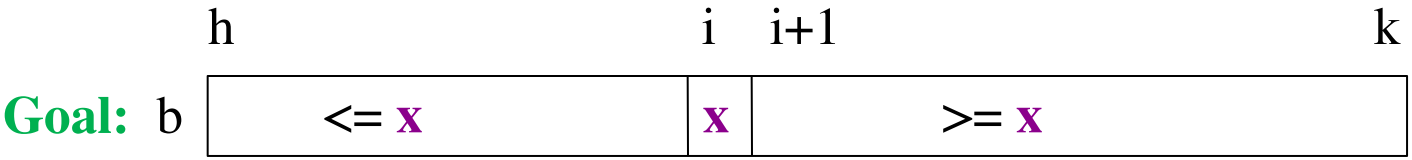
Indices b, h important!
Might partition only part

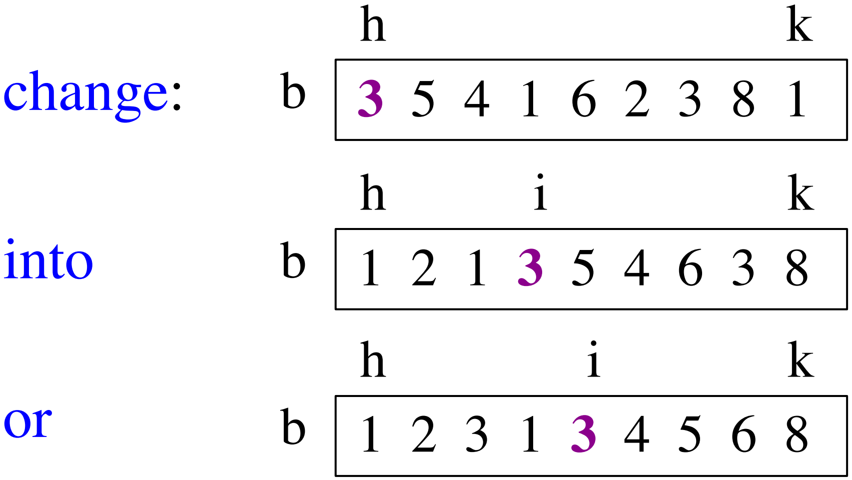
Partition Algorithm

- Given a list segment b[h..k] with some value x in b[h]:



- Swap elements of b[h..k] to get this answer





- x is called the pivot value
    - x is not a program variable
    - denotes value initially in b[h]

Partition Algorithm Implementation

```
def partition(b, h, k):
    """Partition list b[h..k] around a pivot x = b[h]"""
    i = h; j = k+1; x = b[h]

    while i < j-1:
        if b[i+1] >= x:
            # Move to end of block.
            swap(b,i+1,j-1)
            j = j - 1
        else:   # b[i+1] < x
            swap(b,i,i+1)
            i = i + 1

    return i
```

partition(b,h,k), not partition(b[h:k+1])

Remember, slicing always copies the list!

We want to partition the **original** list

Partition Algorithm Implementation

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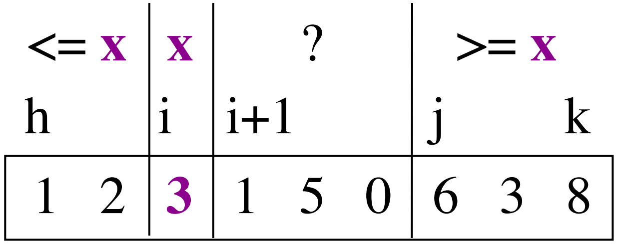

