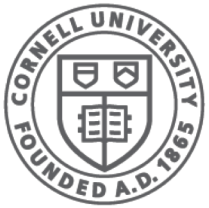
http://www.cs.cornell.edu/courses/cs1110/2019sp

Lecture 15: Recursion

(Sections 5.8-5.10)

CS 1110

Introduction to Computing Using Python



Cornell CIS
COMPUTING AND INFORMATION SCIENCE

[E. Andersen, A. Bracy, D. Gries, L. Lee, S. Marschner, C. Van Loan, W. White]

Recursion

Recursive Function:

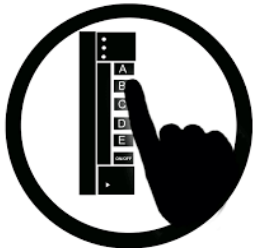
A function that calls itself

(see also Recursive Function)

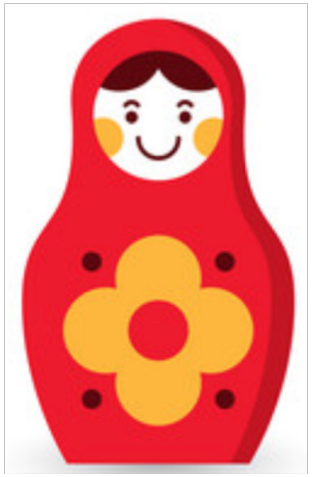
Two parts to every recursive function:

1. A simple case: can be solved easily
2. A complex case: can be made simpler (and simpler, and simpler… until it looks like the simple case)

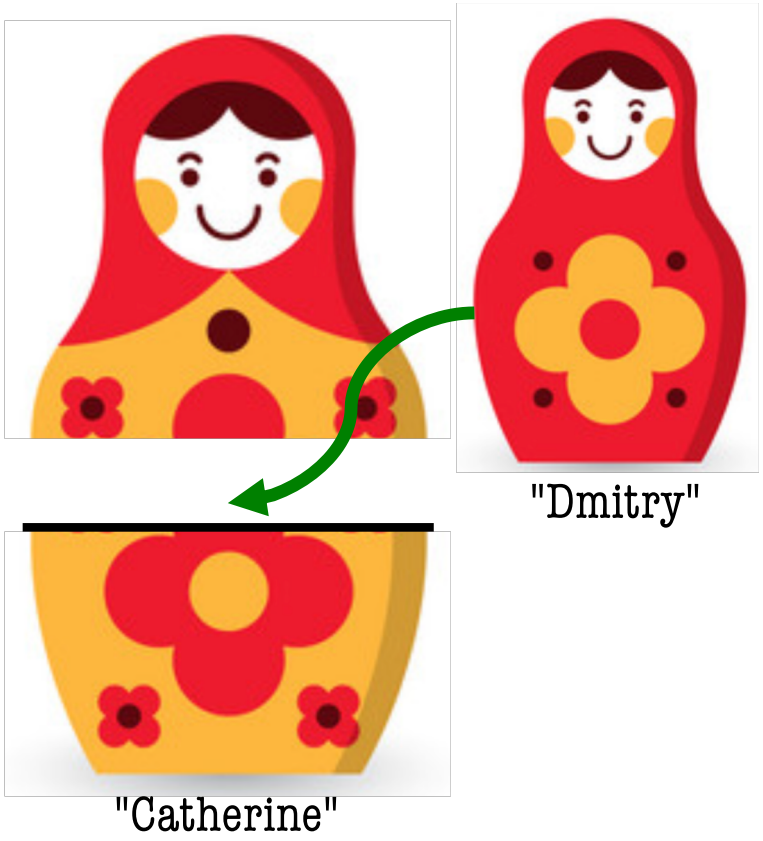
Russian Dolls!



What is the simple case that can be solved easily?

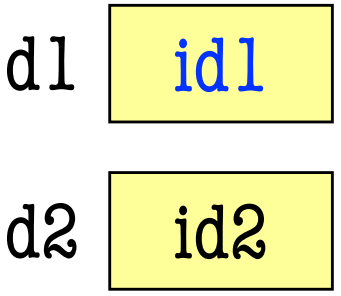


A: The case where the doll has a seam and another doll inside of it.
B: The case where the doll has no seam and no doll inside of it.
C: A & B are both simple
D: I do not know

