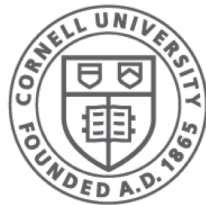


<http://www.cs.cornell.edu/courses/cs1110/2018sp>

# Lecture 25: Sequence Algorithms

CS 1110

Introduction to Computing Using Python



**Cornell CIS**  
COMPUTING AND INFORMATION SCIENCE

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# Announcements

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- Prelim 2 is graded
- A5 is out!
  - Clarification on webpage about what happens when the deck has too few cards in it
- Final Lab is this week:
  - **Reminder:** labs **cannot** be checked off during Wed consulting hours the week after they are released
- Final Exam:
  - May 17<sup>th</sup>, 9am-11:30am
  - **Location:** Barton Hall Central and East

# Box Notation for Sequences

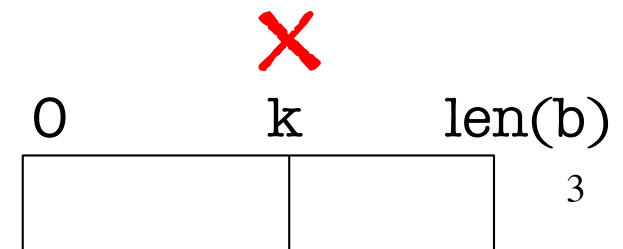
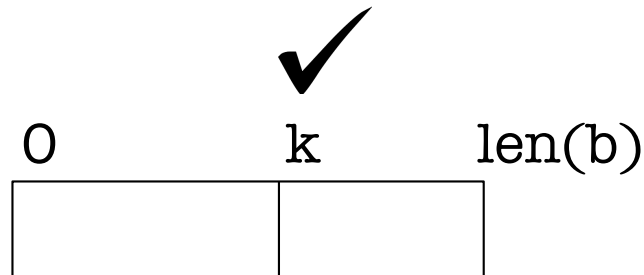
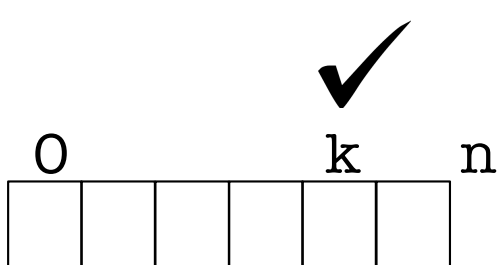


Graphical assertion about sequence b. It asserts that:

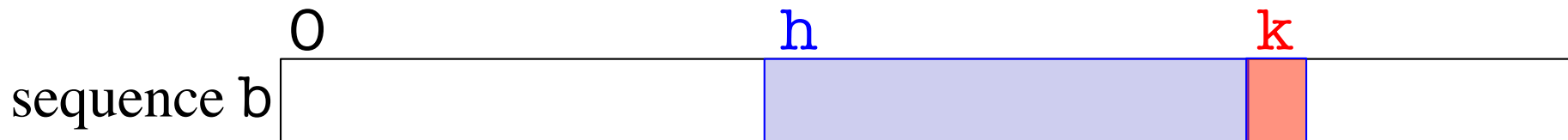
1.  $b[0..k-1]$  is sorted (values are in ascending order)
2. all of  $b[0..k-1]$  is  $\leq$  all of  $b[k..len(b)-1]$

## Pro Tip #1:

index always goes *above a box*, never above a line  
(just like house numbers go on a house not between the houses)



# Q: Indices for Box Notation



Given:

- index **h** of the **first element** of a segment
- index **k** of the **element that follows** that segment,

Questions:

1. How many values are in **segment**  $b[h .. k - 1]$
2. How many values are in  $b[h .. h - 1]$  ?
3. How many values are in  $b[h .. h + 1]$  ?

A: 0  
B: 1  
C: 2  
D:  $k - h$   
E:  $k + h$

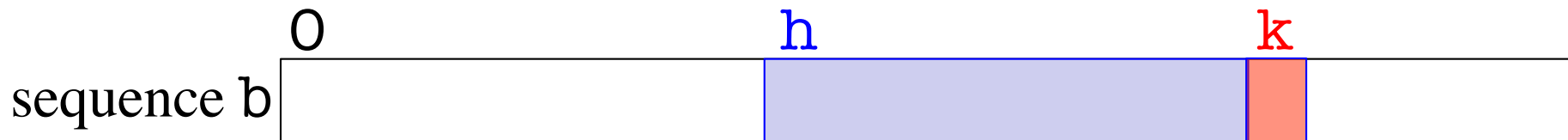
**Pro Tip #2:**

Size is “Follower minus First”

Follower: next thing outside the specified range

# A: Indices for Box Notation

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Given:

- index **h** of the **first element** of a segment
- index **k** of the **element that follows** that segment,

Questions:

1. How many values are in **segment**  $b[h .. k - 1]$  **D**
2. How many values are in  $b[h .. h - 1]$  ? **A**
3. How many values are in  $b[h .. h + 1]$  ? **C**

A: 0  
B: 1  
C: 2  
D:  $k - h$   
E:  $k + h$

# count num adjacent equal pairs (1)

---

Approach #1: compare  $s[k]$  to the character in front of it ( $s[k-1]$ )

