This 150-minute exam has 8 questions worth a total of 100 points. Scan the whole test before starting. Budget your time wisely. Use the back of the pages if you need more space. You may tear the pages apart; we have a stapler at the front of the room.

It is a violation of the Academic Integrity Code to look at any exam other than your own, look at any reference material, or otherwise give or receive unauthorized help.

You will be expected to write Python code on this exam. We recommend that you draw vertical lines to make your indentation clear, as follows:

```python
def foo():
    if something:
        do something
        do more things
        do something last
```

Unless you are explicitly directed otherwise, you may use anything you have learned in this course.

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>8</td>
<td>16</td>
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<tr>
<td><strong>Total:</strong></td>
<td><strong>100</strong></td>
<td></td>
</tr>
</tbody>
</table>

The Important First Question:

1. [2 points] Write your last name, first name, netid, and lab section at the top of each page.
Throughout this exam, there are several questions on sequences (strings and lists). All sequences support slicing. In addition, you may find the following sequence expressions below useful (though not all of them are necessary).

<table>
<thead>
<tr>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>len(s)</code></td>
<td><strong>Returns:</strong> number of elements in sequence <code>s</code>; it can be 0.</td>
</tr>
<tr>
<td><code>x in s</code></td>
<td><strong>Returns:</strong> <code>True</code> if <code>x</code> is an element of sequence <code>s</code>; <code>False</code> otherwise.</td>
</tr>
<tr>
<td><code>s.index(x)</code></td>
<td><strong>Returns:</strong> index of the FIRST occurrence of <code>x</code> in <code>s</code>. Raises a <code>ValueError</code> if <code>x</code> is not found.</td>
</tr>
<tr>
<td><code>x.append(a)</code></td>
<td>(Lists Only) Adds <code>a</code> to the end of list <code>x</code>, increasing length by 1.</td>
</tr>
<tr>
<td><code>x.remove(a)</code></td>
<td>(Lists Only) Removes first occurrence of <code>a</code> in <code>x</code>, decreasing length by 1.</td>
</tr>
<tr>
<td><code>x.extend(y)</code></td>
<td>(Lists Only) Appends each element in <code>t</code> to the end of list <code>x</code>, in order.</td>
</tr>
<tr>
<td><code>x.insert(i,y)</code></td>
<td>(Lists Only) Inserts <code>y</code> at position <code>i</code> in list <code>x</code>. Elements after position <code>i</code> are shifted to the right.</td>
</tr>
</tbody>
</table>

2. [10 points total] **Short Answer**
   (a) [2 points] What value does Python give in the evaluation below? Explain your answer.

   ```python
   >>> a = [1,2,3,4]
   >>> b = a[:]
   >>> a is b
   ???
   ``

   The answer is **false**. The operator `is` checks if the folders are the same. Slicing always creates a copy of the list folder.

   (b) [4 points] Describe the four steps that happen when you call a constructor.

   - It creates a new object (folder) of the class.
   - It puts the folder into heap space
   - It executes the method `__init__` defined in the body of the class. In doing so, it
     - Passes the folder name to that parameter `self`
     - Passes the other arguments in order
     - Executes the commands in the body of `__init__`
   - When done with `__init__` it returns the object (folder) name as final value of the expression (e.g. the constructor call).

   (c) [4 points] Name the three sorting algorithms introduced in class. Which one is the one we prefer to use most of the time and why?

   The algorithms are insertion sort, selection sort, and quicksort. True to its name, we like quicksort best. It generally takes $n \log n$ steps to sort a list of $n$ elements, while the other two take $n^2$ steps.
### 3. [16 points] Classes and Subclasses

Some of you may have worked with the class `GImage` in Assignment 7. This is a subclass of `GRectangle` that allows you to display an image. It has the following primary attributes (there are others, but they are not important for this question).

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Invariant</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>float</td>
<td>x-coordinate of the image center.</td>
</tr>
<tr>
<td>y</td>
<td>float</td>
<td>y-coordinate of the image center.</td>
</tr>
<tr>
<td>width</td>
<td>float &gt; 0</td>
<td>Distance from left side to right side.</td>
</tr>
<tr>
<td>height</td>
<td>float &gt; 0</td>
<td>Distance from bottom edge to top edge.</td>
</tr>
<tr>
<td>source</td>
<td>str or None</td>
<td>Name of the image file.</td>
</tr>
</tbody>
</table>

If `source` is `None`, then the image is blank when drawn.

Similar to what you did in Assignment 7, we want you to make a subclass of `GImage`. This subclass is called `GFilmStrip`. A filmstrip is a sequence of images used in animation. For example, in the illustration to the right, you can see a character in four different poses. Taken together, these poses make it look like the character is running. We can animate this running by changing the `source` attribute in `GImage` to be a different image each animation frame.

The subclass `GFilmStrip` provides new features to make this animation a little easier. Instead of a single source, it has a list of sources. The contents can still be `None`, just as in `GImage`. That would indicate a blank image in that animation frame.

The specification of this class is as follows:

```python
class GFilmStrip(GImage):
    """Instance is an animated sequence of images.

    (MUTABLE) ATTRIBUTES:
    _images [list, whose entries are str or None]: the animation frames
    _frame [int in 0..len(_images)-1]: the current animation frame

    In addition to these invariants, the inherited attribute source must always be equal to animation frame in _images, specified by _frame."""
```

Implement this class on the next page. We have provided you with an outline of the class, but you will need to fill in the blanks where specified. While we have specified the preconditions for `__init__`, you will notice that none of the getters or setters have preconditions. That is because you must figure out what those preconditions are, and enforce them. Remember that your class must enforce all invariants.

**Hint:** The attributes inherited from `GImage` work like they do in Assignment 7, and have invisible setters and getters. Therefore, you never have to enforce the invariants for these attributes. You only need to worry about your two new attributes: `_images` and `_frame`. If you have not finished Assignment 7 yet, you are free to use this class in that assignment.
from game2d import *
class GFilmStrip(GImage):
    """See specification on previous page."""
    def getSize(self):
        """Returns the number of images in this filmstrip"""
        return len(self._images)
    def getFrame(self):
        """Returns the position number of the active animation frame"""
        return self._frame
    def setFrame(self,frame):
        """Sets the frame number and makes that image the active one in source"""
        assert type(frame) == int
        assert 0 <= frame and frame < len(self._images)
        self._frame = frame
        self.source = self._images[self._frame]
    def getImage(self,n):
        """Returns the image at position n (where n is an index of _images)"""
        assert type(n) == int and 0 <= n and n < len(self._images)
        return self._images[n]
    def setImage(self,n,image):
        """Sets the image at position n (where n is an index of _images)"""
        assert image is None or type(image) == str
        assert type(n) == int
        assert 0 <= n and n < len(self._images)
        self._images[n] = image
    def __init__(self, width, height, size, frame=0): # FILL IN
        """Creates a new GFilmstrip centered at (0,0) with all blank images.
        The initializer does not assign any images, but it must create the list of
        size elements. Initially the list only contains None values. Images are set
        later with setImage.
        This initializer has four parameters: width, height, size, and frame. size
        is the number of images that the filmstrip can hold, while frame is the
        starting frame. The last parameter is OPTIONAL and defaults to 0.
        Preconditions: width and height are floats > 0. size is an int > 0, while
        frame is an int in 0..size-1."""
        GImage.__init__(self,x=0,y=0,width=width,height=height)
        assert type(size) == int
        assert size > 0
        self._images = [None]*size
        self setFrame(frame)
4. [14 points] **Recursion and Iteration**

Some of you may remember Pascal’s Triangle from your algebra class. This is the triangle that gives you the coefficients when you raise the polynomial \((x + 1)\) to a power. This triangle is shown to the right. We refer to each row of the triangle as a level, and a level can be represented as a list of ints. The row \([1]\) is the 0th level, row \([1, 1]\) is the 1st level, row \([1, 2, 1]\) is the 2nd level and so on.

As you can see to the right, we compute each level by looking at the previous level. We put new 1s at the start and end of the list. For the values in-between, we add two adjacent elements from the previous list. As a result, each level has exactly one more element than the previous level.

Implement the function below which computes the specified level of the Pascal triangle. You do not need to enforce the precondition.

**Important:** Your solution *must* use recursion, but you are also allowed to use a loop as well. However, you are restricted to exactly one loop. If you need more than one loop, you are not using recursion properly.