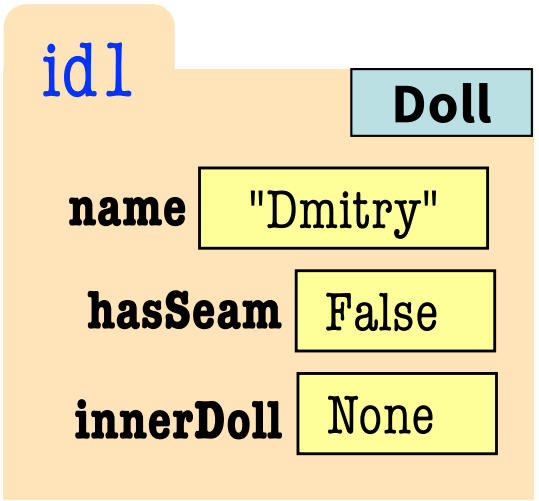


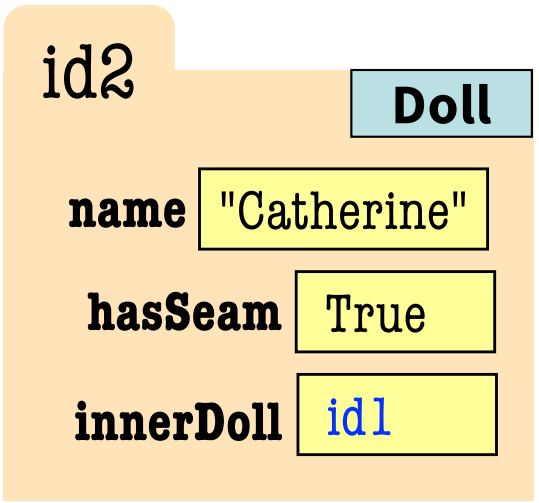
Russian Dolls!

Global Space



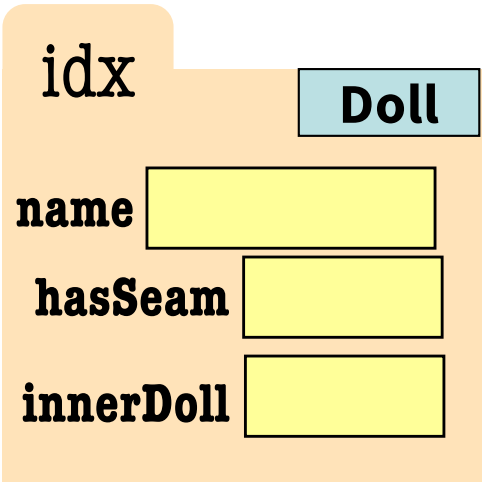
Heap Space





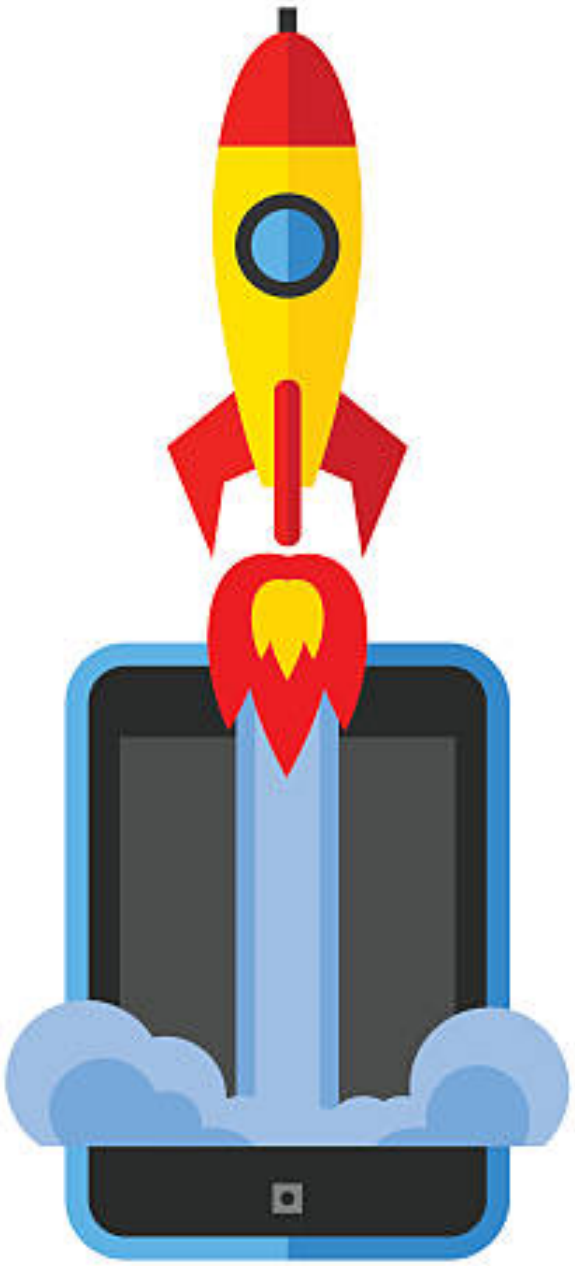

```
import russian
d1 = russian.Doll("Dmitry", None)
d2 = russian.Doll("Catherine", d1)
```

```
def open_doll(d):
    """Input: a Russian Doll
     Opens the Russian Doll d """
    print("My name is "+ d.name)
    if d.hasSeam:
        inner = d.innerDoll
        open_doll(inner)
    else:
        print("That's it!")
```



