http://www.cs.cornell.edu/courses/cs1110/2018sp

Lecture 15: Recursion

(Sections 5.8-5.10)

CS 1110

Introduction to Computing Using Python

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Announcements

Prelim 1

- Graded. 2 Emails went out (1 from gradescope, 1 from CMS) and some #s released
- **Regrade requests**: we are processing them
- **Q3**: we are re-evaluating them

Lab 8: alive!

(There was no Lab 7.)

Recursion

- **Recursive Function**:

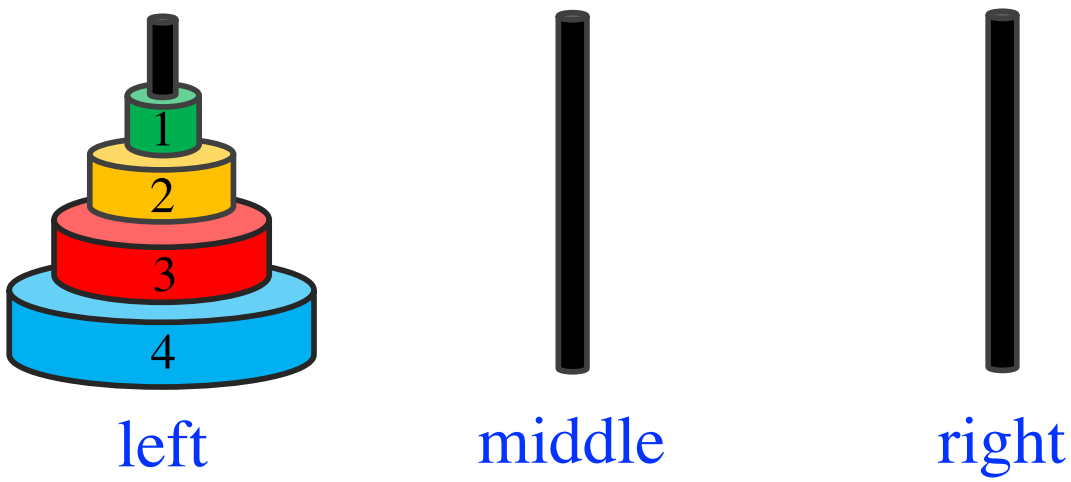
  A function that calls itself (directly or indirectly)

Examples

- **Blast-off**
- **Gift in gift**
- **Family Trees**
- Towers of Hanoi
- Deblanking

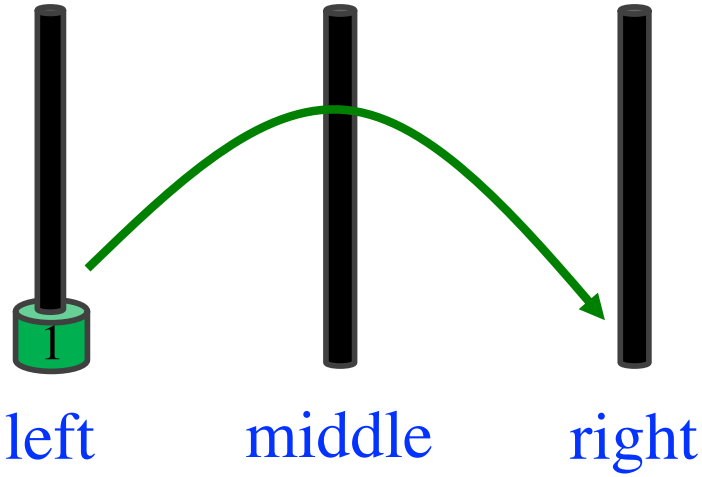
Tower of Hanoi

- Three towers: *left*, *middle*, and *right*
- *n* disks of unique sizes on *left*
- **Goal**: move all disks from *left* to *right*
- Cannot put a larger disk on top of a smaller disk



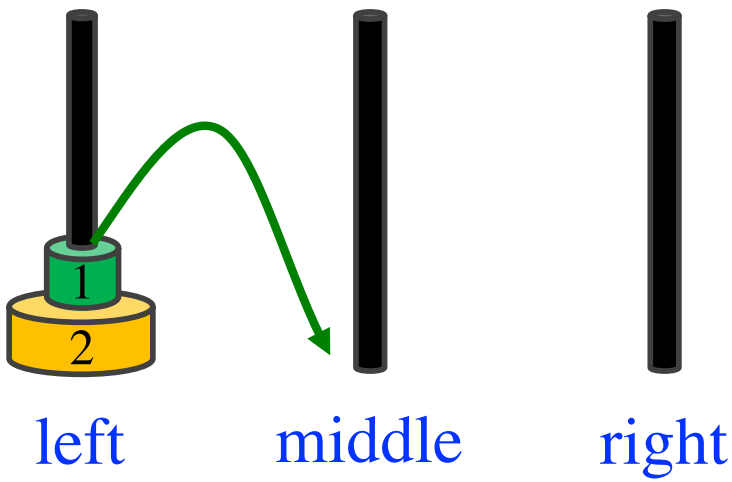
1 Disc: Easy!

1. Move from *left* to *right*



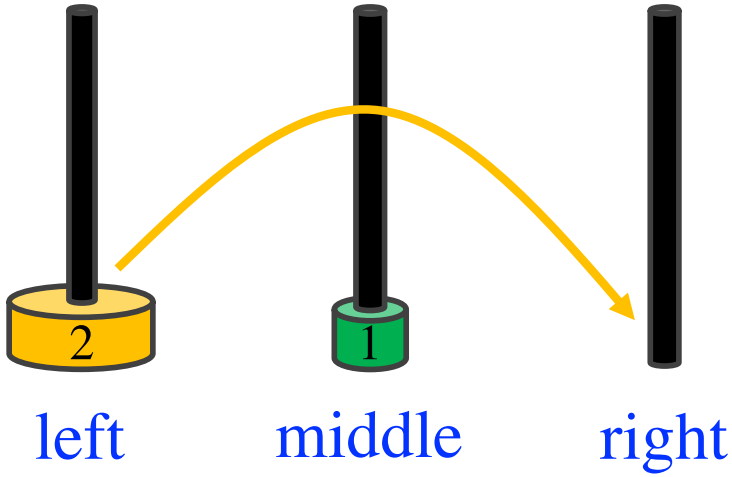
2 Discs: Step 1

1. Move from *left* to *middle*

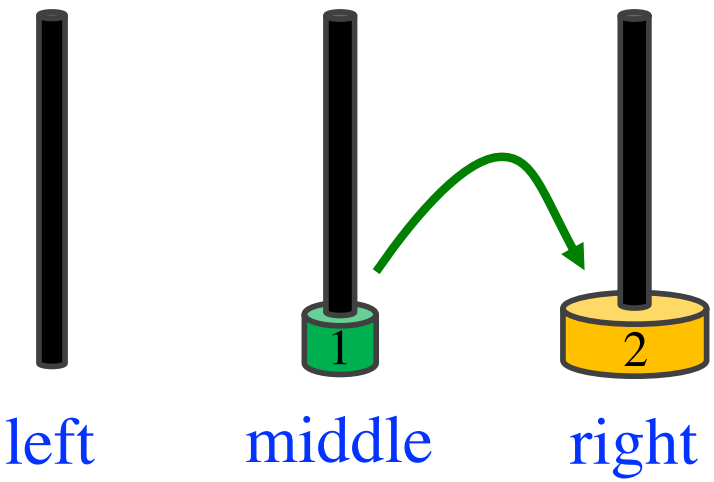


2 Discs: Step 2

1. Move from *left* to *middle*
2. Move from *left* to *right*



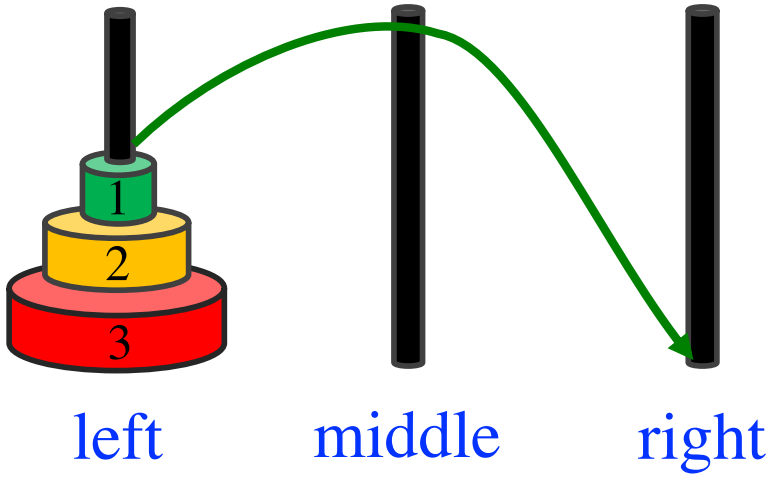
2 Discs: Step 3 (final)