```python
def pascal_level(n):
    
    # Returns: the nth level of the pascal triangle
    # Precondition: n is an int >= 0.

    # Must handle both base cases separately
    if n == 0:
        return [1]
    elif n == 1:
        return [1, 1]

    # Recursive call to access previous level
    last = pascal_level(n-1)
    accum = [1]

    # Compute the interior points
    for x in range(len(last)-1):
        accum.append(last[x]+last[x+1])
    accum.append(1)

    return accum
```

5. [14 points] **Call Frames**

Each level of Pascal’s triangle is a palindrome; it is the same backwards and forwards. It also has the additional property that values are always increasing as you go towards the center. The function below tests whether or not a list has these two properties. For example, while the lists [2, 3, 2] and [1, 4, 4, 1] are Pascal-ish, the lists [2, 3] and [4, 1, 4] are not.

```python
def ispascalish(b):
    if len(b) < 2:
        return True  # 1
    if (len(b) > 2 and b[0] >= b[1]):  # 2
        return False
    return b[0] == b[-1] and ispascalish(b[1:-1])  # 5
```

On this page and the next, diagram the execution of the assignment statement

```python
>>> x = ispascalish([1,3,1,3,1])
```

You need a new diagram every time a frame is added or erased, or an instruction counter changes. **However, do not draw a frame for **len**. We have done the first step for you. There are eight more diagrams to draw. You may write *unchanged* if a space does not change.
6. [18 points total] **2-Dimensional Lists**

A matrix is a rectangular table of floats. It is a nonempty 2D list of floats where all rows have the same length. You add matrices by adding all of floats in the same position. For example,

\[
\begin{bmatrix}
1.5, & 2.0 \\
2.5, & -1.0
\end{bmatrix}
+ \begin{bmatrix}
5.0, & 0.0 \\
0.0, & 5.0
\end{bmatrix}
= \begin{bmatrix}
6.5, & 2.0 \\
2.5, & 4.0
\end{bmatrix}
\]

The *dimension* of a matrix is \( n \times m \) where \( n \) is the number of rows and \( m \) the number of columns. Implement the two functions below. **You do not need to enforce the preconditions.**

(a) [7 points]
```python
def sum_matrix(matrix1, matrix2):
    """Returns: new matrix that is sum of matrix1 and matrix2.
    Precondition: matrix1 and matrix2 are matrices with the same dimension"
    accum = []
    # Process each row of the two matrices
    for x in range(len(matrix1)):
        row = []
        # Add each column
        for y in range(len(matrix1[x])):
            row.append(matrix1[x][y]+matrix2[x][y])
        accum.append(row)
    return accum
```

(b) [11 points]
```python
def ismatrix(m):
    """Returns: true if m is a matrix (nonempty, rectangular with floats).
    Precondition: NONE (m can be anything)"
    # Manually check the problematic cases
    if (type(m) != list):
        return False
    elif (len(m) == 0):
        return False
    elif (type(m[0]) != list):
        return False
    # Compare everything to size of the first row
    size = len(m[0])
    for row in m:
        if type(row) != list or len(row) != size:
            return False
        for x in row:
            if type(x) != float:
                return False
    return True
```
7. [10 points] Testing

Consider the function ismatrix from the previous question. In the space below, provide at least six different test cases to verify that this function is working correctly. For each test case provide: (1) the function input, (2) the expected output, and (3) an explanation of what makes this test significantly different.

Unlike previous testing functions, you actually wrote the function this time. As in Assignment 3, your test cases need to have enough coverage to include every possible if-branch. While we did accept other answers, here are the six tests we were thinking of.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Output</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m = 2.0 )</td>
<td>False</td>
<td>Not a list</td>
</tr>
<tr>
<td>( m = [1.5, 2.0] )</td>
<td>False</td>
<td>Not a 2D list</td>
</tr>
<tr>
<td>( m = [[1.5], [2.0, 3.5]] )</td>
<td>False</td>
<td>Not rectangular</td>
</tr>
<tr>
<td>( m = [[1.5, 2], [2.0, 3.5]] )</td>
<td>False</td>
<td>Not all floats</td>
</tr>
<tr>
<td>( m = [[1.5]] )</td>
<td>True</td>
<td>The simplest, “degenerate” matrix</td>
</tr>
<tr>
<td>( m = [[1.5, 2.0], [2.0, 3.5]] )</td>
<td>True</td>
<td>Regular matrix with many elements</td>
</tr>
</tbody>
</table>

8. [16 points total] Loop Invariants

On the next page are two variations of the Dutch National Flag algorithm shown in class. The one on the left has been completed for you. The second algorithm on the right is identical to the first except that it has a different loop invariant. It is currently missing the code for initialization, the loop condition, and the body of the loop (all other code is provided).

(a) [2 points] Draw the horizontal notation representation for the loop invariant on the left.

