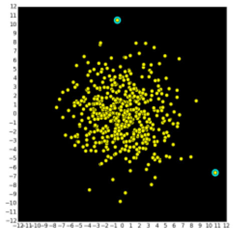
15. Functions and Lists

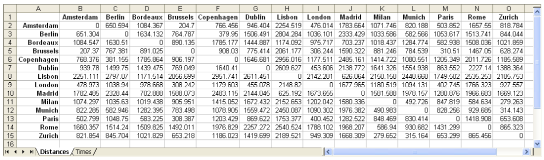
Topics:
- Subscripting
- Map
- Searching a list
- Example 1: Clouds of points
- Example 2: Selection Sort

Example 1: Computing the Diameter of a Cloud of Points



500 Points. Which two are furthest apart and what is their separation?

Same Problem: What's the Biggest Number in This Table?



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1		Amsterdam	Berlin	Bordeaux	Brussels	Copenhagen	Dublin	Lisbon	London	Madrid	Milan	Munich	Paris	Rome	Zurich
2	Amsterdam	0	650.594	1084.367	204.7	766.456	946.404	2254.519	476.014	1783.664	1071.746	820.188	503.852	1657.55	818.784
3	Berlin	651.304	0	1634.132	764.787	379.95	1506.491	2804.284	1036.101	2333.429	1033.586	582.566	1053.617	1513.741	844.044
4	Bordeaux	1084.547	1630.51	0	890.135	1785.177	1444.887	1174.092	975.717	703.237	1018.437	1284.774	582.938	1508.036	1021.859
5	Brussels	207.37	767.381	891.025	0	908.03	775.414	2061.177	306.244	1590.322	881.246	784.539	310.51	1467.05	628.274
6	Copenhagen	768.376	381.155	1785.864	906.197	0	1646.681	2956.016	1177.511	2485.161	1414.722	1080.551	1205.349	2011.726	1185.589
7	Dublin	939.78	1499.75	1439.475	769.049	1640.41	0	2609.627	453.606	2138.772	1641.326	1554.938	863.552	2227.14	1388.364
8	Lisbon	2251.111	2797.07	1171.514	2056.699	2951.741	2611.451	0	2142.281	626.064	2150.158	2448.668	1749.502	2535.253	2185.753
9	London	478.973	1038.94	978.668	308.242	1179.603	455.078	2148.82	0	1677.965	1180.519	1094.131	402.745	1766.323	927.557
10	Madrid	1782.485	2328.44	702.888	1588.073	2483.115	2144.045	625.192	1673.655	0	1581.588	1978.157	1280.876	1966.683	1669.123
11	Milan	1074.297	1035.63	1019.438	905.951	1415.052	1672.432	2152.653	1202.042	1580.336	0	492.726	847.819	584.634	279.263
12	Munich	822.285	582.946	1282.395	783.498	1078.905	1559.472	2450.087	1090.302	1976.382	490.983	0	828.256	929.685	314.143
13	Paris	502.799	1048.75	583.225	308.387	1203.429	869.622	1753.377	400.452	1282.522	848.469	830.414	0	1418.908	653.608
14	Rome	1660.357	1514.24	1509.825	1492.011	1976.829	2257.272	2540.524	1788.102	1968.207	586.94	930.682	1431.299	0	865.323
15	Zurich	821.854	845.704	1021.829	653.218	1186.023	1419.699	2189.521	949.309	1668.309	279.652	315.164	653.299	865.456	0
16															

Which two cities are furthest apart and what is their separation?

Example 2: Sorting a List of Numbers

Before:

`x -->` | 50 | 40 | 10 | 80 | 20 | 60 |

After:

`x -->` | 10 | 20 | 40 | 50 | 60 | 80 |

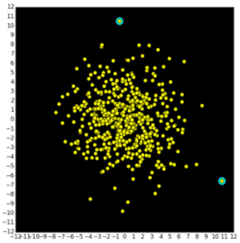
Sorting Algorithms

There are many sorting algorithms:

Selection Sort
Insertion Sort
Bubble Sort
Merge Sort
Quick Sort

A great venue for practicing list-based computing and for studying such things as efficiency and recursion (which we will do later).

Example 1: Computing the Diameter of a Cloud of Points



We will develop a module PointCloud

It Will Have Three Functions

`MakeCloud(n,sigma)`

This generates two lists x and y that define the coordinates of the points in the cloud.