Examples

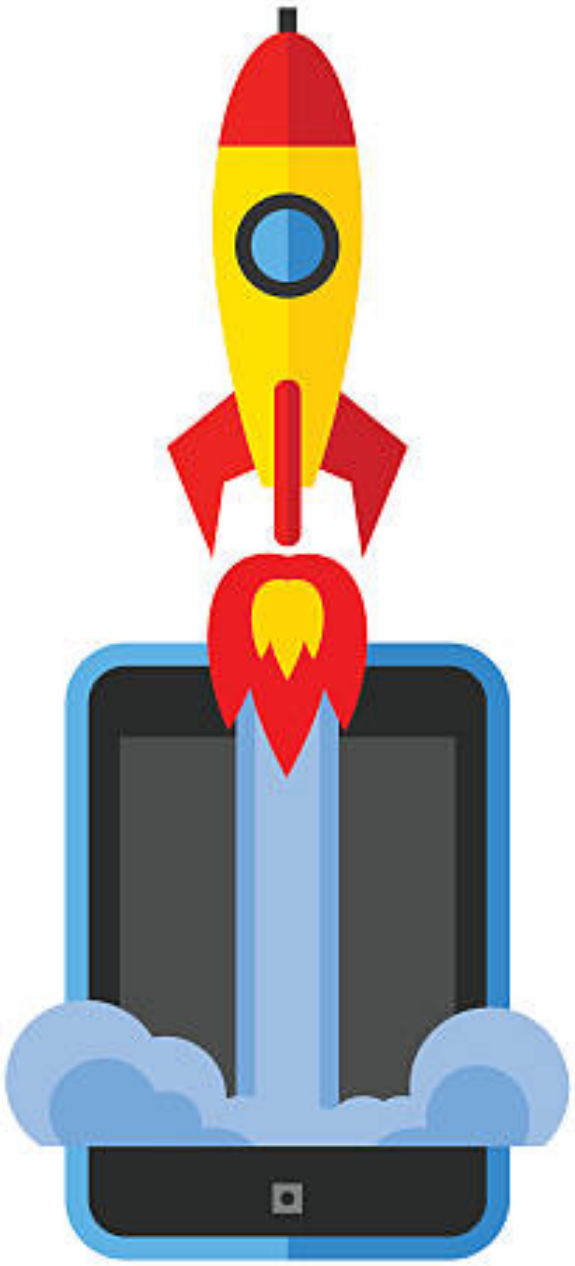
- Russian Dolls
- **Blast Off!**
- Towers of Hanoi



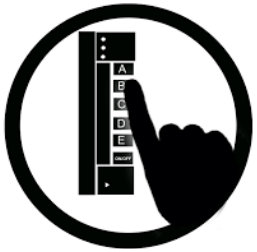
Blast Off!

```
blast_off(5) # must be a positive int
5
4
3
2
1
BLAST OFF!

blast_off(0)
BLAST OFF!
```



Blast Off!

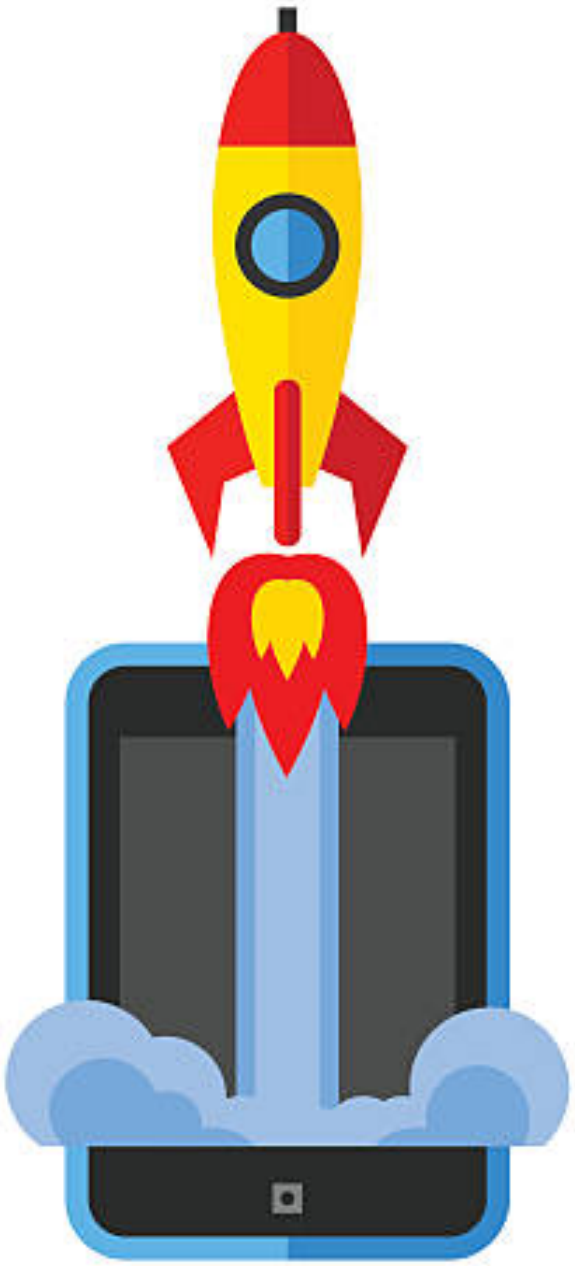


```
blast_off(5) # must be a positive int
5
4
3
2
1
BLAST OFF!


blast_off(0)
BLAST OFF!
```

What is the simple case that can be solved easily?

A: negative n
B: positive n
C: n == 0
D: n == 1
E: I do not know.

