17. Searching and Sorting

Topics:
- Linear Search
- Binary Search
- Measuring Execution Time
- The Divide and Conquer Framework
- Merge Sort

Search

Examples:

Is this song in that playlist?

Is this number in that phone book?

Is this name in that phone book?

Is this fingerprint in that archive of fingerprints?

Is this photo in that yearbook?

More on Using Phone Books

The Manhatten phone book has 1,000,000+ entries.

How is it possible to locate a name by examining just a <u>tiny</u> , <u>tiny</u> fraction of those entries?

There must be a great search algorithm behind the scenes.

Linear Search

LinSearch: The Spec

```
def LinSearch(x,a):
    """ Returns an int k with the
    property that a[k]==x is True.
    If no such k exists, then
    k==-1.

    PreC: a is a nonempty list of
    ints and x is an int.
    """
```

Linear Search

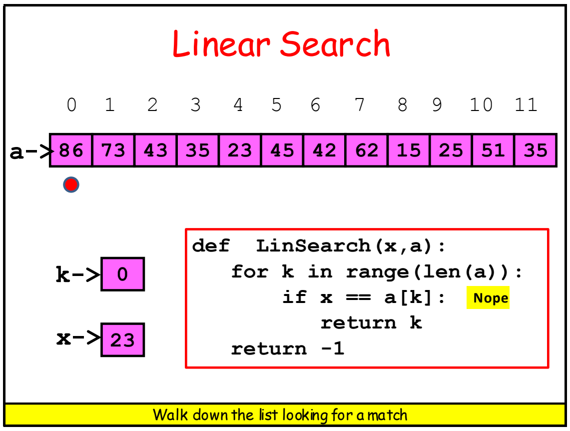
	0	1	2	3	4	5	6	7	8	9	10	11
a->	86	73	43	35	23	45	42	62	15	25	51	35

k-> 0

x-> 23

```
def  LinSearch(x,a):
   for k in range(len(a)):
       if x == a[k]:
          return k
   return -1
```

Walk down the list looking for a match



Linear Search

	0	1	2	3	4	5	6	7	8	9	10	11
a->	86	73	43	35	23	45	42	62	15	25	51	35

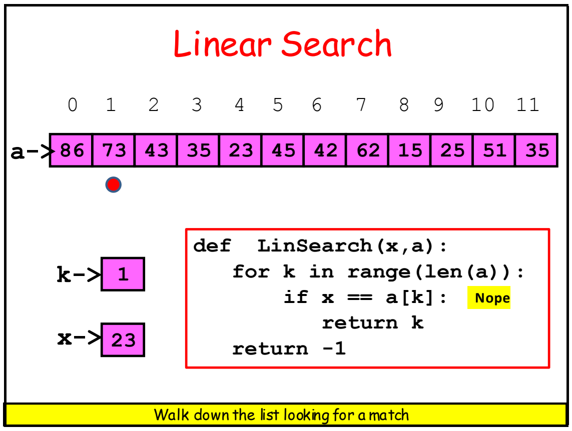
k-> 0

x-> 23

```
def  LinSearch(x,a):
    for k in range(len(a)):
        if x == a[k]:
            return k
    return -1
```

Nope

Walk down the list looking for a match



Linear Search

	0	1	2	3	4	5	6	7	8	9	10	11
a->	86	73	43	35	23	45	42	62	15	25	51	35

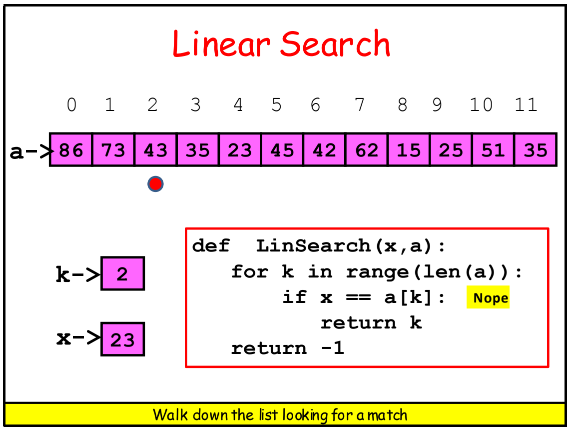
k-> 1

x-> 23

```
def  LinSearch(x,a):
    for k in range(len(a)):
        if x == a[k]:
            return k
    return -1
```