`Diameter(x,y)`

This will compute the diameter of the cloud using the (x,y) coordinates of its points.

`ShowCloud(x,y)`

This will use simpleGraphicsE to display the cloud and highlight the "diameter points".

The Function MakeCloud

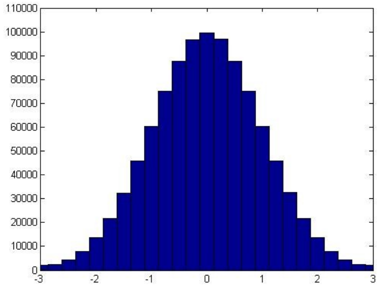
```
from random import normalvariate as randn

def MakeCloud(n,sigma):
     x=[]
     y=[]
     for k in range(n):
          r = randn(0,sigma)
          x.append(r)
          r = randn(0,sigma)
          y.append(r)
     return (x,y)
```

New Feature

The normal distribution

Generating floats from the Normal Distribution



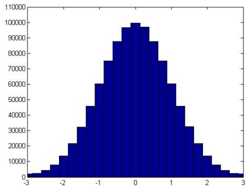
Generating floats from the Normal Distribution

If `mu` and `sigma` (positive) are floats, then

```
x = random.normalvariate(mu,sigma)
```

assigns to `x` a "random" float sampled from the normal distribution with mean mu and standard deviation sigma

Generating floats from the Normal Distribution



This is a histogram of the numbers this generates:

```
for k in range(10**6):
    r = randn(0,1)
```

Mean = 0
Standard deviation = 1

MakeCloud Returns Two Lists

```
from random import normalvariate as randn

def MakeCloud(n,sigma):
     x=[]
     y=[]
     for k in range(n):
          r = randn(0,sigma)
          x.append(r)
          r = randn(0,sigma)
          y.append(r)
     return (x,y)
```

New Feature

A function that returns more than one thing.

Note the parentheses

MakeCloud Returns Two Lists

```
>>> (x,y) = MakeCloud(3,1)
>>> print x
>>> print y
```

```
[-2.328, -0.044, -0.241]

[ 2.737,  2.078, -1.272]
```

Note the parentheses

MakeCloud

```
from random import normalvariate as randn

def MakeCloud(n,sigma):
    x=[]
    y=[]
    for k in range(n):
        r = randn(0,sigma)
        x.append(r)
        r = randn(0,sigma)
        y.append(r)
    return x,y
```

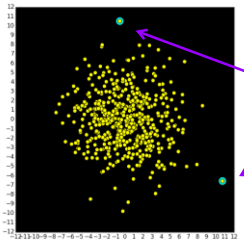
Old Stuff

x and y start out as empty lists.

Repeatedly generate a random number and append to x

Ditto for y

The Diameter Function: What It Computes



The "diameter points" and the distance between them

Input: lists x and y that define the yellow dots

Diameter: Formal Specs

```
def Diameter(x,y):
  """ Returns (d,imax,jmax) where d is a
  float that is the diameter of a cloud of
  points defined by lists x and y. imax and
  jmax are ints that are the indices of the
  diameter points.

  The diameter of a cloud of points is the
  maximum distance between any two points in
  the cloud. The two points for which this
  occurs are called diameter points.

  PreC: x and y are lists of floats with the
  same length.
```

Diameter: The Implementation

```
def Diameter(x,y):
   d = 0
   n = len(x)
   for i in range(n):
       for j in range(n):
          dx = x[i]-x[j]
          dy = y[i]-y[j]
          dij = sqrt(dx**2+dy**2)
          if dij>d:
             d = dij
             imax = i
             jmax = j
   return (d,imax,jmax)
```

New Feature

Nested Loops

Nested Loops

In this situation we have a loop whose body contains a loop

```
for blahblahblah
    ▨
```

and ▨ contains a loop.