1. Move from *left* to *middle*
2. Move from *left* to *right*
3. Move from *middle* to *right*

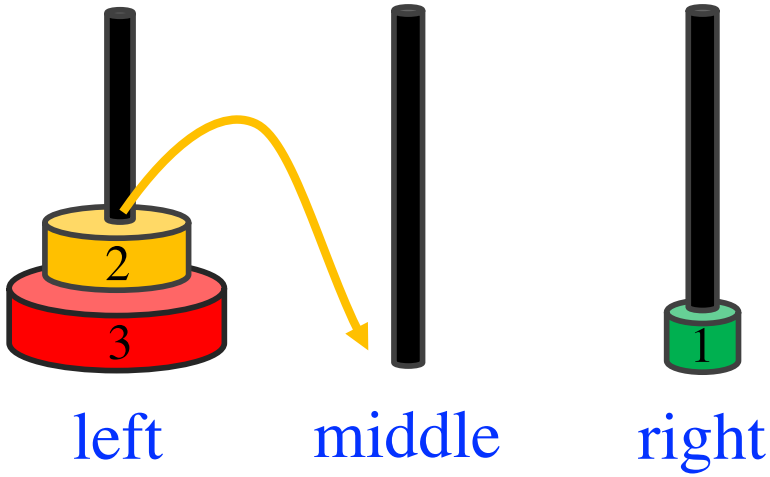
3 Discs: Step 1

1. Move from *left* to *right*

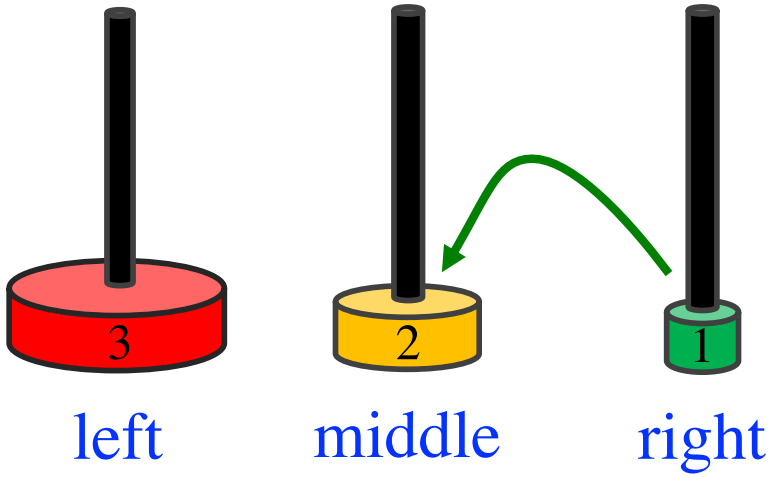


3 Discs: Step 2

1. Move from *left* to *right*
2. Move from *left* to *middle*

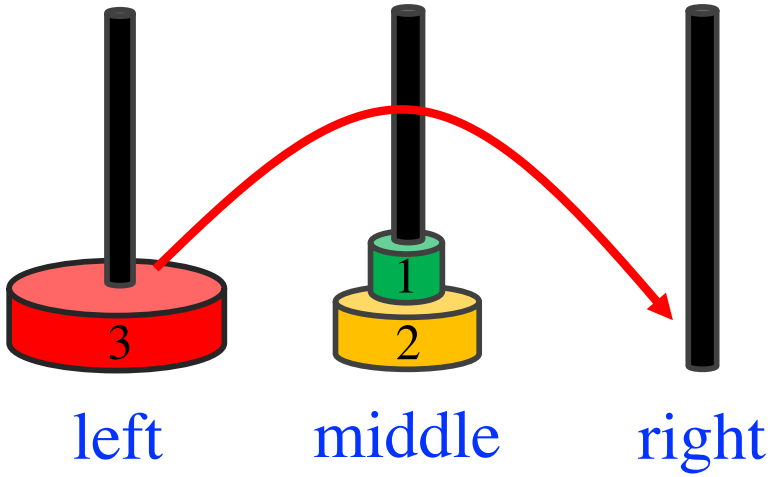


3 Discs: Step 3



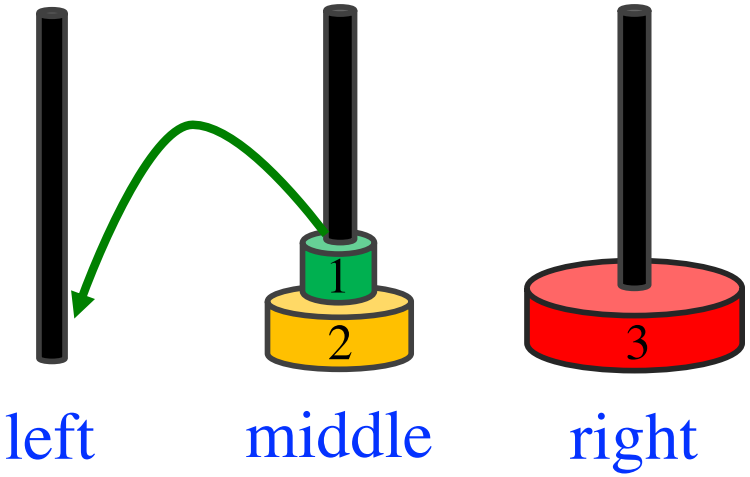
1. Move from *left* to *right*
2. Move from *left* to *middle*
3. Move from *right* to *middle*

3 Discs: Step 4



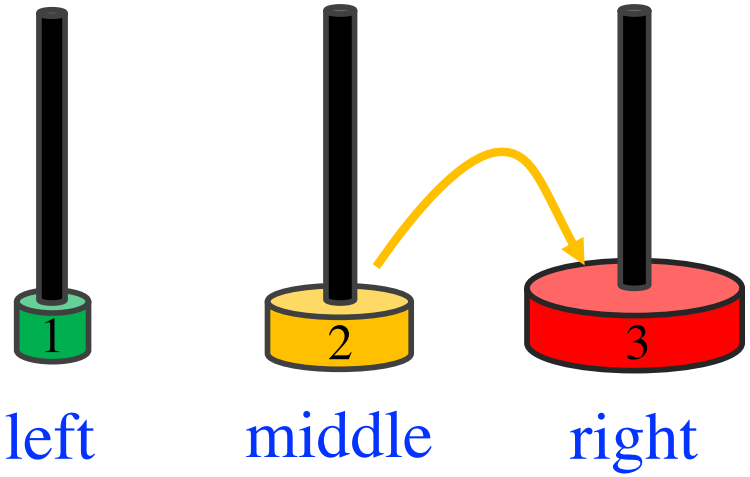
1. Move from *left* to *right*
2. Move from *left* to *middle*
3. Move from *right* to *middle*
4. Move from *left* to *right*

3 Discs: Step 5



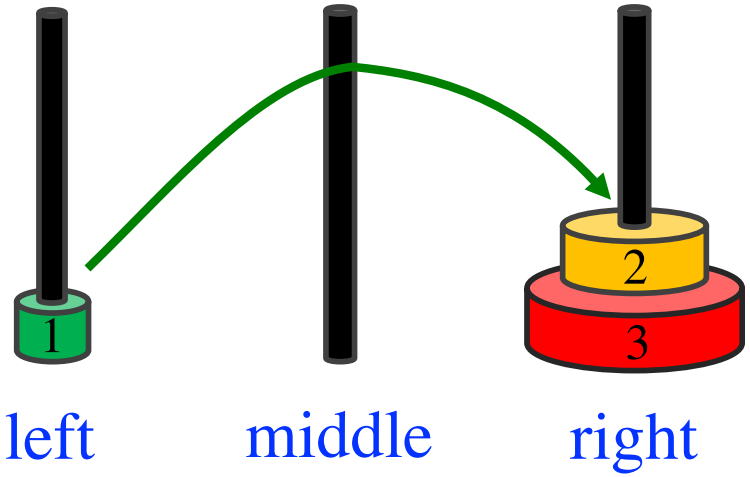
1. Move from *left* to *right*
2. Move from *left* to *middle*
3. Move from *right* to *middle*
4. Move from *left* to *right*
5. Move from *middle* to *left*

3 Discs: Step 6



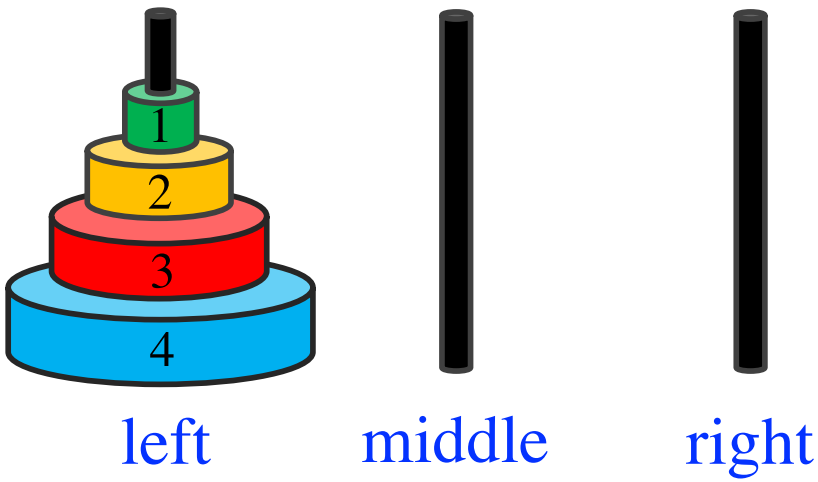
1. Move from *left* to *right*
2. Move from *left* to *middle*
3. Move from *right* to *middle*
4. Move from *left* to *right*
5. Move from *middle* to *left*
6. Move from *middle* to *right*

3 Discs: Step 7 (final)



1. Move from *left* to *right*
2. Move from *left* to *middle*
3. Move from *right* to *middle*
4. Move from *left* to *right*
5. Move from *middle* to *left*
6. Move from *middle* to *right*
7. Move from *left* to *right*

4 Discs: Oh, boy...



BUT, we already know how to solve Hanoi for 3 towers.