Nope

Walk down the list looking for a match



Linear Search

	0	1	2	3	4	5	6	7	8	9	10	11
a->	86	73	43	35	23	45	42	62	15	25	51	35

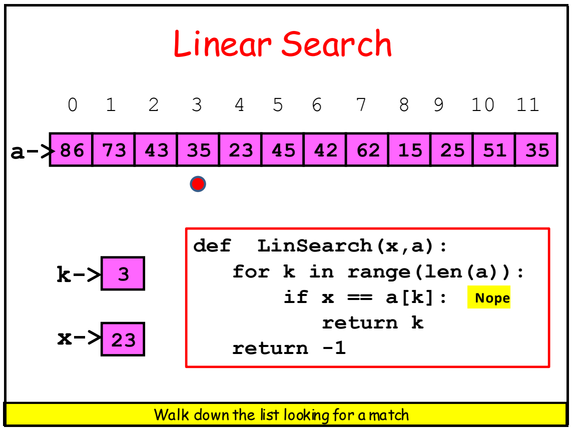
k-> 2

x-> 23

```
def  LinSearch(x,a):
    for k in range(len(a)):
        if x == a[k]:
            return k
    return -1
```

Nope

Walk down the list looking for a match



Linear Search

	0	1	2	3	4	5	6	7	8	9	10	11
a->	86	73	43	35	23	45	42	62	15	25	51	35

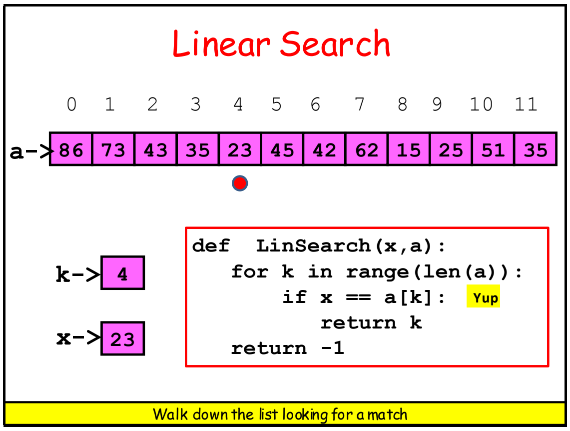
k-> 3

x-> 23

```
def  LinSearch(x,a):
    for k in range(len(a)):
        if x == a[k]:
            return k
    return -1
```

Nope

Walk down the list looking for a match



Linear Search

	0	1	2	3	4	5	6	7	8	9	10	11
a->	86	73	43	35	23	45	42	62	15	25	51	35

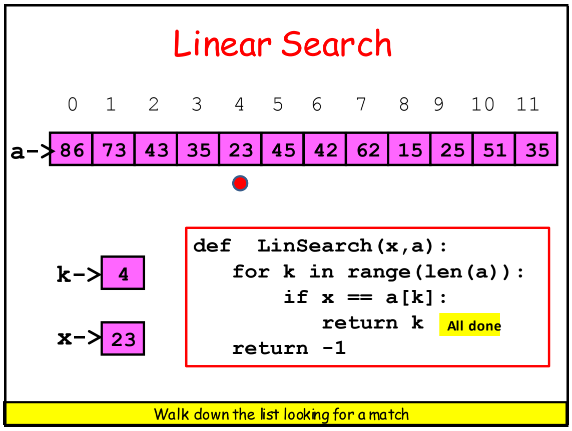
k-> 4

x-> 23

```
def  LinSearch(x,a):
    for k in range(len(a)):
        if x == a[k]:
            return k
    return -1
```