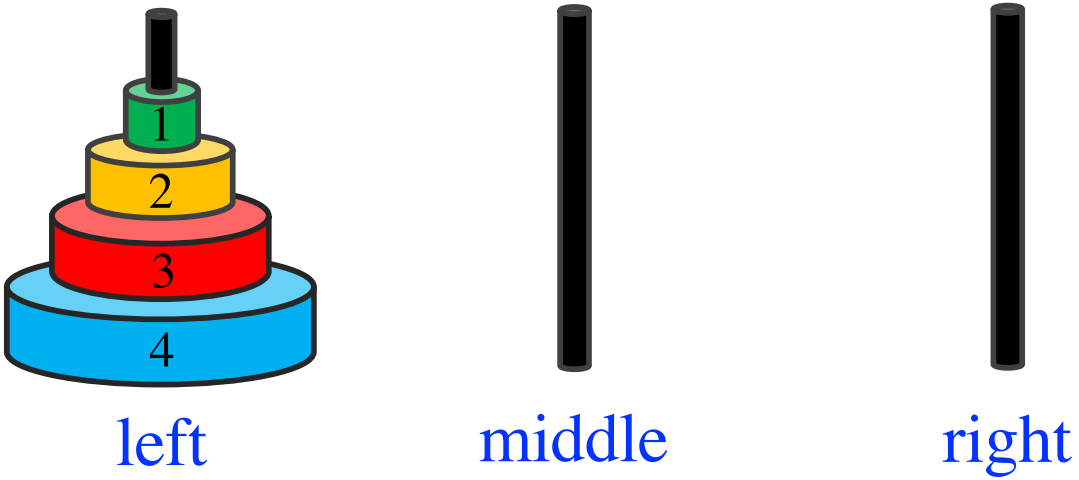


Blast Off!

```
def blast_off(n):
    """Input: a positive int
     Counts down from n to Blast-Off!
     """
    if (n == 0):
        print("BLAST OFF!")
    else:
        print(n)
        blast_off(n-1)
```

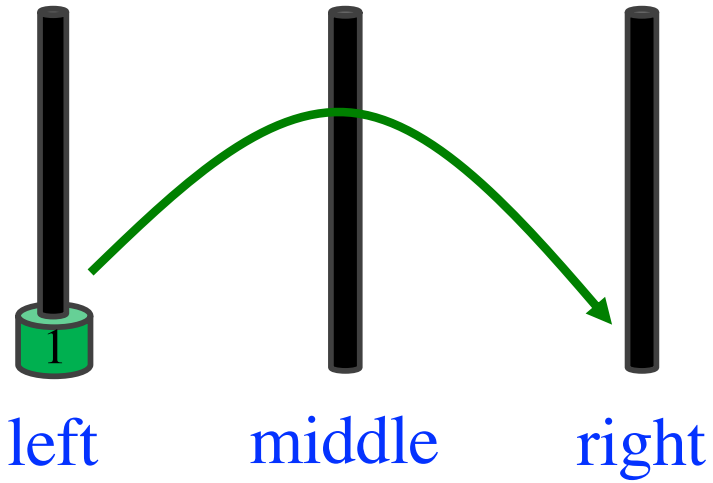
Tower of Hanoi

- Three towers: *left*, *middle*, and *right*
- *n* disks of unique sizes on *left*
- **Goal**: move all disks from *left* to *right*
- Cannot put a larger disk on top of a smaller disk