Sooo….

Divide and Conquer

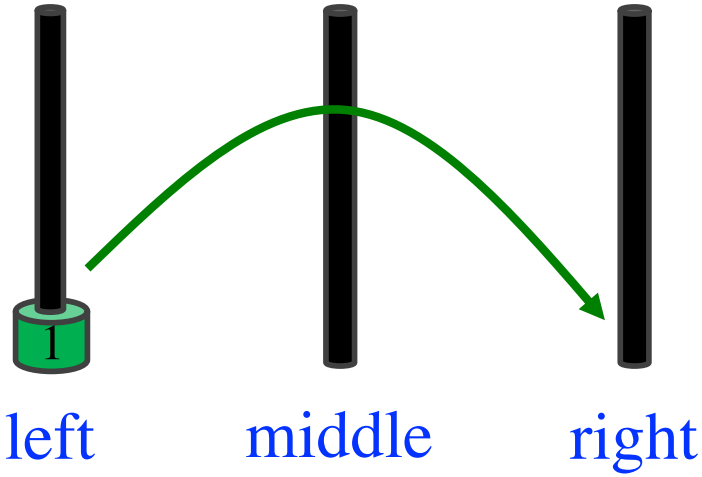
Goal: Solve really big problem P

Idea: Split into smaller problems, solve, combine

3 Steps:

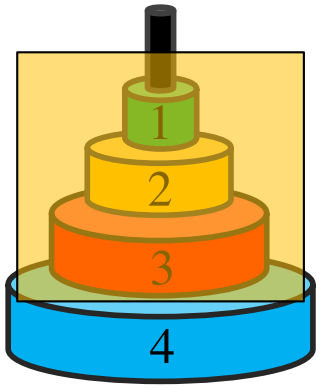
1. Decide what to do for simple cases
2. Decide how to break up the task
3. Decide how to combine your work

Decide what to do for simple cases



Move from *left* to *right*

Decide how to break up the task

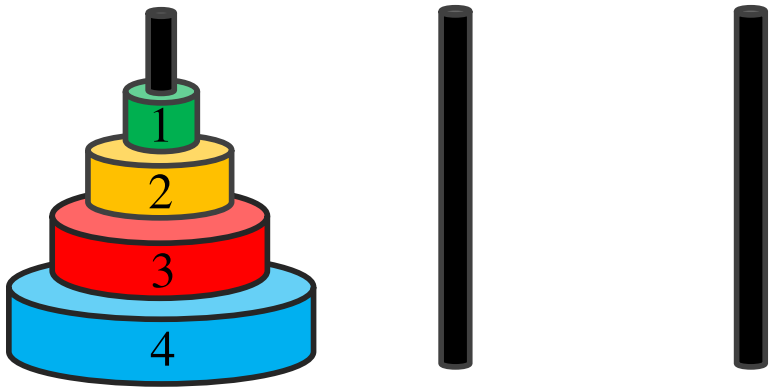


left

← Simpler than the original task, slowly becoming the "simple case"

← Simple case

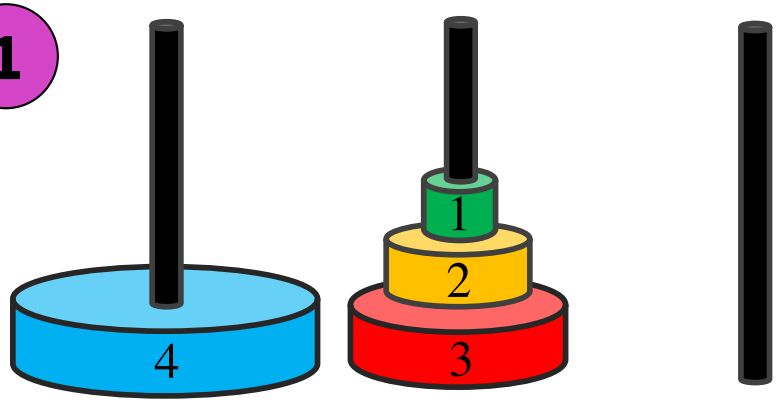
Decide how to combine your work



Hanoi(4)

=

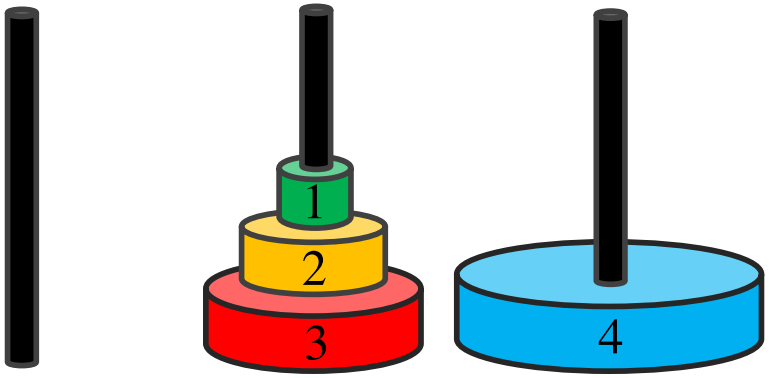
1



Hanoi(3)

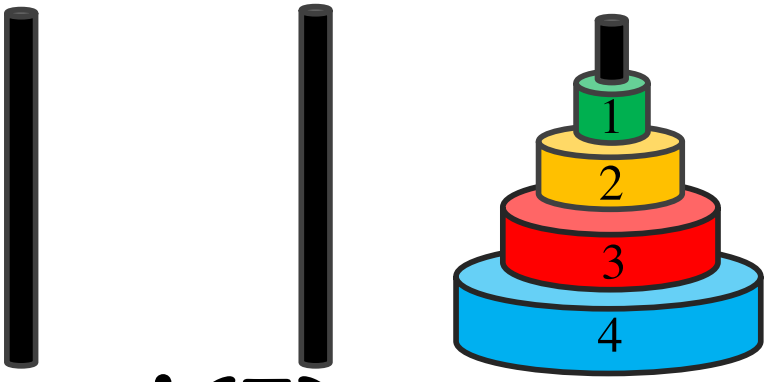
(uncover the big one)

2



move the big one

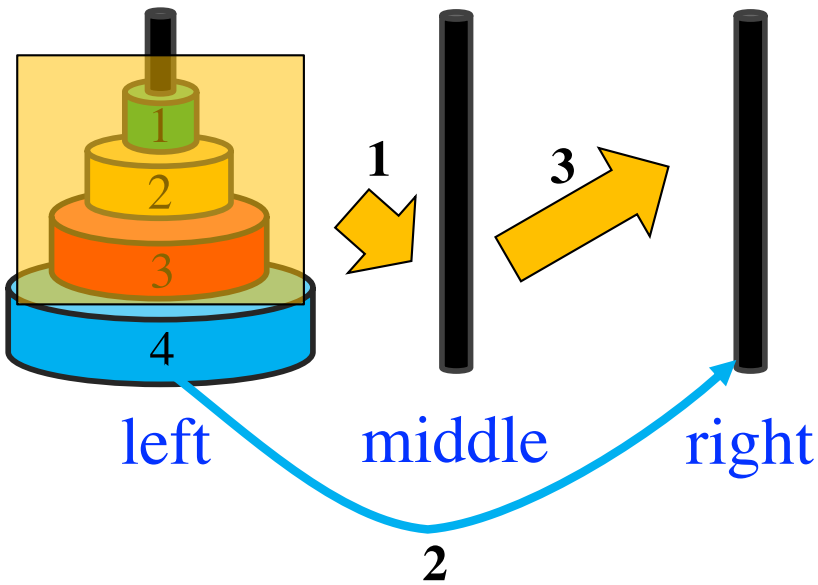
3



Hanoi(3)

(cover the big one)

4 Discs: High-level Idea



1. Move top three disks from *left* to *middle*
2. Move largest disk from *left* to *right*
3. Move top three disks from *middle* to *right*

solve_hanoi(source, target, other, n_disks)

"""Prints instructions for how to move n_disks (sorted small to large, going down) from the source peg to the target peg, using the other peg if needed. """

```
if (n== 1):
    print("move from "+source+" to "+target)
else:
    # need to move top n_disks-1 disks from source to
    # other... luckily, I have a function that does that!
    solve_hanoi(source, other, target, n_disks-1)

    # move the last disk to the target, that's easy!
    print("move from "+source+" to "+target)

    # now put everything back on the last disk at target
    solve_hanoi(other, target, source, n_disks-1)
```