Yup

Walk down the list looking for a match



Linear Search

	0	1	2	3	4	5	6	7	8	9	10	11
a->	86	73	43	35	23	45	42	62	15	25	51	35

k-> 4

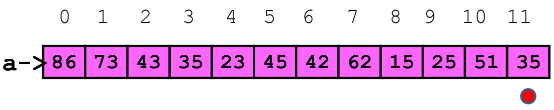
x-> 23

```
def  LinSearch(x,a):
    for k in range(len(a)):
        if x == a[k]:
            return k
    return -1
```

All done

Walk down the list looking for a match

Linear Search: No Match Case



0	1	2	3	4	5	6	7	8	9	10	11
86	73	43	35	23	45	42	62	15	25	51	35

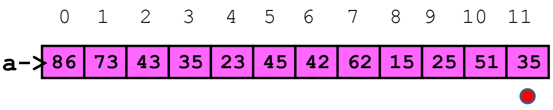
k-> 11

x-> 7

```
def  LinSearch(x,a):
     for k in range(len(a)):
         if x == a[k]:  Nope
            return k
     return -1
```

Walk down the list looking for a match

Linear Search: No Match Case



0	1	2	3	4	5	6	7	8	9	10	11
86	73	43	35	23	45	42	62	15	25	51	35

x-> 7

```
def  LinSearch(x,a):
     for k in range(len(a)):
         if x == a[k]:
            return k
     return -1  Yup
```

Return -1 if no match

Linear Search: While Implementation

```
def  LinSearchW(x,a):
   k=0
   while k<len(a)  and a[k]!=x:
       k+=1
   if k==len(a):
       return -1
   else:
       return k
```

Binary Search

Now we assume that the list
to be searched is sorted
from little to big.

```
a = [10,20,40,60,90]

a = ['brown','dog','fox','lazy','quick','the']
```

Back to Using Phone Books

The Ithaca phone book has 10,000+ entries.

The Manhatten phone book has 1,000,000+ entries. But it does not take 100 x longer to look something up. Why?

Key Idea: Repeated Halving

To Derek Jeter's number…

```
B = phone book
while (B is longer than 1 page):
   1. P = middle page of B
   2. Let Q be the first name on P
   3. if  'Jeter" comes before Q:
          Rip away the 2nd half of B
      else:
          Rip away the 1st half of B.

Scan remaining page P line-by-line for 'Jeter'
```

What Happens to Phone Book Length?

Original:	3000 pages
After 1 rip:	1500 pages
After 2 rips:	750 pages
After 3 rips:	375 pages
After 4 rips:	188 pages
After 5 rips:	94 pages
After 12 rips:	1 page

Binary Search

The idea of repeatedly halving the size of the "search space" is the main idea behind the method of binary search.

An item in a sorted array of length n can be located with approximately $\log_2 n$ comparisons.

$\log_2 8 = 3$ $\log_2 64 = 7$ $\log_2 2^{**}k = k$

What is $\log_2(n)$?

n	$\text{ceil}(\log_2(n))$
10	4
100	7
1000	10
10000	14
100000	17
1000000	20

`BinSearch`: The Spec

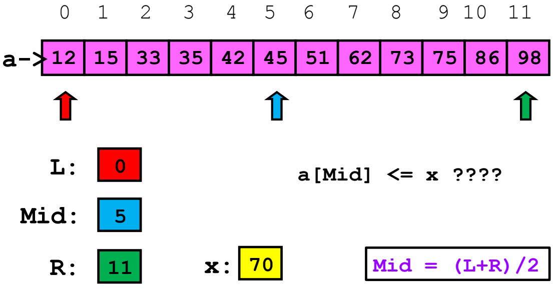
```
def BinSearch(x,a):
    """ Returns an int k with the
    property that a[k]==x is True.
    If no such k exists, then
    k==-1.

    PreC: a is a nonempty list of
    ints that is sorted from smallest
    to largest. x is an int.
    """
```

Example:
Does this List have an Element With Value Equal to 70?

0	1	2	3	4	5	6	7	8	9	10	11
12	15	33	35	42	45	51	62	73	75	86	98

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 0

Mid: 5

R: 11

x: 70

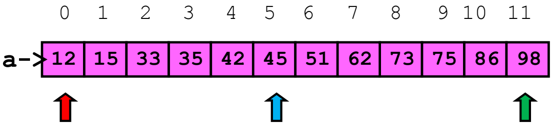
a[Mid] <= x ????