Nested Loops: A Simple Example

```
for i in range(2):
     for j in range(3):
         print i,j
     print 'Inner'
print 'Outer'
```

Nested Loops: A Simple Example

```
for i in range(2):
     for j in range(3):
         print i,j
     print 'Inner'

print 'Outer'
```

Execute the loop body with i=0

Nested Loops: A Simple Example

```
for i in range(2):
     for j in range(3):
         print i,j
     print 'Inner'

print 'Outer'
```

```
0  0
0  1
0  2
Inner
```

Execute the loop body with i=0

Nested Loops: A Simple Example

```
for i in range(2):
     for j in range(3):
         print i,j
     print 'Inner'

print 'Outer'
```

```
0  0
0  1
0  2
Inner
```

Execute the loop body with i=1

Nested Loops: A Simple Example

```
for i in range(2):
     for j in range(3):
         print i,j
     print 'Inner'

print 'Outer'
```

```
0  0
0  1
0  2
Inner
1  0
1  1
1  2
Inner
```

Execute the loop body with i=1

Nested Loops: A Simple Example

```
for i in range(2):
     for j in range(3):
         print i,j
     print 'Inner'

print 'Outer'
```

```
0  0
0  1
0  2
Inner
1  0
1  1
1  2
Inner
```

Go to the next statement after the loop body.

Nested Loops: A Simple Example

```
for i in range(2):
    for j in range(3):
        print i,j
    print ‘Inner’

print ‘Outer’
```

```
0  0
0  1
0  2
Inner
1  0
1  1
1  2
Inner
Outer
```

Go to the next statement after the loop body.

Back to Diameter

When developing nested-loop solutions, it is essential to apply the methodology of step-wise refinement, perhaps preceded by a small example

Aspects of our problem

- Must check all possible pairs of points.
- Look at their separation distance
- What's the largest among these distances?

Suppose There Are 3 points

From	To	Dist
(x[0],[y[0])	(x[0],y[0])	0
(x[0],[y[0])	(x[1],y[1])	7
(x[0],[y[0])	(x[2],y[2])	9
(x[1],[y[1])	(x[0],y[0])	7
(x[1],[y[1])	(x[1],y[1])	0
(x[1],[y[1])	(x[2],y[2])	10
(x[2],[y[2])	(x[0],y[0])	9
(x[2],[y[2])	(x[1],y[1])	10
(x[2],[y[2])	(x[2],y[2])	0

Number of possibilities.: 9 = 3x3

Suppose There Are 3 points

From	To	Dist
(x[0],[y[0])	(x[0],y[0])	0
(x[0],[y[0])	(x[1],y[1])	7
(x[0],[y[0])	(x[2],y[2])	9
(x[1],[y[1])	(x[0],y[0])	7
(x[1],[y[1])	(x[1],y[1])	0
(x[1],[y[1])	(x[2],y[2])	10
(x[2],[y[2])	(x[0],y[0])	9
(x[2],[y[2])	(x[1],y[1])	10
(x[2],[y[2])	(x[2],y[2])	0

Number of possibilities.: 9 = 3x3

And now, stepwise refinement in action....

First Solution

```
d = 0
n = len(x)
for i in range(n):
    # Examine the distance from
    # (x[i],y[i]) to every other point
```

Second Solution

```
d = 0
n = len(x)
for i in range(n):
    for j in range(n):
        # Examine the distance from
        # (x[i],y[i]) to (x[j],y[j])
```

Third Solution

```
d = 0
n = len(x)
for i in range(n):
    for j in range(n):
        dx = x[i]-x[j]
        dy = y[i]-y[j]
        dij = sqrt(dx**2+dy**2)
        # Compare dij to d revising
        # the latter if necessary
```

Fourth Solution

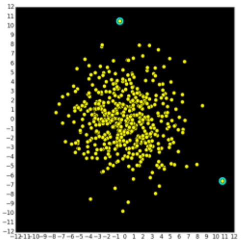
```
d = 0
n = len(x)
for i in range(n):
    for j in range(n):
        dx = x[i]-x[j]
        dy = y[i]-y[j]
        dij = sqrt(dx**2+dy**2)
        if dij>d:
            d = dij
            imax = i
            jmax = j
return (d,imax,jmax)
```

Fourth Solution

```
d = 0
n = len(x)
for i in range(n):
    for j in range(n):
        dx = x[i]-x[j]
        dy = y[i]-y[j]
        dij = sqrt(dx**2+dy**2)
        if dij>d:
            d = dij
            imax = i
            jmax = j
return (d,imax,jmax)
```

We have to "remember" where the max separation occurs.