Recursion vs Iteration

- **Recursion** is ***provably equivalent*** to **iteration**
  - Iteration includes **for-loop** and **while-loop** (later)
  - Anything can do in one, can do in the other
- But some things are easier with recursion
  - And some things are easier with iteration
- Will **not** teach you when to choose recursion
- We just want you to ***understand the technique***

Examples

- Blast-off
- Gift in gift
- Family Trees
- Towers of Hanoi
- **Deblanking**

Divide and Conquer

Goal: Solve problem P on a piece of data

Divide and Conquer

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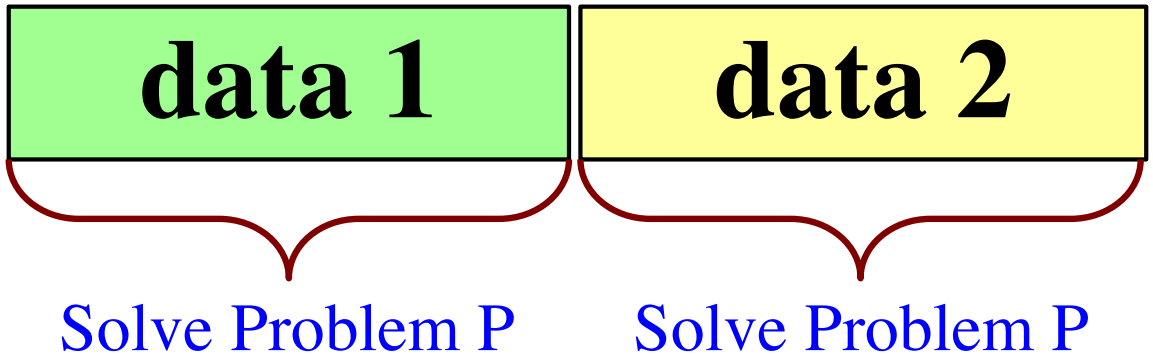
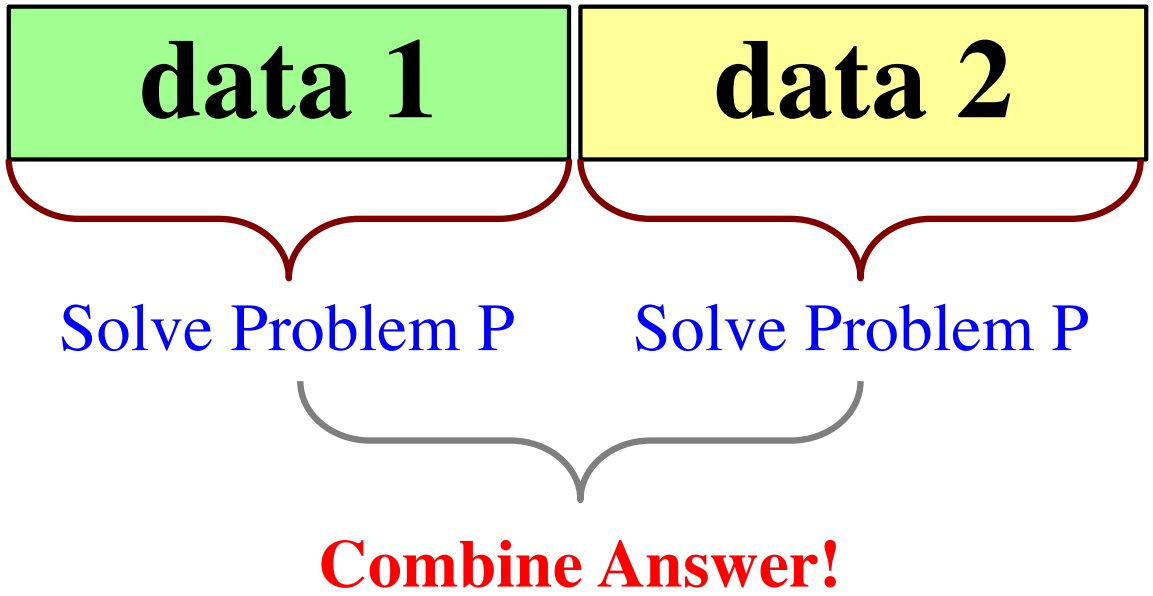
Idea: Split data into two parts and solve problem



Divide and Conquer

Goal: Solve problem P on a piece of data

Idea: Split data into two parts and solve problem



Exercise: Remove Blanks from a String

```
def deblank(s):
    """Returns: s but with its blanks removed"""
```

1. Decide what to do on "small" data

- If it is the **empty string**, nothing to do

```
if s == '':
    return s
```

- If it is a **single character**, delete it if a blank

```
if s == ' ':     # There is a space here
    return ''  # Empty string
else:
    return s
```

Exercise: Remove Blanks from a String

```
def deblank(s):
    """Returns: s but with its blanks removed"""
```

2. Decide how to break it up

```
left = deblank(s[0])      # A string with no blanks
right = deblank(s[1:])    # A string with no blanks
```

3. Decide how to combine the answer

```
return left+right         # String concatenation
```

Putting it All Together

```
def deblank(s):
    """Returns: s w/o blanks"""
    if s == '':
        return s
    elif len(s) == 1:
        if s[0] == ' ':
            return ''
        else:
            return s
    left = deblank(s[0])
    right = deblank(s[1:])
    return left+right
```

Handle simple cases

Break up the data

Combine answers

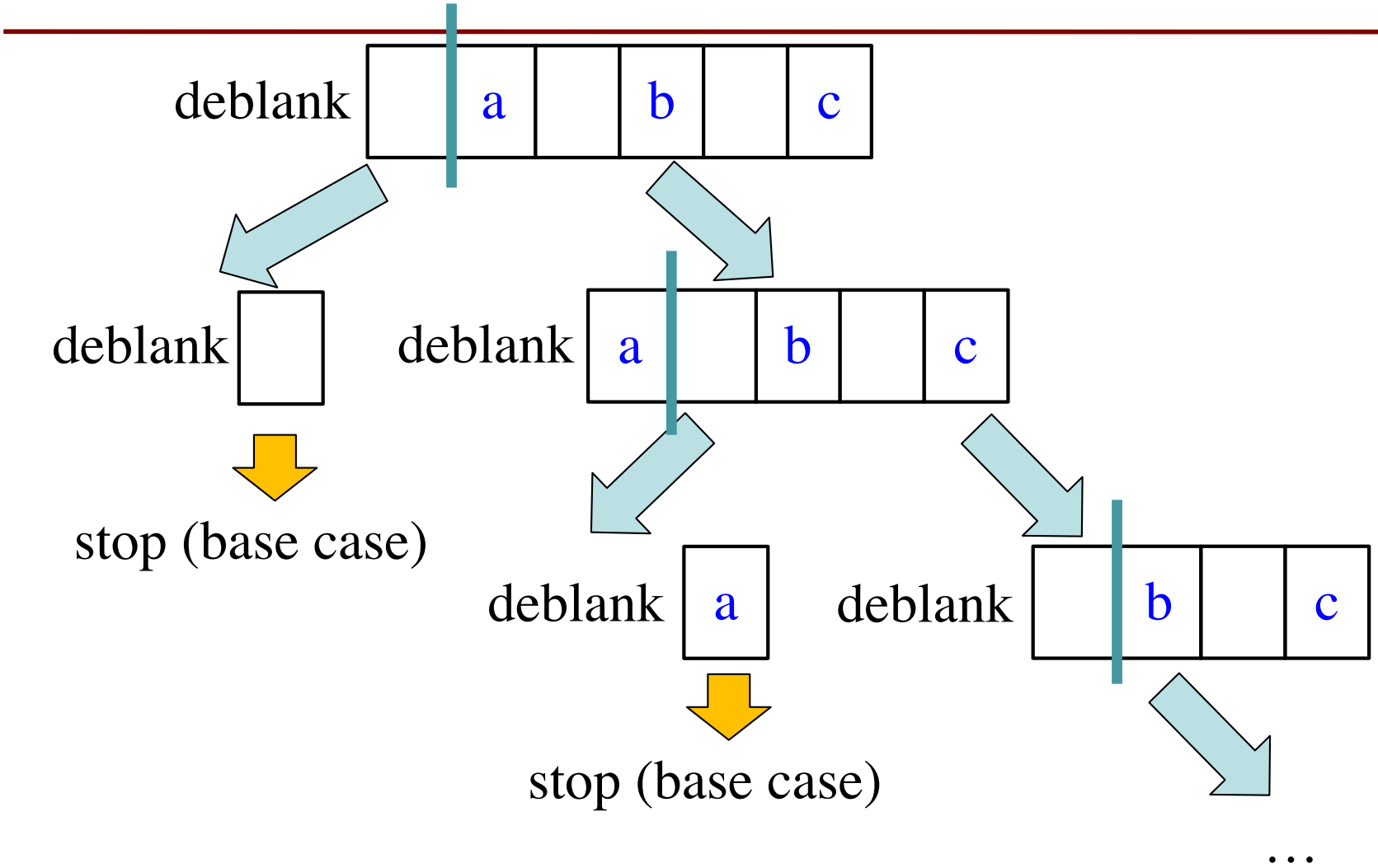
Putting it All Together

```
def deblank(s):
    """Returns: s w/o blanks"""
    if s == '':
        return s
    elif len(s) == 1:
        if s[0] == ' ':
            return ''
        else:
            return s
    left = deblank(s[0])
    right = deblank(s[1:])
    return left+right
```