Mid = (L+R)/2

The Midpoint Computations

L	R	(L+R)/2
0	11	5
2	6	4
1	100	50

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 0

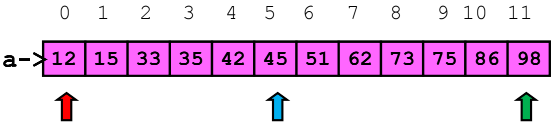
Mid: 5

R: 11

x: 70

`a[Mid] <= x ????`

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 0

Mid: 5

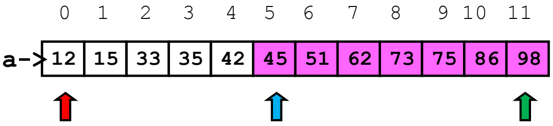
R: 11

x: 70

`a[Mid] <= x`

Yes!
So throw away
The "left half"

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 0

Mid: 5

R: 11

x: 70

`a[Mid] <= x`

Yes!
So throw away
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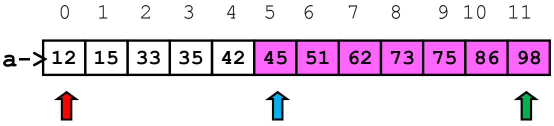
Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 0

Mid: 5

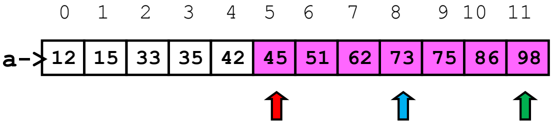
R: 11

x: 70

`a[Mid] <= x`

Revise L and Mid

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 5

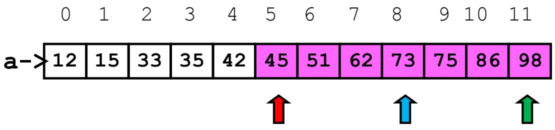
Mid: 8

R: 11

x: 70

`a[Mid] <= x ???`

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 5

Mid: 8

R: 11

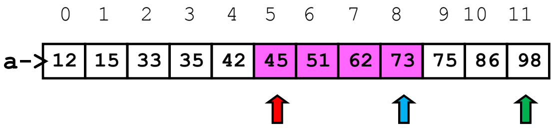
x: 70

`a[Mid] <= x`

No

So throw away the
"right half"