1 Disc: Easy!

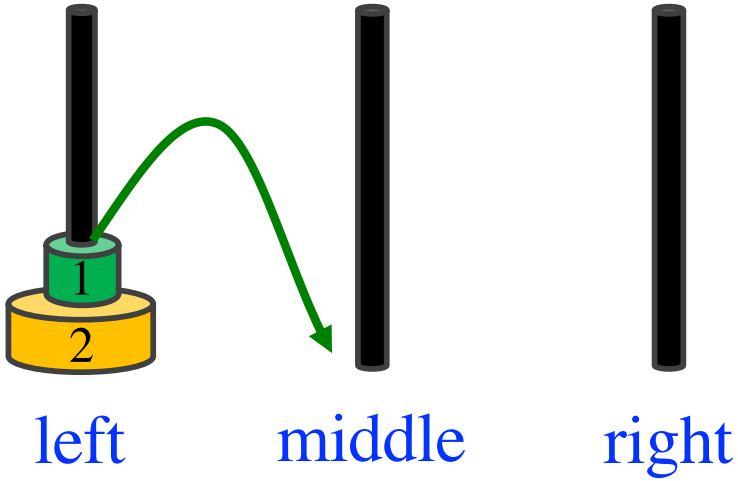
1. Move from *left* to *right*



Solving for 1 tower is easy! ***That's the simple case!***

2 Discs: Step 1

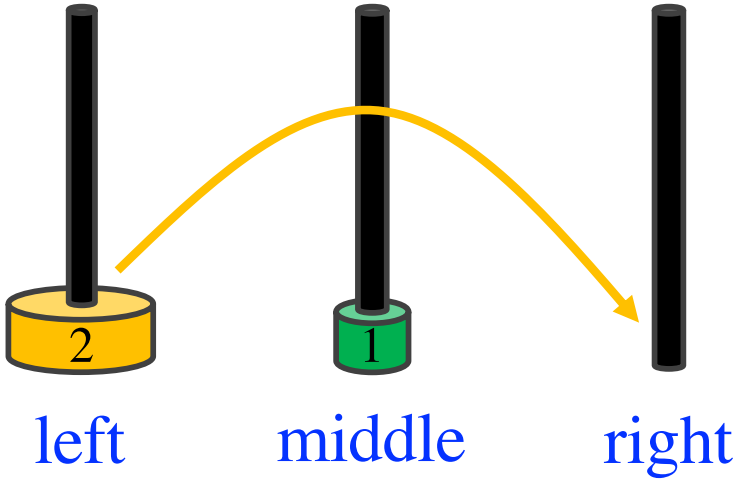
1. Move from *left* to *middle*



Thought: If I could get Disk 1 off of Disk 2, I could move Disk 2 to where it's supposed to go…. Moving 1 disk is easy!

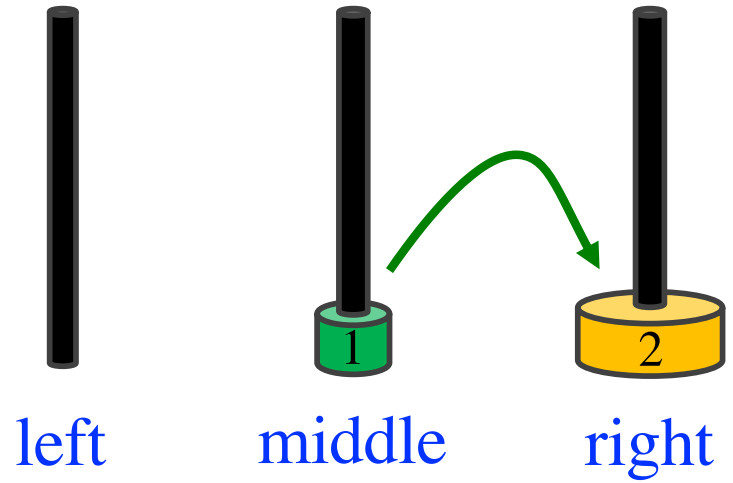
2 Discs: Step 2

1. Move from *left* to *middle*
2. Move from *left* to *right*



Thought: Now that Disk 1 is gone, I can move Disk 2 to where it's supposed to go.

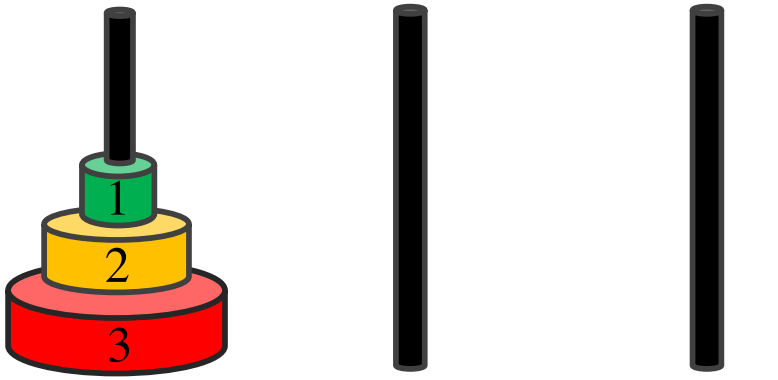
2 Discs: Step 3 (final)



1. Move from *left* to *middle*
2. Move from *left* to *right*
3. Move from *middle* to *right*

Thought: Now that Disk 2, is where it's supposed to be, all I have to do is move Disk 1. Moving 1 disk is easy!

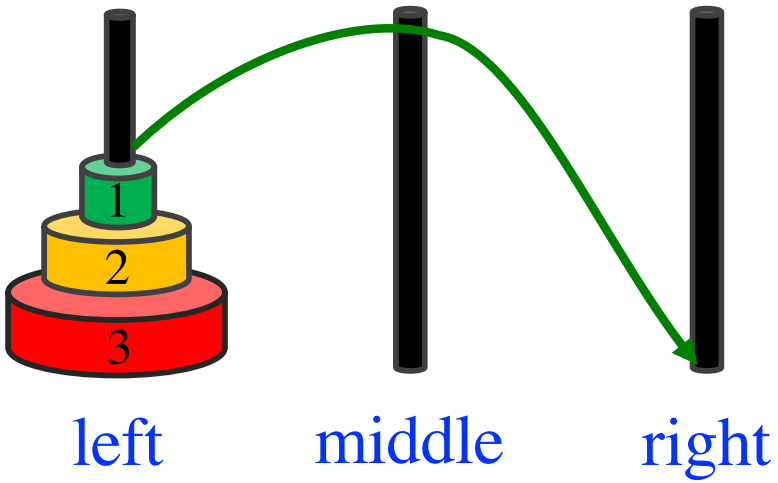
3 Discs!



Thought: If I could get Disks 1 & 2 off of Disk 3, I could move Disk 3 to where it's supposed to go…. And I know how to move 2 Disks from the previous slide!