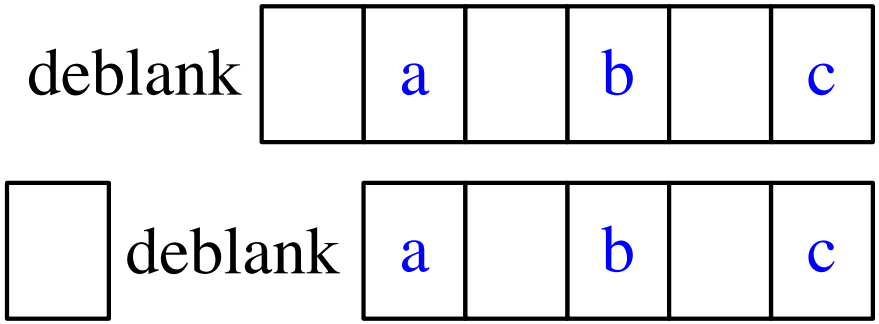
Base Case

Recursive Case

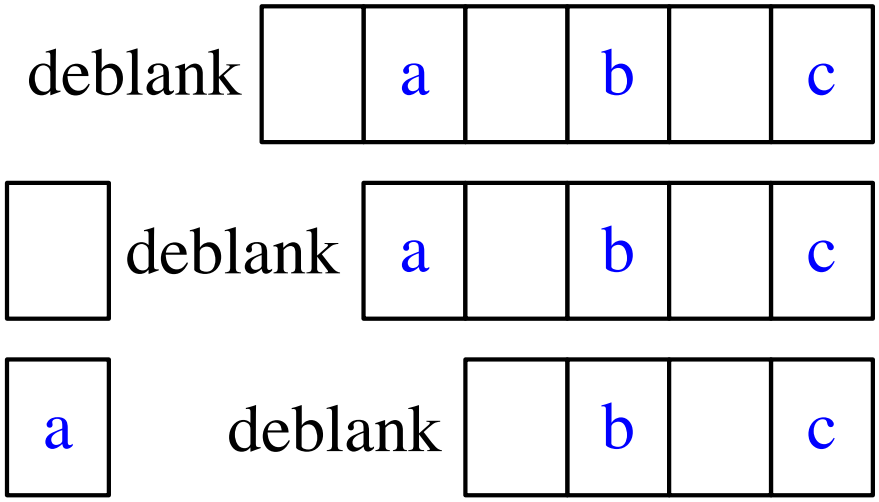
Following the Recursion



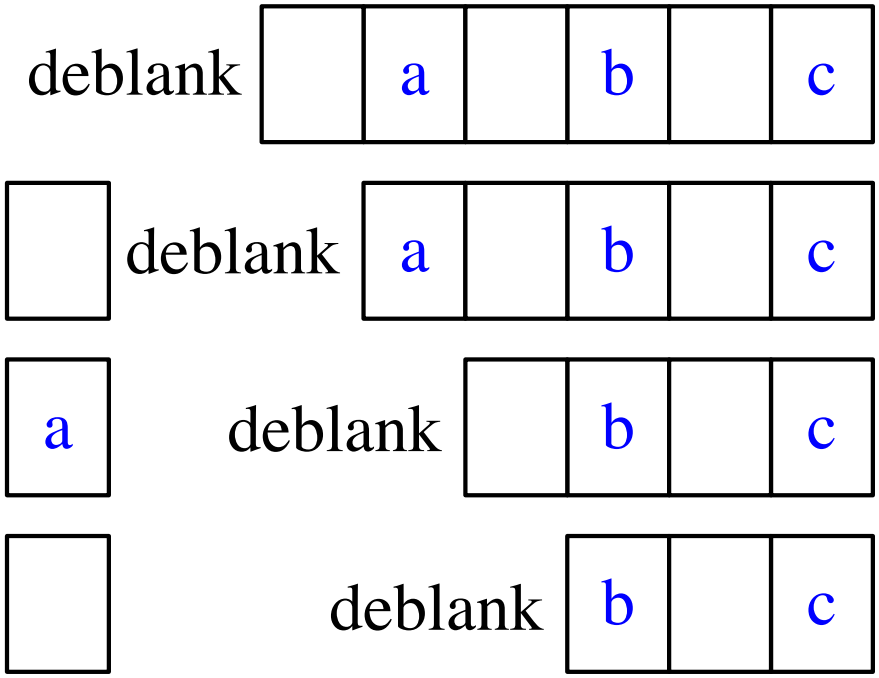
Breaking it up (1)



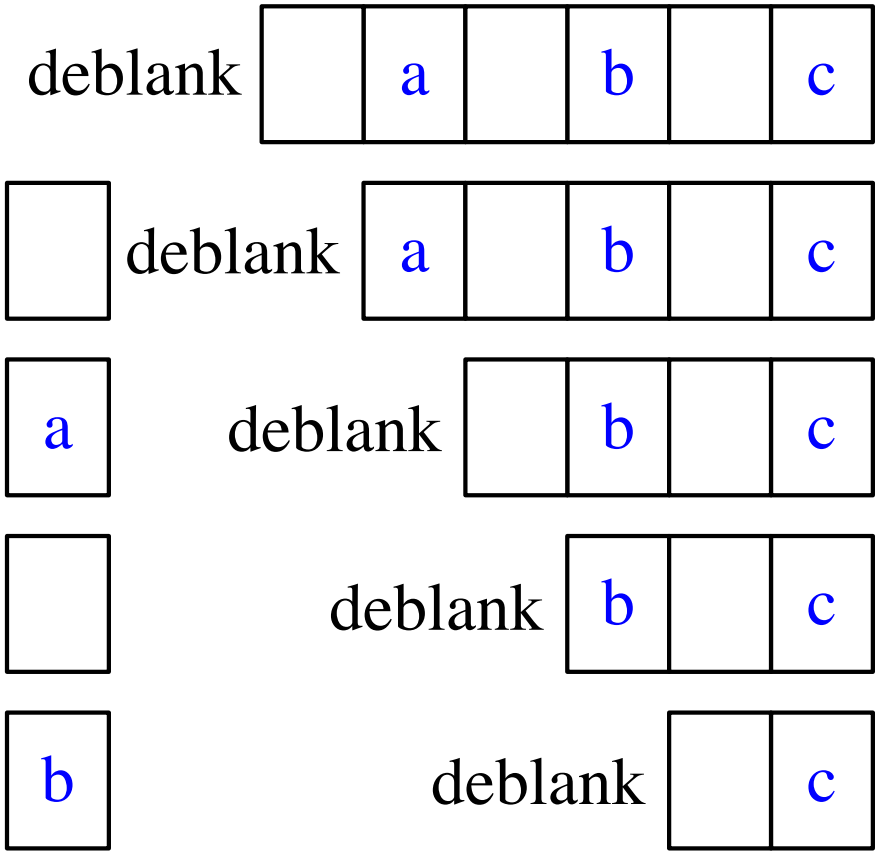
Breaking it up (2)



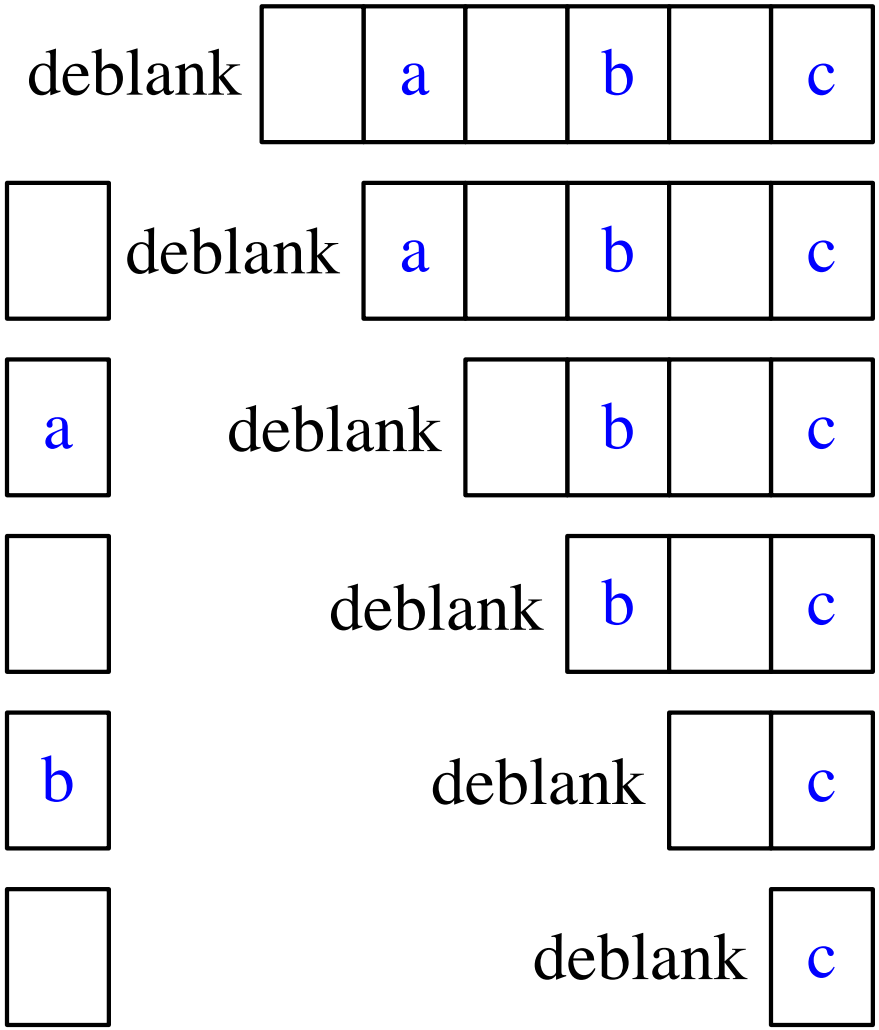
Breaking it up (3)