Let's Look For x = 70



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 5

Mid: 8

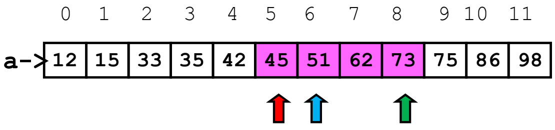
R: 11

x: 70

`a[Mid] <= x`

Revise R and Mid

Let's Look For x = 70



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 5

Mid: 6

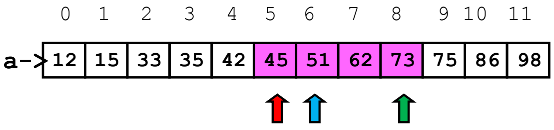
R: 8

x: 70

`a[Mid] <= x`

Revise R and Mid

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 5

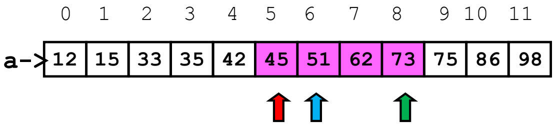
Mid: 6

R: 8

x: 70

`a[Mid] <= x ????`

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 5

Mid: 6

R: 8

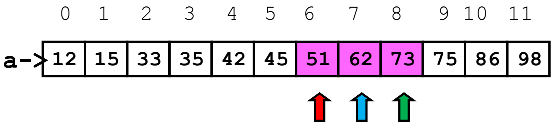
x: 70

`a[Mid] <= x`

Yes

Throw away the
Left half

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 6

Mid: 7

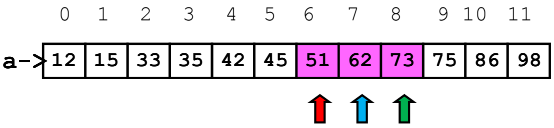
R: 8

x: 70

`a[Mid] <= x`

Yes

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 6

Mid: 7

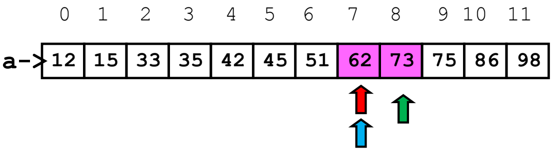
R: 8

x: 70

`a[Mid] <= x`

Throw away the left half

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 6

Mid: 7

R: 8

x: 70

Done! At this point we just compare **x** with `a[L]` and `a[L+1]`.

What We Just Did

```
L = 0
R = len(a)-1
while R-L > 1:
    # a[L]<=x<=a[R]
     Mid = (L+R)/2
     if x <= a[mid]:
         R = Mid
     else:
         L = Mid
```

A Loop Invariant

Note that a[L]<=x<=a[R} remains True throughout the loop

What We Just Did

```
L = 0
R = len(a)-1
while R-L > 1:
    # a[L]<=x<=a[R]
     Mid = (L+R)/2
     if x <= a[mid]:
         R = Mid
     else:
         L = Mid
```

What is the situation when the loop terminates?

What We Just Did

```
L = 0
R = len(a)-1
while R-L > 1:
    # a[L]<=x<=a[R]
     Mid = (L+R)/2
     if x <= a[mid]:
         R = Mid
     else:
         L = Mid
```

R-L<=1 implies R = L+1

After the Loop Ends

This is True: `a[L]<=x<=a[L+1]`

After the Loop Ends

```
if x==a[L]:
   return L
elif x==a[L+1]:
   return L+1
else:
   return -1
```

Measuring Execution Time

We now have two ways to search a list:

```
LinSearch(x,a)
BinSearch(x,a)
```

Intuition: BinSearch much faster.

Can we quantify this with a "stop watch"?

The `timeit` Module

This module can be used to time how long it takes to execute a chunk of code.

Typical chunk = some function of interest.

This is called benchmarking.

Benchmarking

Let's benchmark `LinSearch(x,a)` and `BinSearch(x,a)`.

Compare how long it takes when `len(a)` equals 1000, 10000, 100000, and 1000000.

Our intuition tells us that as `len(a)` increases, `BinSearch` will be dramatically faster.

BinSearch vs LinSearch

```
      n        tBin       tLin        tLinW
---------------------------------------
   1000      0.0007    0.0064     0.0119
  10000      0.0009    0.0668     0.1203
 100000      0.0011    0.8296     1.2082
1000000      0.0015   17.7388    13.9341
```

```
tBin  = time for BinSearch
tLin  = time for LinSearch (for loop version)
tLinW = time for LinSearch (while-loop version)
```

BinSearch vs LinSearch

```
      n          tLin/tBin
------------------------
   1000              9
  10000             74
 100000            754
1000000           7095
```

Reporting ratios is more illuminating since we do not really care about the time units in this informal comparison

Using the `timeit` Module

We show how this module was use to get the results on the previous slides.

Our LinSearch vs BinSearch example is very typical: is one function faster than another?

A Benchmarking Framework

```
from timeit import *
S = """

  Set-up code

"""
B = """

  Code to Benchmark

"""
p = 10; m = 100
t = min(Timer(B,setup=S).repeat(p,m))
```

Yes, these are doc strings.

The Set-Up and Bench Codes

```
from random import randint as randi
from ShowSearch import BinSearch
n = 10000
s = [randi(0,10*n) for i in range(n)]
s.sort()
x = s[n/2]
```

```
k=BinSearch(x,s)
```

The set-up code is run once.

It is not timed.

It just sets up the code to be timed.

A Benchmarking Framework

```
from timeit import *
S = """

  Set-up code

"""
B = """

  Code to Benchmark

"""
p = 10; m = 100
t = min(Timer(B,setup=S).repeat(p,m))
```

An “experiment” consists of running the blue code m times.