```
  h  i  t  j  k
b | < 0 | = 0 | ??? | > 0
```

(b) [2 points] Draw the horizontal notation representation for the loop invariant on the right.

```
  h  t  i  j  k
b | ??? | < 0 | = 0 | > 0
```
(c) [12 points] Add the missing code to the function on the right. Like the function on the left, you may use the helper function \texttt{swap(b,n,m)} to swap two positions in the list. It is not enough for your code to be correct. \textbf{To get credit}, you must satisfy the loop invariant.

\begin{verbatim}
def dnf(b,h,k):
    """Returns: Divisions i, j of dnf Rearranges the list into negatives, 0s, and positives according to DNF.
    Pre: h, k are positions in list b.""
    # Make invariant true at start
    i = h-1
    t = h-1
    j = k+1
    # inv: b[h..i] < 0, b[i+1..t] = 0, # b[t+1..j-1] ???, and b[j..k] > 0

    while t < j-1:
        if b[t+1] < 0:
            swap(b,i+1,t+1)
            t = t+1
            i = i+1
        elif b[t+1] == 0:
            t = t+1
        else: # b[t+1] > 0
            swap(b,j-1,t+1)
            j = j-1

    # post: b[h..i] < 0, b[i+1..t] = 0, # and b[j..k] > 0
    return (i,j)

def dnf(b,h,k):
    """Returns: Divisions i, j of dnf Rearranges the list into negatives, 0s, and positives according to DNF.
    Pre: h, k are positions in list b.""
    # Make invariant true at start
    t = k+1
    i = k+1
    j = k
    # inv: b[h..t-1] ???, b[t..i-1] < 0, # b[i..j] = 0, and b[j+1..k] > 0

    while h < t:
        if b[t-1] > 0:
            swap(b,t-1,j)
            if i < j+1:
                swap(b,t-1,i-1)
            j = j-1
            i = i-1
            t = t-1
        elif b[t-1] == 0:
            swap(b,t-1,i-1)
            i = i-1
            t = t-1
        else: # b[t-1] < 0
            t = t-1

    # post: b[h..i] < 0, b[i+1..j-1] = 0, # and b[j..k] > 0
    return (i,j)
\end{verbatim}