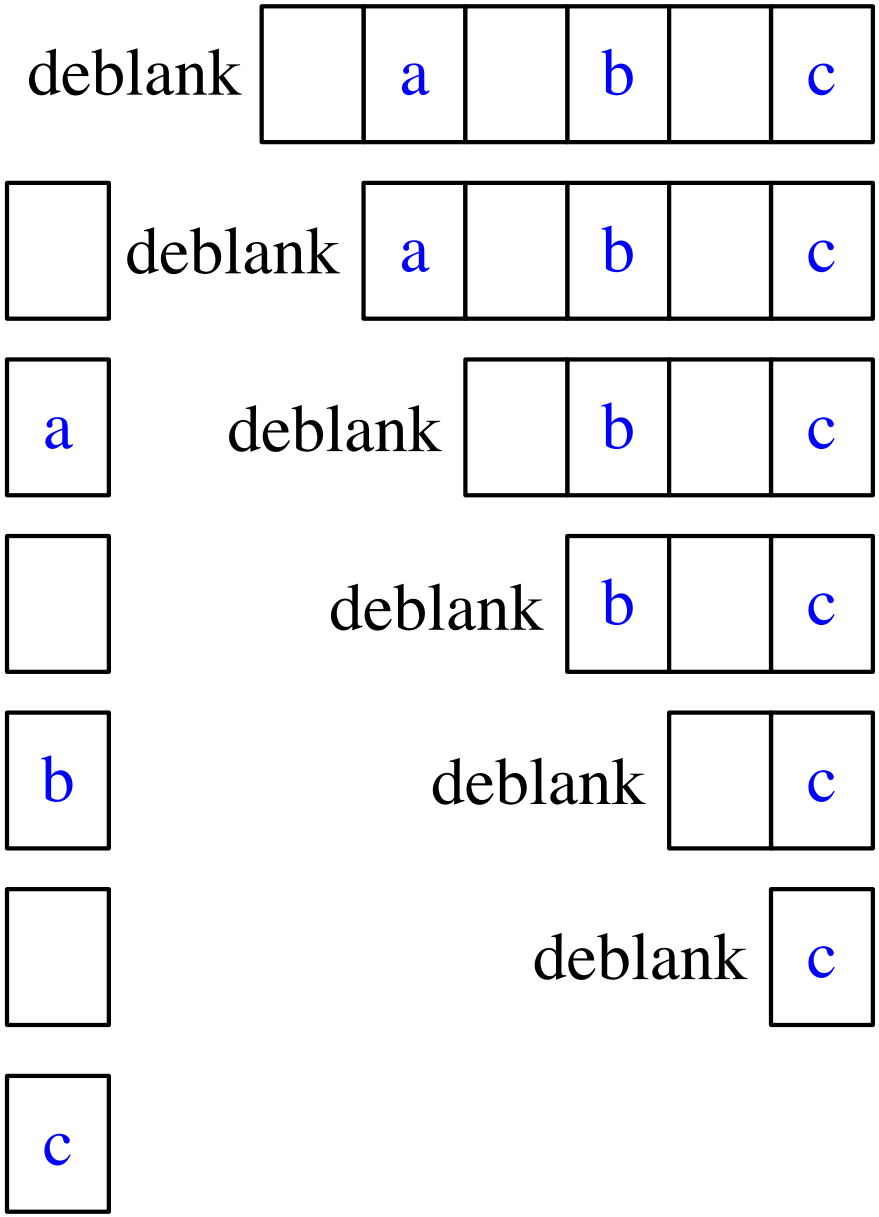
Breaking it up (4)



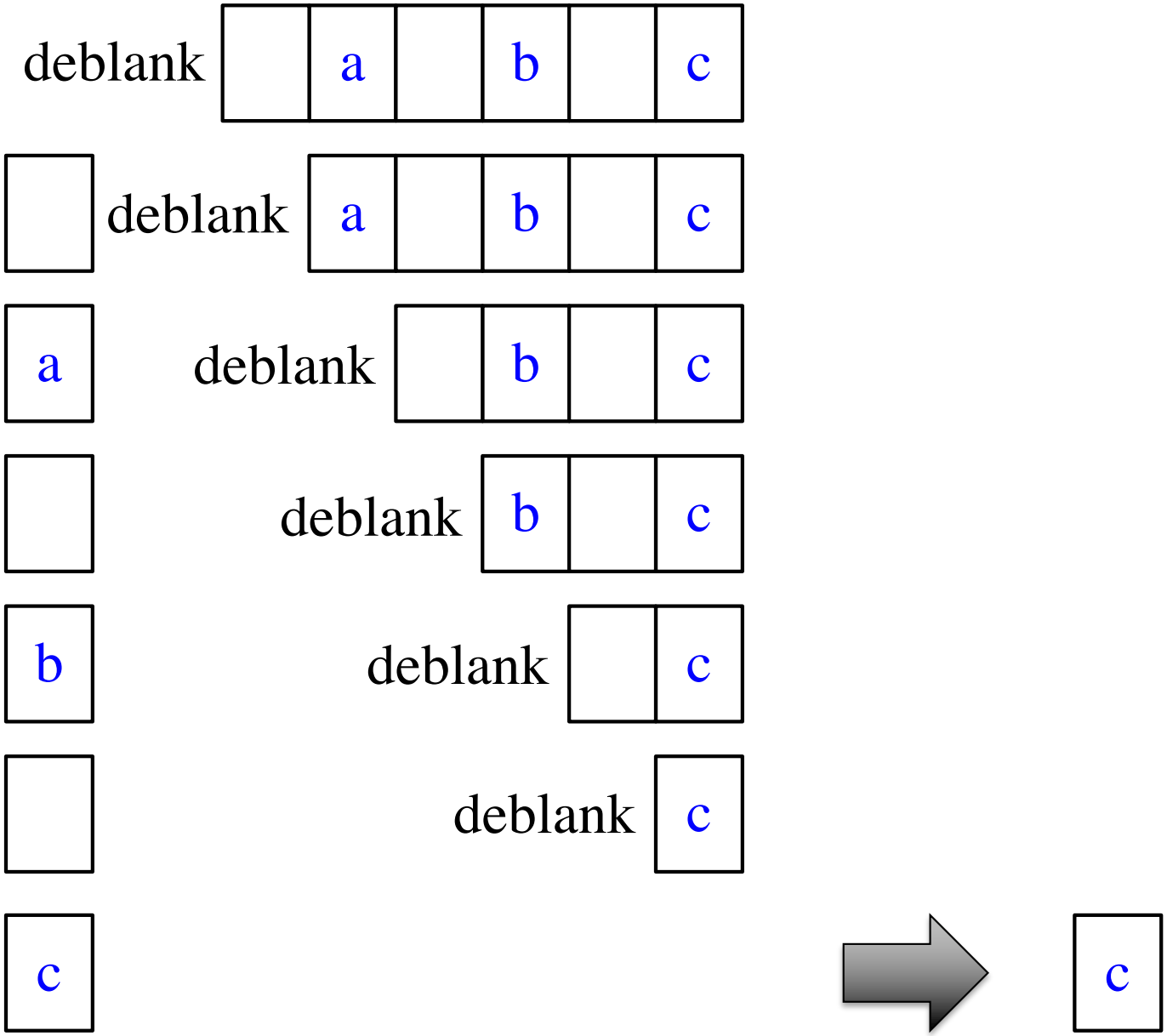
Breaking it up (5)