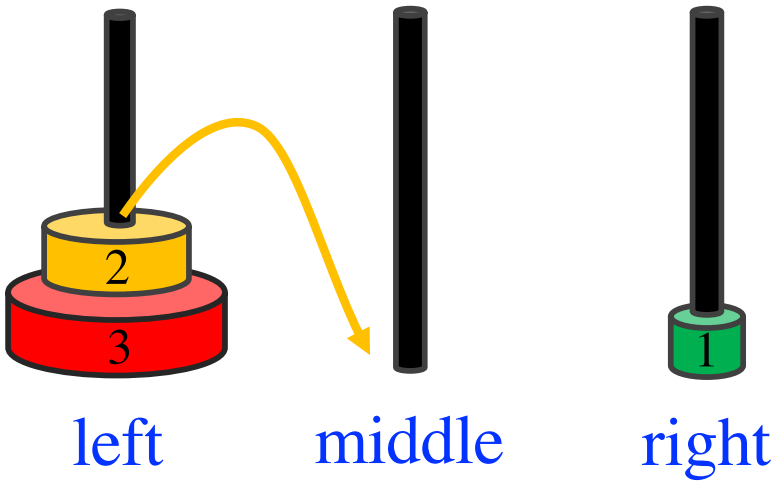
3 Discs: Moving Disks 1&2 off of Disk 3 (1)

1. Move from *left* to *right*

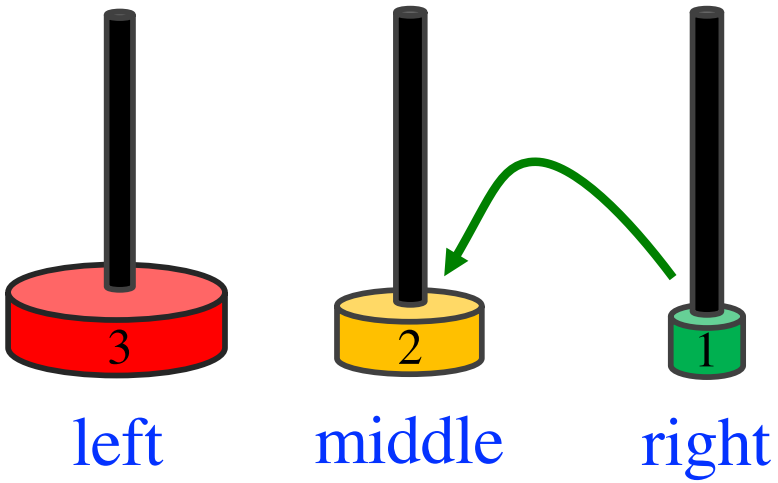


3 Discs: Moving Disks 1&2 off of Disk 3 (2)

1. Move from *left* to *right*
2. Move from *left* to *middle*

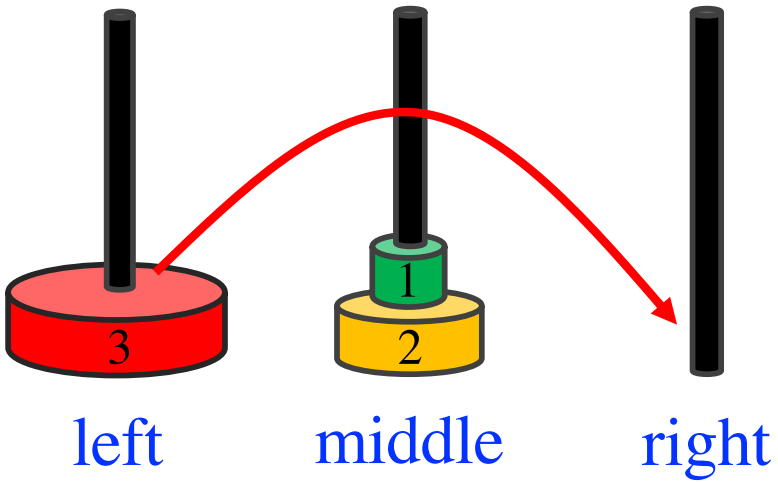


3 Discs: Moving Disks 1&2 off of Disk 3 (3)



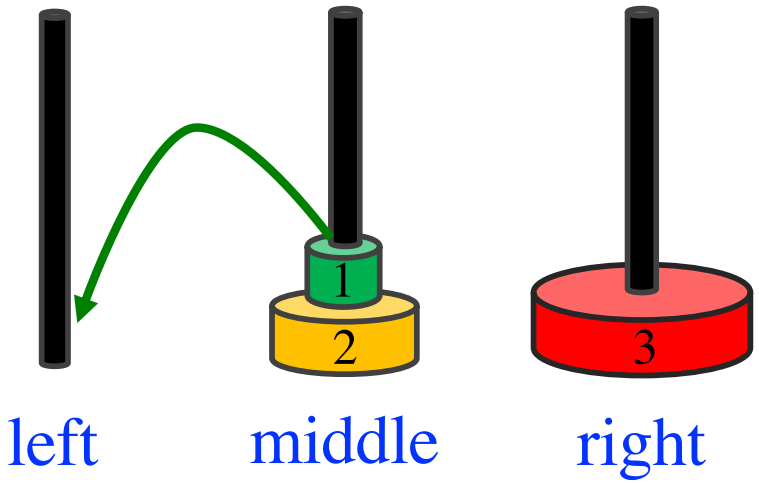
1. Move from *left* to *right*
2. Move from *left* to *middle*
3. Move from *right* to *middle*