<= x		x	?			>= x		
h		i	i+1			j		k
1	2	3	1	5	0	6	3	8

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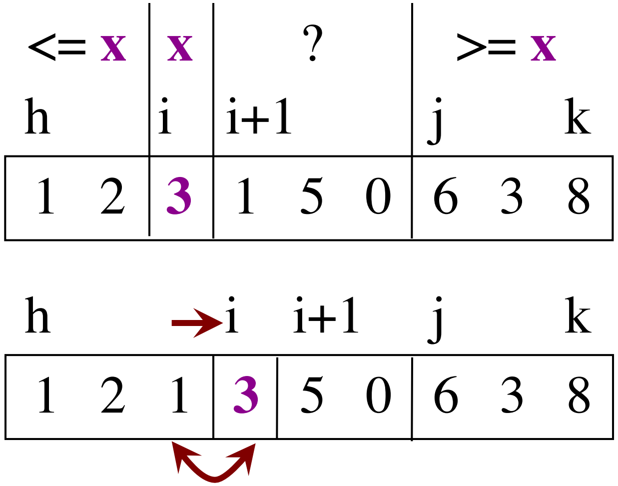
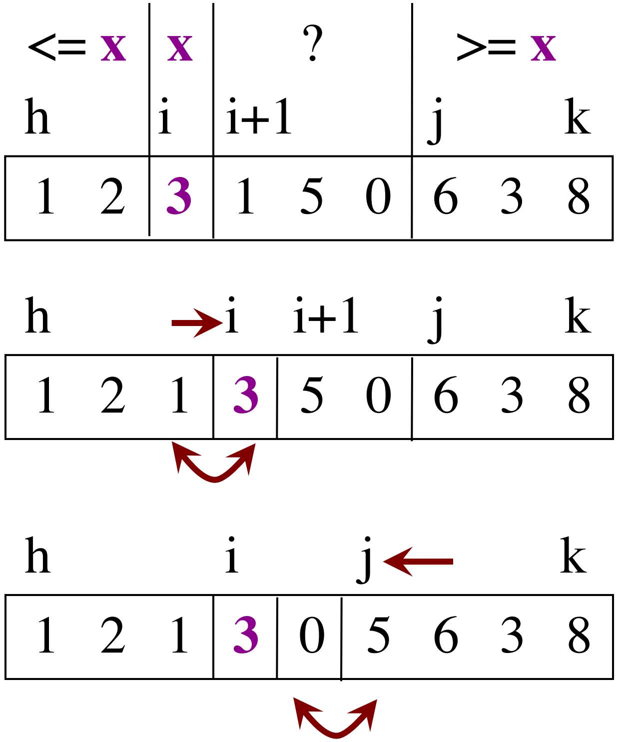


Partition Algorithm Implementation

```
def partition(b, h, k):
    """Partition list b[h..k] around a pivot x = b[h]"""
    i = h; j = k+1; x = b[h]

    while i < j-1:
        if b[i+1] >= x:
            # Move to end of block.
            swap(b,i+1,j-1)
            j = j - 1
        else:   # b[i+1] < x
            swap(b,i,i+1)
            i = i + 1

    return i
```



Partition Algorithm Implementation

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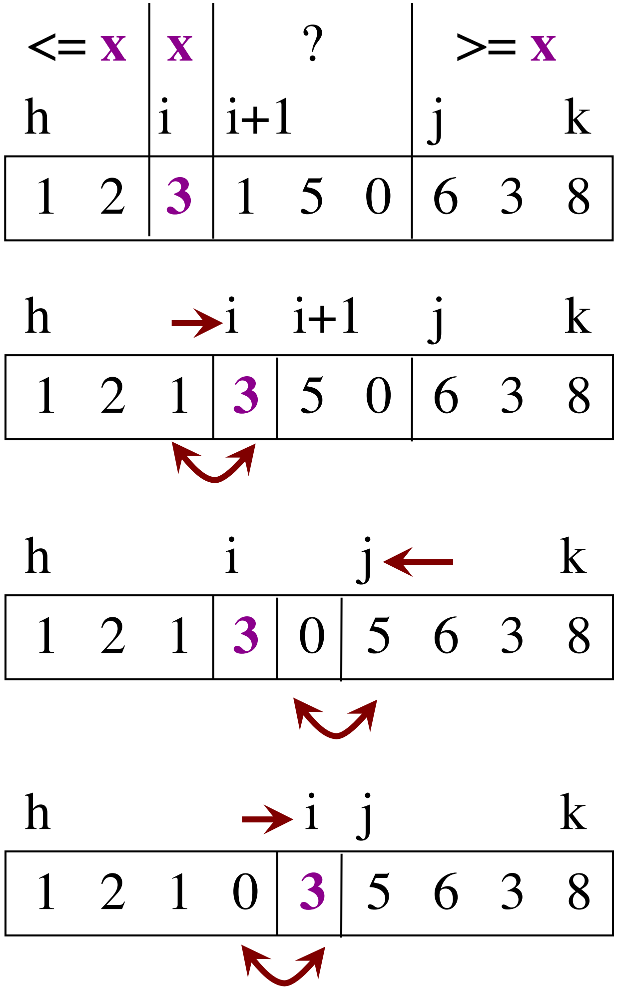
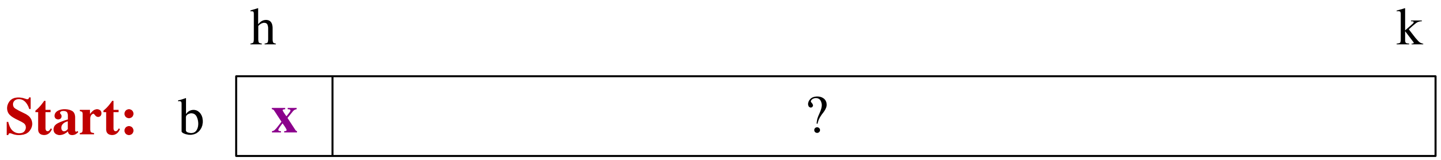