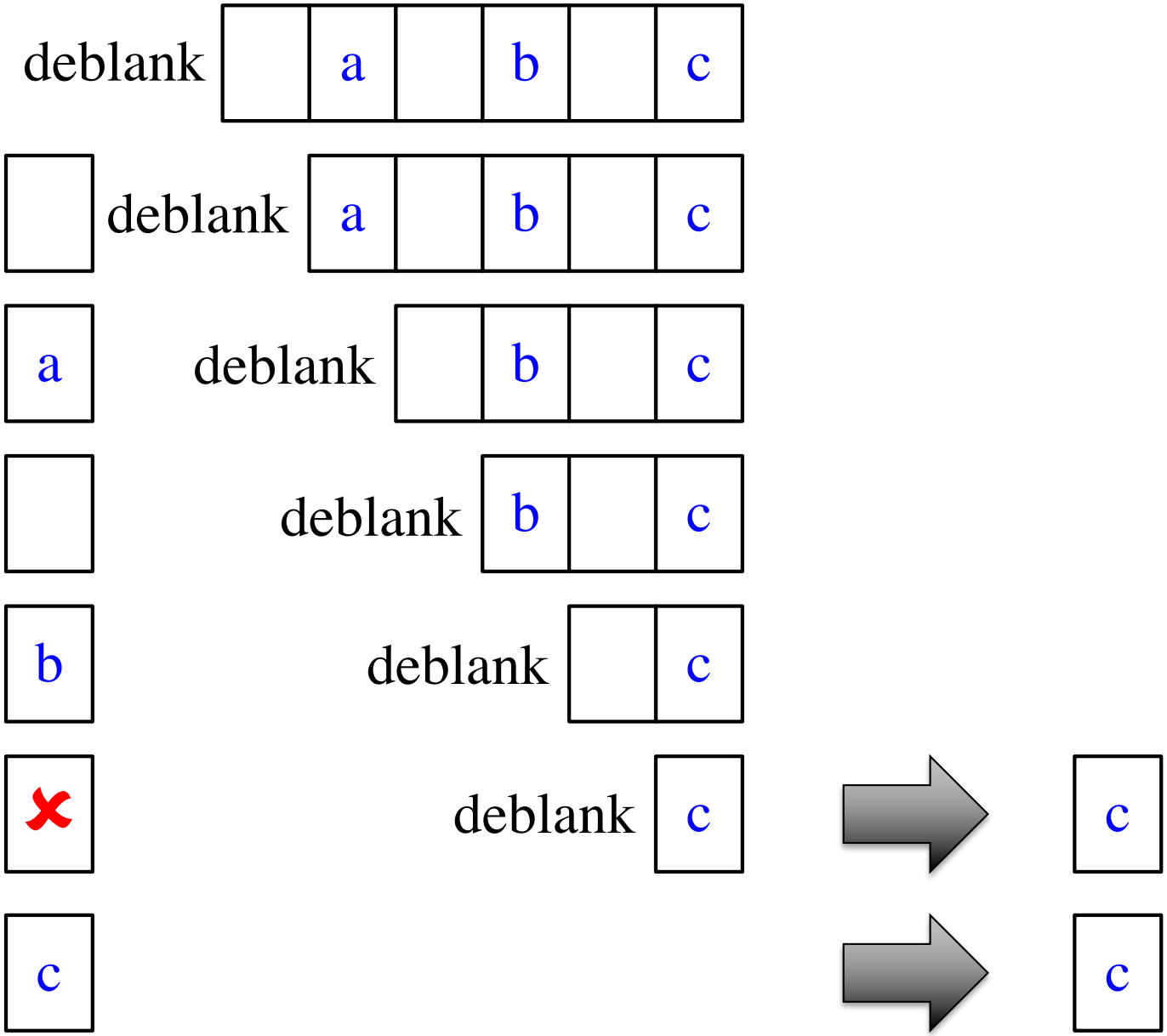
Breaking it up (6)



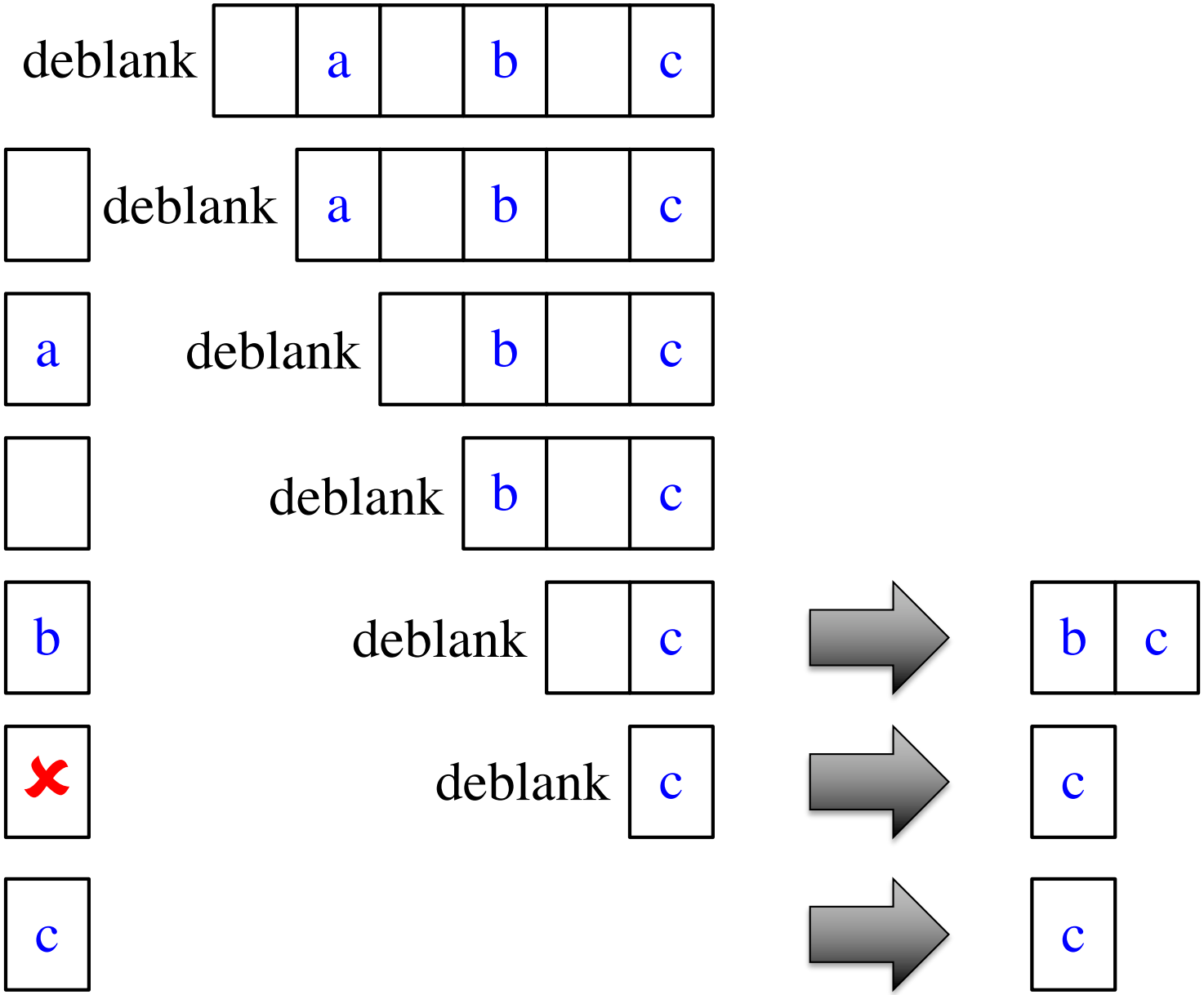
Combining Left+Right (1)



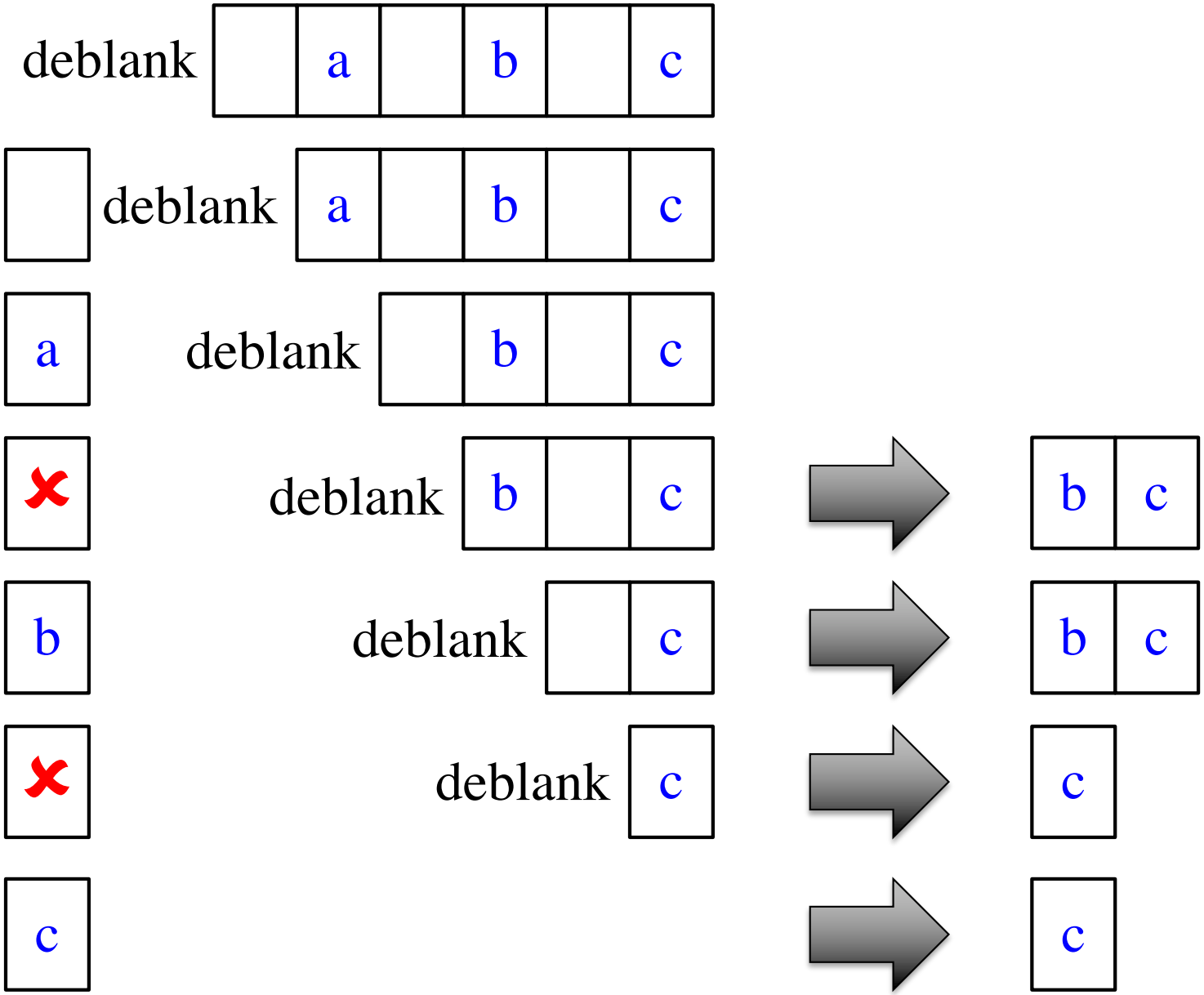
Combining Left+Right (2)