The stopwatch will time how long it takes to do one experiment

Larger values necessary if the blue code executes very quickly

---

A Benchmarking Framework

```
from timeit import *
S = """

  Set-up code

"""
B = """

  Code to Benchmark

"""
p = 10; m = 100
t = min(Timer(B,setup=S).repeat(p,m))
```

Timer returns a length-p list. Each element is the stopwatch time for 1 experiment

This helps control for other stuff that may be running on your computer.

A Benchmarking Framework

```
from timeit import *
S = """

  Set-up code

"""
B = """

  Code to Benchmark

"""
p = 10; m = 100
t = min(Timer(B,setup=S).repeat(p,m))
```

In general, it is best to take the mininum as the most reliable. The benchmark time is assigned to t

This helps control for other stuff that may be running on your computer.

Why Benchmarking is Important

Confirms/refutes what our intuition might say about efficiency.

Makes us sensitive to the various issues that affect efficiency.

Steers us away from simplistic comparisons of different methods that can be used on the same problem.

MergeSort

Binary Search is an example of a "divide and conquer" approach top probkem solving.

A method for sorting a list that features this strategy is MergeSort

Motivation

You are asked to sort a list but you have two "helpers": H1 and H2.

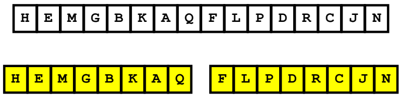
Idea:

1. Split the list in half and have each helper sort one of the halves.

2. Then merge the two sorted lists into a single larger list.

This idea can be repeated if H1 has two helpers and H2 has two helpers.

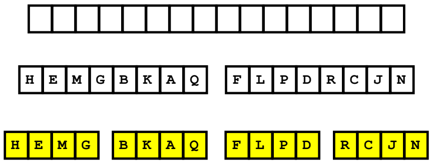
Subdivide the Sorting Task



H E M G B K A Q F L P D R C J N

H E M G B K A Q   F L P D R C J N

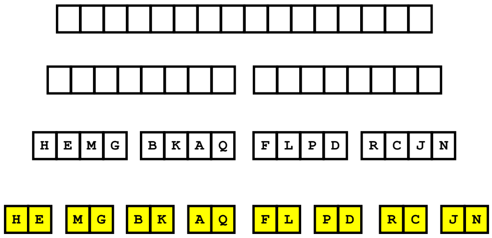
Subdivide Again



H E M G B K A Q   F L P D R C J N

H E M G   B K A Q   F L P D   R C J N

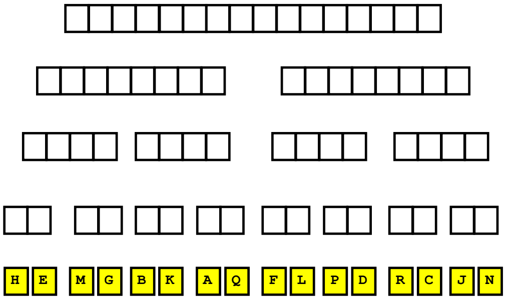
And Again



H E M G   B K A Q   F L P D   R C J N

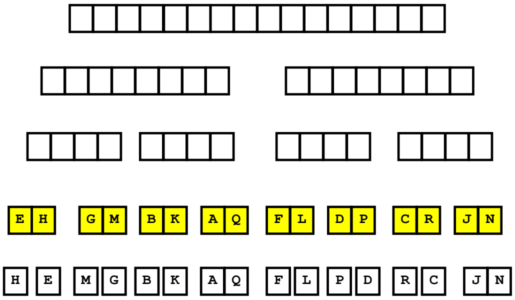
H E   M G   B K   A Q   F L   P D   R C   J N