# set `n_pair` to # adjacent equal pairs in `s`

```
n_pair = 0
```

```
k = 1
```

```
while k < len(s):  
    if s[k-1] == s[k]:  
        n_pair += 1  
    k = k + 1
```

# count num adjacent equal pairs (2)

---

Approach #1: compare  $s[k]$  to the character in front of it ( $s[k-1]$ )

# set  $n\_pair$  to # adjacent equal pairs in  $s$

pre: seq s 

0	n
<i>? (unknown values)</i>	

 $n \geq 0, n\_pair = 0$

$n\_pair = 0$

$k = 1$

INV: seq s 

0	k	n
<i>processed</i>	<i>? (unknown)</i>	

 $n\_pair = \text{num adjacent pairs in } s[0..k-1]$

**while**  $k < \text{len}(s)$ :

    if  $s[k-1] == s[k]$ :

$n\_pair += 1$

$k = k + 1$

post: seq s 

0	n
<i>processed</i>	

 $n\_pair = \text{num adjacent pairs in } s[0..n-1]$

# find the max of a seq (1)

---

**Task: find the maximum of a sequence s**

```
k = 1
```

```
big = s[0]
```

```
while k < len(s):
```

```
    big = max(big, s[k])
```

```
    k = k + 1
```



# find the max of a seq (2)

**Task: find the maximum of a sequence s**

pre: s 

0		n
↙	? (unknown values)	

 $n > 0$ , big = s[0]

big is the max of this segment (s[0])

k = 1

big = s[0]

inv: s 

0		k		n
	<i>big is max of this segment</i>		?	

 $n > 0$ , big = s[0..k-1]

while k < len(s):

big = max(big, s[k])

k = k + 1

post: s 

0		n
	<i>big is the max of this segment</i>	

 k = n, big = max of s[0..n-1]

# Developing Algorithms on Sequences

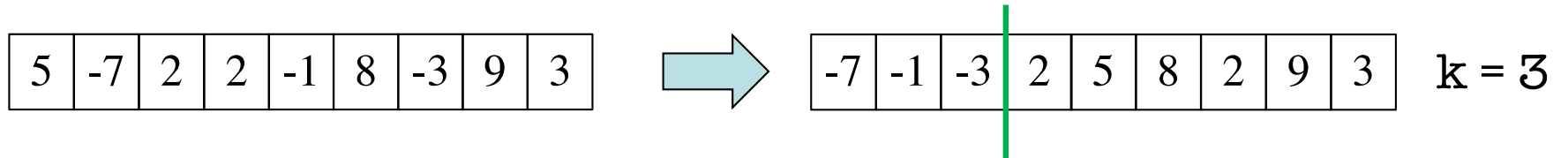
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- Specify the algorithm by giving its **precondition** and **postcondition** as pictures.
- Draw the **invariant** by drawing another picture that “moves from” the **precondition** to the **postcondition**
  - The invariant is true at the beginning and at the end
- The four loop design questions
  1. How does loop start (how to make the invariant true)?
  2. How does it stop (is the postcondition true)?
  3. How does the body make progress toward termination?
  4. How does the body keep the invariant true?

# Task: separate + from - in a list

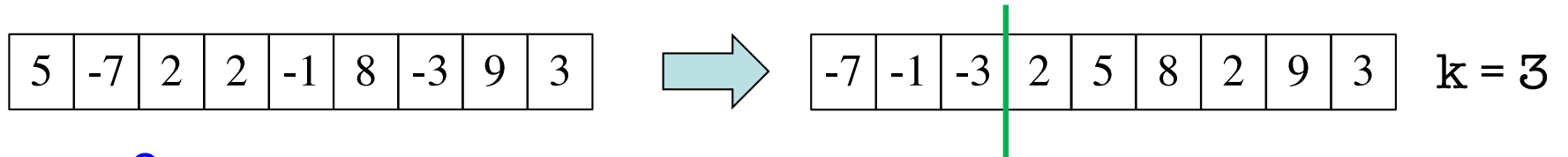
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Task: Put negative values before nonnegative ones and return the split index



# Invariants: separate + from - in a list

Task: Put negative values before nonnegative ones and return the split index



pre:  $s$   $\begin{matrix} 0 & & n \\ \boxed{\text{? (unknown)}} \\ n \geq 1 \end{matrix}$

$k = 0$

$j = n$

inv:  $s$   $\begin{matrix} 0 & & k \rightarrow & \leftarrow j & & n \\ \boxed{< 0 \quad \quad \quad ? \quad \quad \quad \geq 0} \end{matrix}$

$s[0..k-1]$  negative

$s[j..n-1]$  zero or +

$s[k..j-1]$  unknown

while  $k < j$ :

<body goes here>

post:  $s$   $\begin{matrix} 0 & & k & & n \\ \boxed{< 0 \quad \quad \quad \geq 0} \\ k = j \end{matrix}$

# Body: separate + from - in a list

k = 0

j = n

inv: s 

< 0	?	>= 0
-----	---	------

s[0..k-1] negative  
s[j..n-1] zero or +  
s[k..j-1] unknown

while k < j:

if s[k] < 0: # kth elem stays where it is

k = k + 1

elif s[j-1] >= 0: # (j-1)th elem stays where it is

j = j - 1

else: # both elements in the wrong place

swap(s, k, j-1)

k = k + 1

j = j - 1

post: s 

< 0	>= 0
-----	------

k = j