3 Discs: Move Disk 3 to the Goal



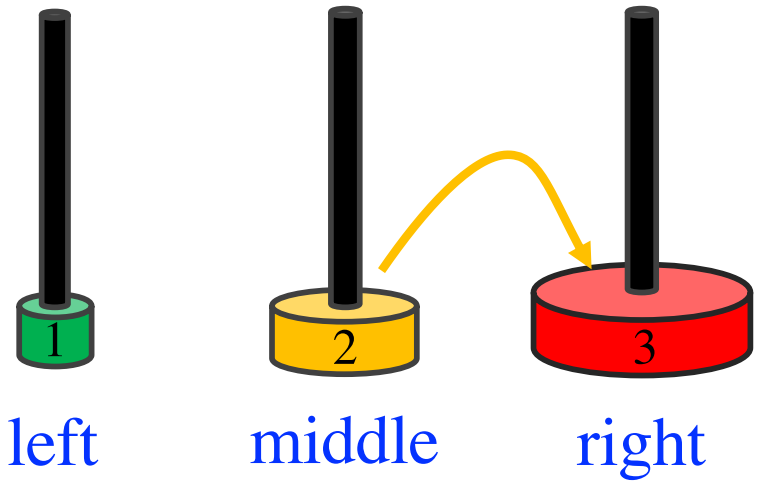
1. Move from *left* to *right*
2. Move from *left* to *middle*
3. Move from *right* to *middle*
4. Move from *left* to *right*

3 Discs: Moving Disks 1&2 to the Goal (1)



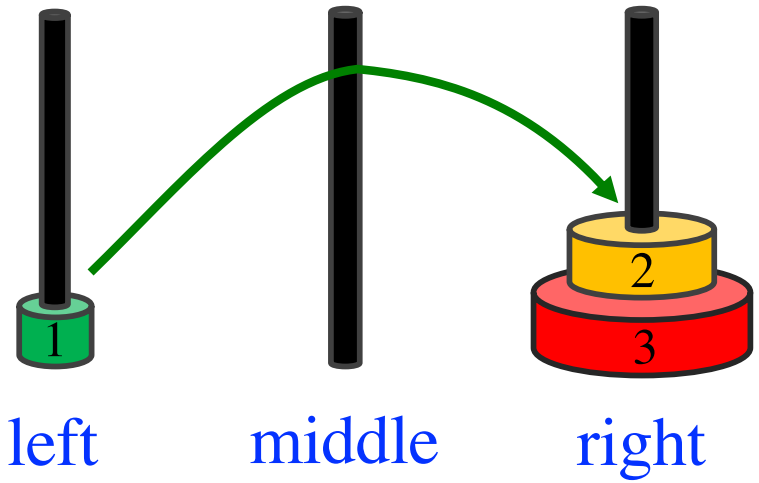
1. Move from *left* to *right*
2. Move from *left* to *middle*
3. Move from *right* to *middle*
4. Move from *left* to *right*
5. Move from *middle* to *left*

3 Discs: Moving Disks 1&2 to the Goal (2)



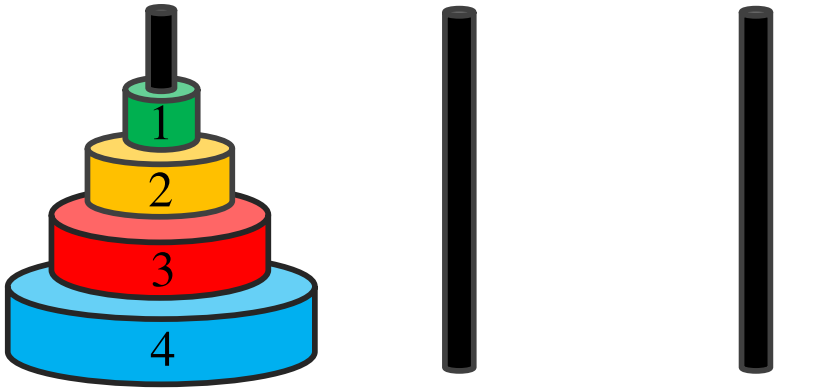
1. Move from *left* to *right*
2. Move from *left* to *middle*
3. Move from *right* to *middle*
4. Move from *left* to *right*
5. Move from *middle* to *left*
6. Move from *middle* to *right*

3 Discs: Moving Disks 1&2 to the Goal (3)



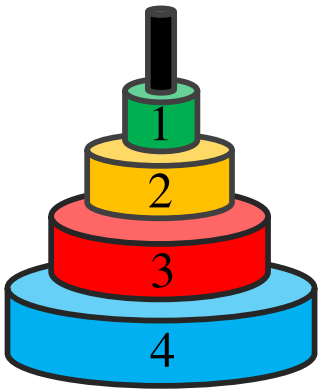
1. Move from *left* to *right*
2. Move from *left* to *middle*
3. Move from *right* to *middle*
4. Move from *left* to *right*
5. Move from *middle* to *left*
6. Move from *middle* to *right*
7. Move from *left* to *right*

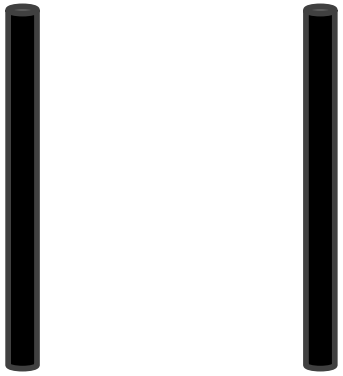
4 Discs: Oh, boy...