And One Last Time



H E　M G　B K　A Q　F L　P D　R C　J N

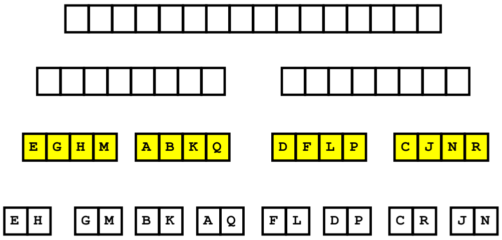
Now Merge



E H　G M　B K　A Q　F L　D P　C R　J N

H E　M G　B K　A Q　F L　P D　R C　J N

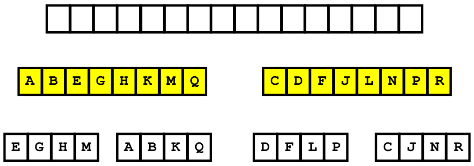
And Merge Again



E G H M　A B K Q　D F L P　C J N R

E H　G M　B K　A Q　F L　D P　C R　J N

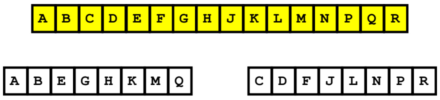
And Again



A B E G H K M Q　C D F J L N P R

E G H M　A B K Q　D F L P　C J N R

And One Last Time



A B C D E F G H J K L M N P Q R

A B E G H K M Q　C D F J L N P R

Done!

A B C D E F G H J K L M N P Q R

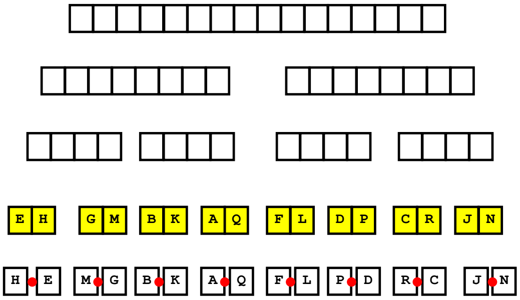
Done!

Let's write a function to do this making use of

```
def Merge(x,y):
     """ Returns a float list that is the
     merge of sorted lists x and y.

     PreC: x and y are lists of floats
     that are sorted from small to big.
     """
```

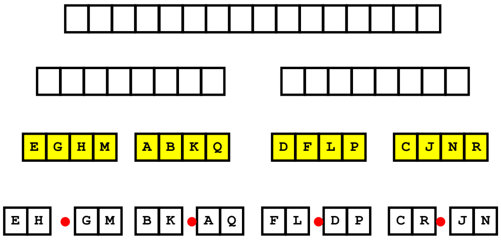
8 Merges Producing length-2 lists



Handcoding the n =16 case

```
A0 = Merge(a[0],a[1])
A1 = Merge(a[2],a[3])
A2 = Merge(a[4],a[5])
A3 = Merge(a[6],a[7])
A4 = Merge(a[8],a[9])
A5 = Merge(a[10],a[11])
A6 = Merge(a[12],a[13])
A7 = Merge(a[14],a[15])
```

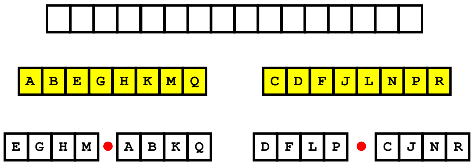
4 Merges Producing Length-4 lists



Handcoding the n =16 case

```
B0 = Merge(A0,A1)
B1 = Merge(A2,A3)
B2 = Merge(A4,A5)
B3 = Merge(A6,A7)
```

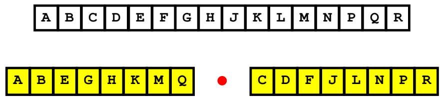
2 Merges Producing Length-8 Lists



Handcoding the n =16 case

```
C0 = Merge(B0,B1)
C1 = Merge(B2,B3)
```

1 Merge Producing a Length-16 List



A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q	R

A	B	E	G	H	K	M	Q

●

C	D	F	J	L	N	P	R

All Done!

```
D0 = Merge(C0,C1)
```

For general n, it can be handled using recursion.

Recursive Merge Sort

```
def MergeSort(a):
     n = length(a)
     if n==1:
          return a
     else:
          m  = n/2
          u0 = list(a[:m])
          u1 = list(a[m:])
          y0 = MergeSort(u0)
          y1 = MergeSort(u1)
          return Merge(y0,y1)
```

A function can call Itself!