Why is this Useful?

- Will use this algorithm to replace inner loop
  - The inner loop cost us n swaps every time
- Can this reduce the number of swaps?
  - Worst case is k-h swaps
  - This is n if partitioning the whole list
  - But less if only partitioning part
- **Idea:** Break up list and partition only part?
  - This is **Divide-and-Conquer!**

Sorting with Partitions

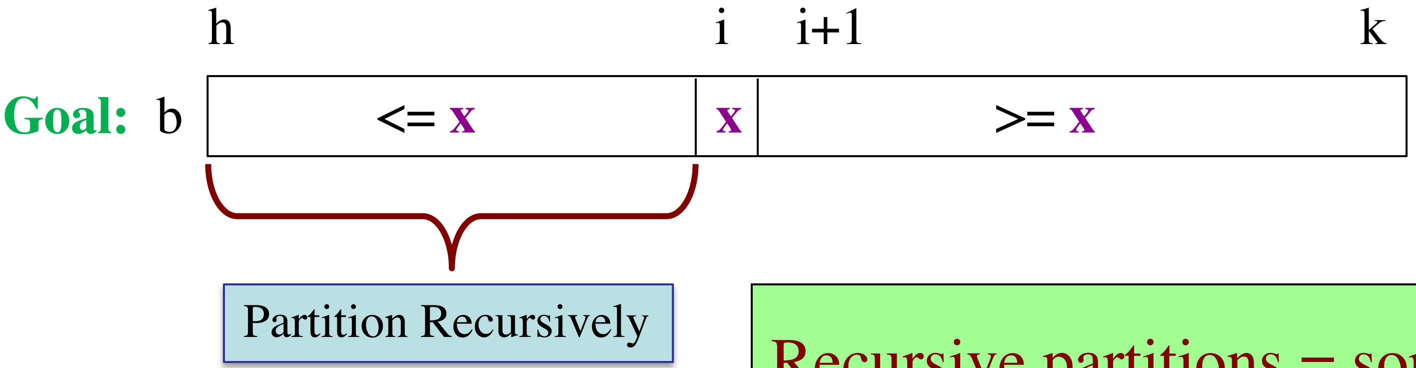
- Given a list segment b[h..k] with some value x in b[h]:



h k

Start: b | x | ?

- Swap elements of b[h..k] to get this answer



h i i+1 k

Goal: b | <= **x** | **x** | >= **x**

Partition Recursively

Recursive partitions = sorting

- Called **QuickSort** (why???)
- Popular, fast sorting technique

Sorting with Partitions

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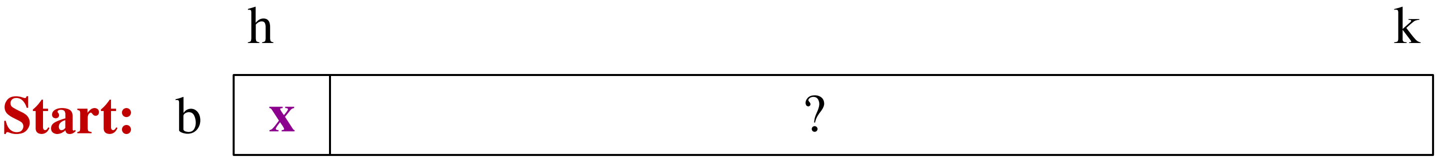