Thought: If I could get Disks 1&2&3 off of Disk 4, I could move Disk 4 to where it's supposed to go…. And I know how to move 3 Disks from the previous slide!

Rely on the solution for the simpler case

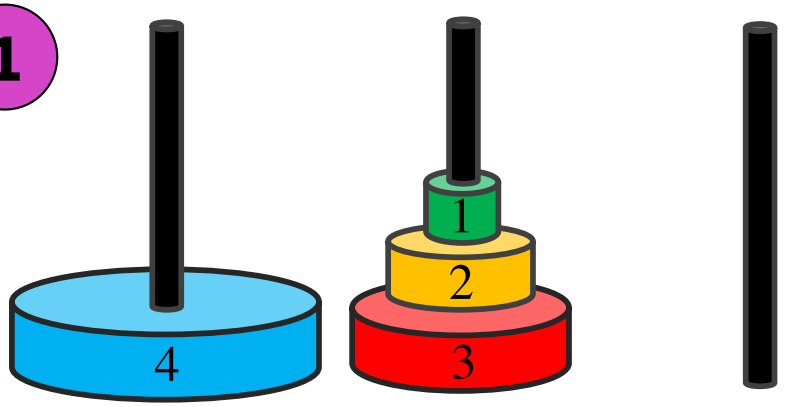




Hanoi(4,L→R)

=

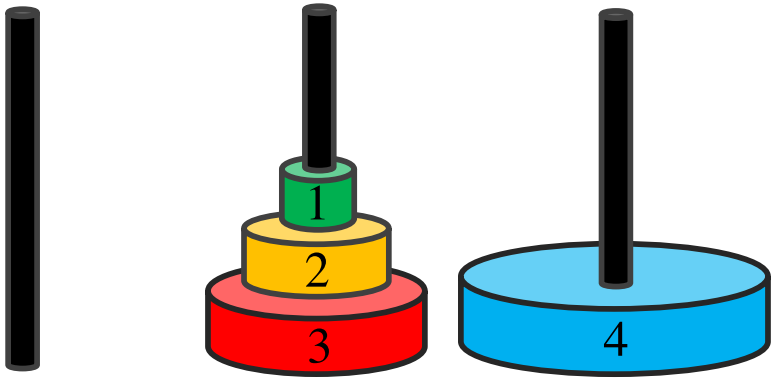
1



Hanoi(3,L→M)

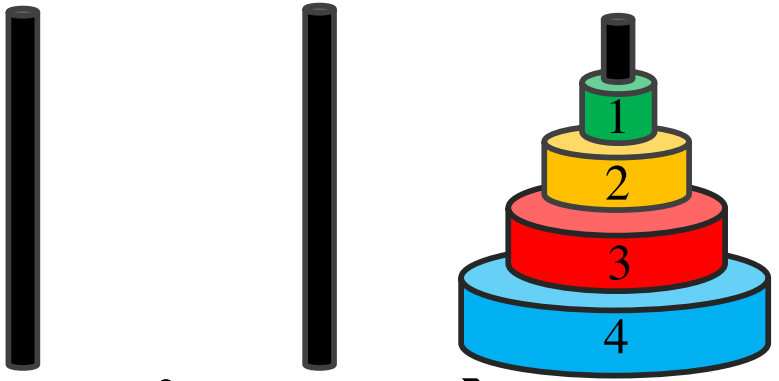
(uncover the big one)

2



move the big one

3



Hanoi(3,M→R)

(cover the big one)

solve_hanoi(n, start, goal, temp)

"""Prints instructions for how to move n disks (sorted small to large, going down) from the start peg to the goal peg, using the temp peg if needed. """

```
if n == 1:
    print("move from "+start+" to "+goal)
else:
    # need to move top n-1 disks from start to temp so that I can move
    # the bottom disk to goal... luckily, I have a function that does that!
1   solve_hanoi(n-1, start, temp, goal)

    # move the bottom disk from start to goal
2   print("move from "+ start +" to "+ goal)

    # now put everything back on the last disk at goal
3   solve_hanoi(n-1, temp, goal, start)
```

Divide and Conquer

Goal: Solve really big problem P

Idea: Split into simpler problems, solve, combine

3 Steps:

1. Decide what to do for simple cases
2. Decide how to break up the task
3. Decide how to combine your work

Recursion vs Iteration

- **Recursion** is ***provably equivalent*** to **iteration**
  - Iteration includes **for-loop** and **while-loop** (later)
  - Anything can do in one, can do in the other
- But some things are easier with recursion
  - And some things are easier with iteration
- Will **not** teach you when to choose recursion
- We just want you to ***understand the technique***