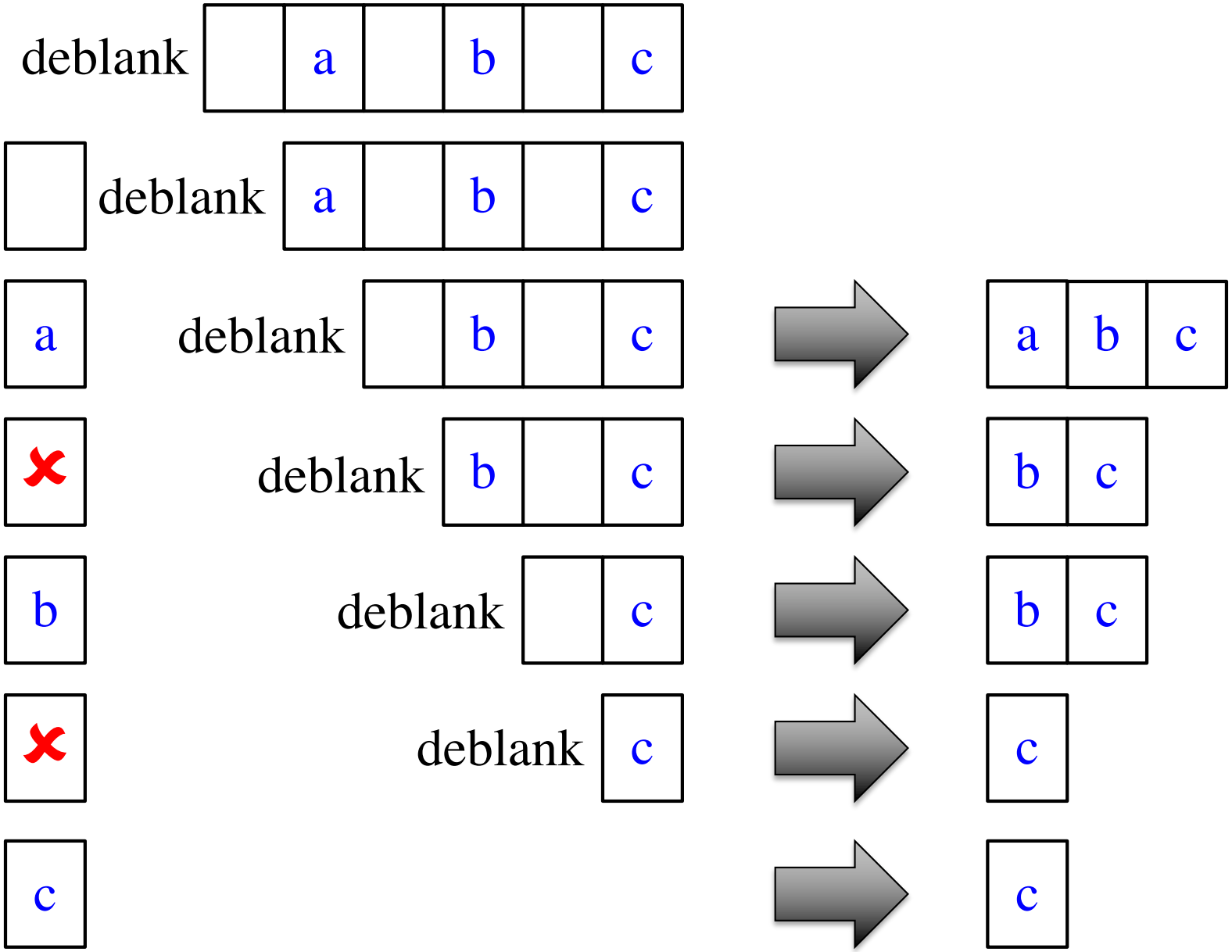
Combining Left+Right (3)



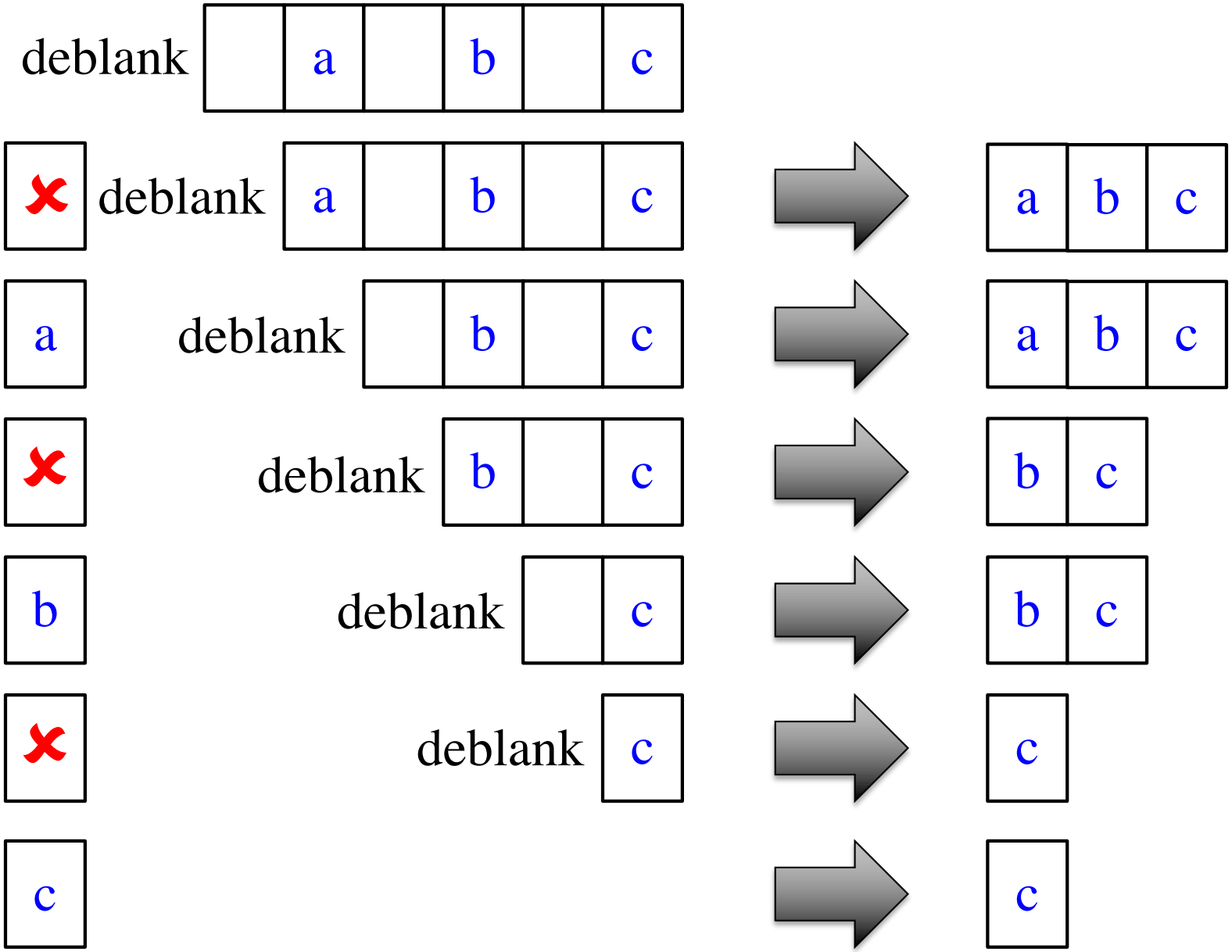
Combining Left+Right (4)



Combining Left+Right (5)



Combining Left+Right (6)



Combining Left+Right (7)