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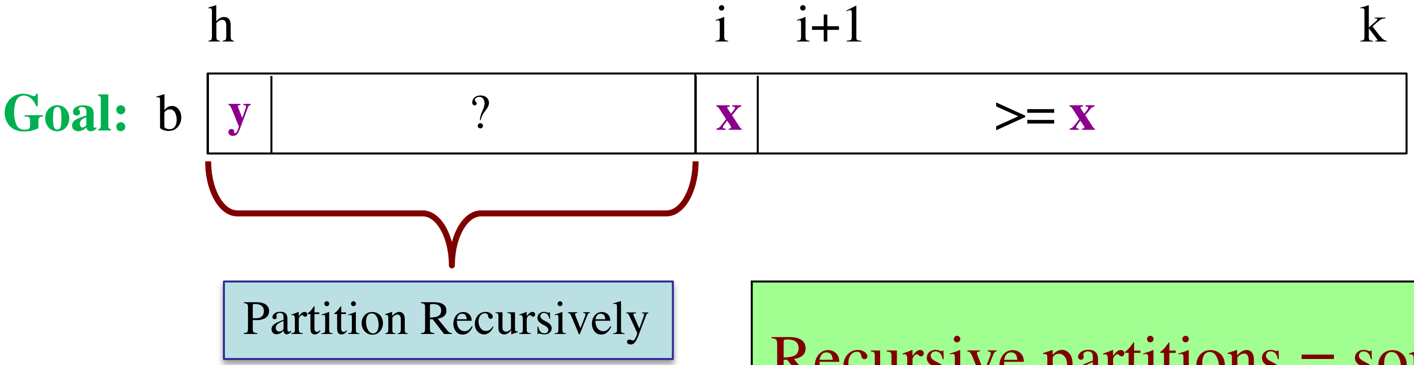

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Sorting with Partitions

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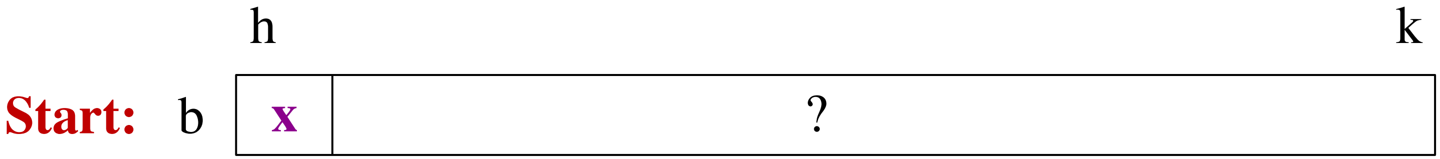

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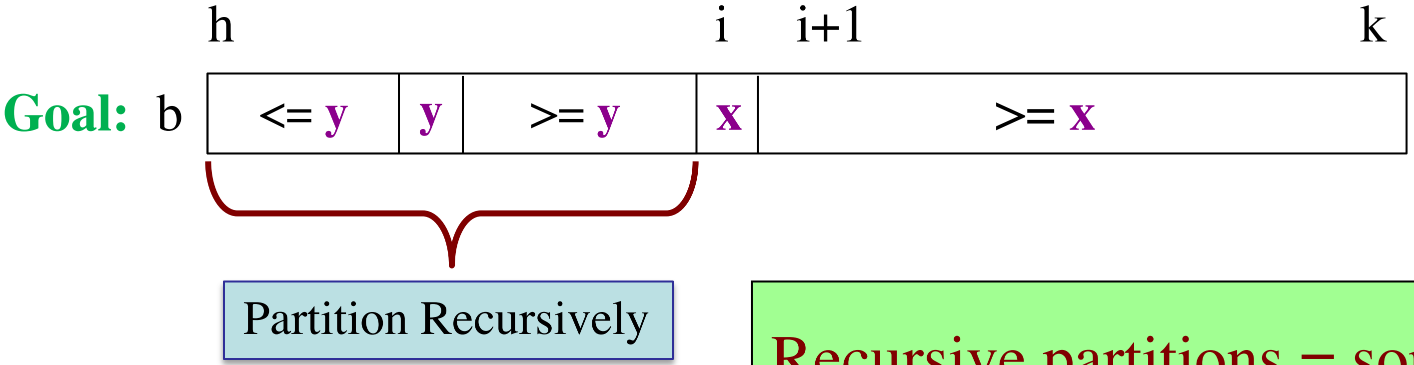

Recursive partitions = sorting

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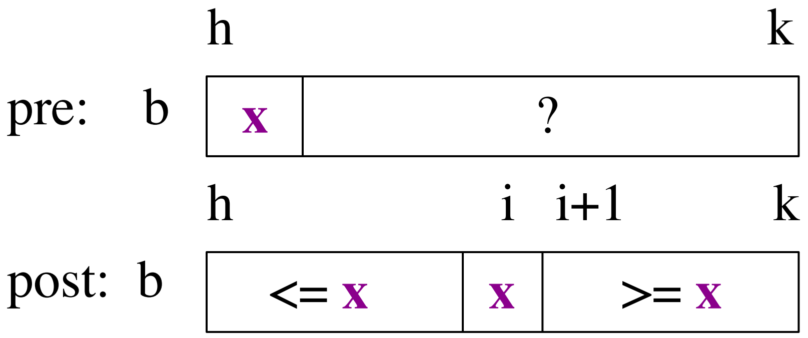
QuickSort

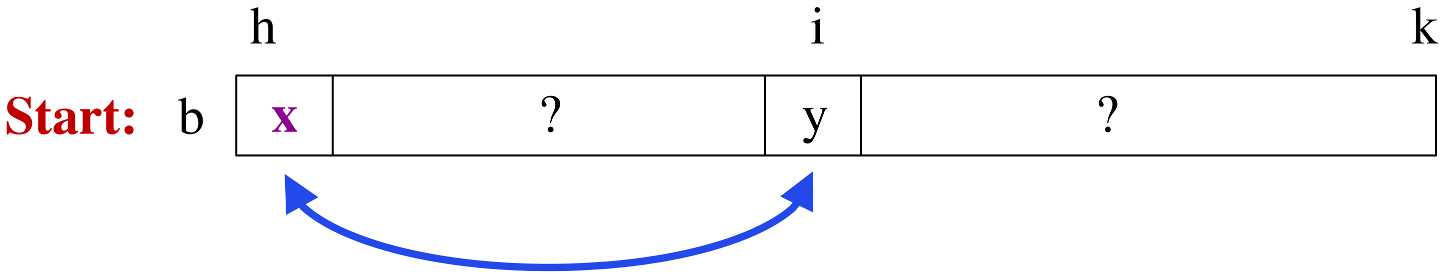
```
def quick_sort(b, h, k):
    """Sort the array fragment b[h..k]"""
    if  b[h..k] has fewer than 2 elements:
        return
    j = partition(b, h, k)
    # b[h..j–1] <= b[j] <= b[j+1..k]
    # Sort b[h..j–1]  and  b[j+1..k]
    quick_sort (b, h, j–1)
    quick_sort (b, j+1, k)
```

- **Worst Case:** array already sorted
  - Or almost sorted
  - n^2 in that case
- **Average Case:** array is scrambled
  - n log n in that case
  - Best sorting time!



So Does that Solve It?

- Worst case still seems bad! Still n2
    - Only happens in small number of cases
    - Just happens that case is common (already sorted)
- Can greatly reduce issue with randomization
    - Swap start with random element in list
    - Now pivot is random and already sorted unlikely



h i k

Start: b | x | ? | y | ?

So Does that Solve It?

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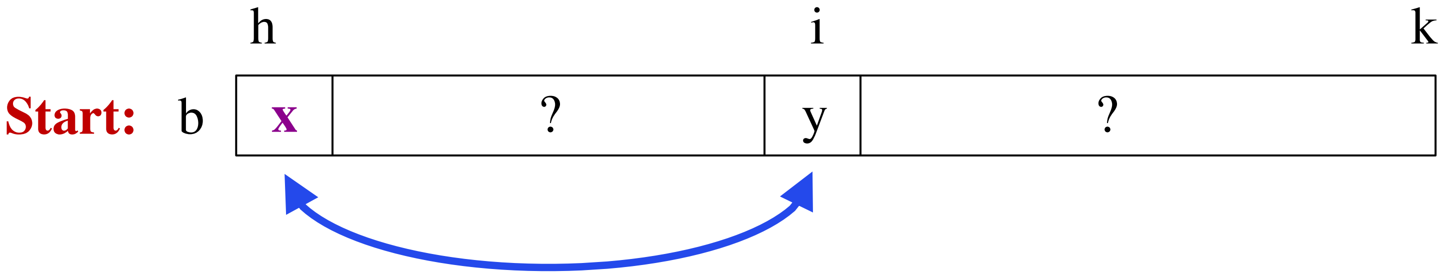
Makes it “good enough”
for most applications



Can We Do Better?

- There is guaranteed n log n sorting algorithm
  - Called **merge sort** (beyond scope of course)
  - Used heavily in large databases
  - But it has high overhead (slower on small data)
- What does the `sort()` method use?
  - Uses **Timsort** (invented by Tim Peters in 2002)
  - Combination of insertion sort and merge sort
  - Insertion on small data, merge sort on large

Can We Do Better?

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Quicksort is 1959!