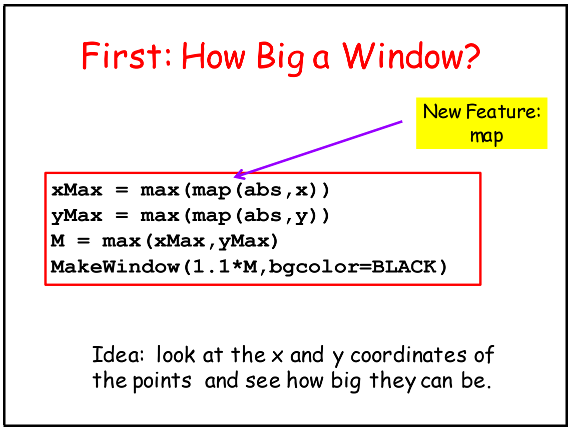
Next Up: ShowCloud



ShowCloud: Specs

```
def ShowCloud(x,y):
    """ Displays a point cloud
    defined by x and y and highlights
    the two points  that define
    its diameter.

    PreC: x and y are lists of
    floats with the same length.
    """
```

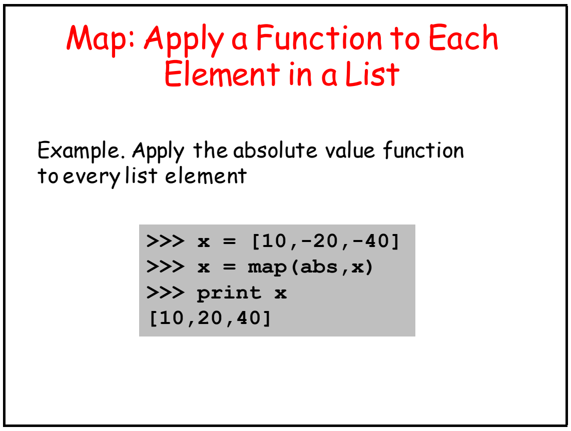


First: How Big a Window?

New Feature: map

```
xMax = max(map(abs,x))
yMax = max(map(abs,y))
M = max(xMax,yMax)
MakeWindow(1.1*M,bgcolor=BLACK)
```

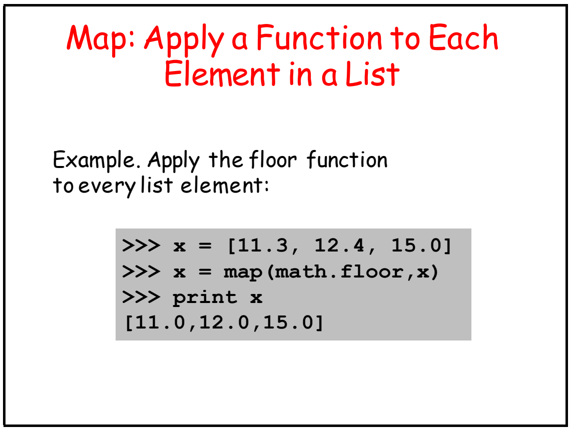
Idea: look at the x and y coordinates of the points and see how big they can be.



Map: Apply a Function to Each Element in a List

Example. Apply the absolute value function to every list element

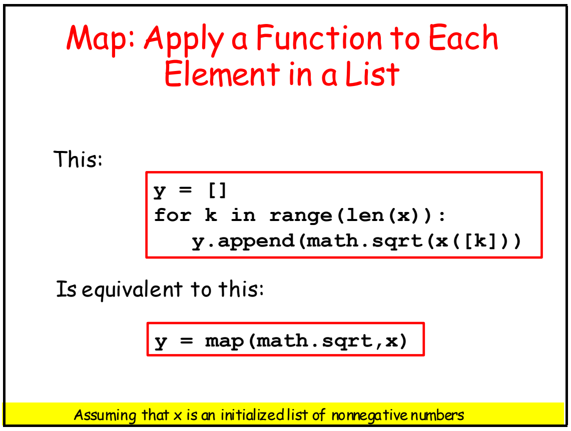
```
>>> x = [10,-20,-40]
>>> x = map(abs,x)
>>> print x
[10,20,40]
```



Map: Apply a Function to Each Element in a List

Example. Apply the floor function to every list element:

```
>>> x = [11.3, 12.4, 15.0]
>>> x = map(math.floor,x)
>>> print x
[11.0,12.0,15.0]
```



Map: Apply a Function to Each Element in a List

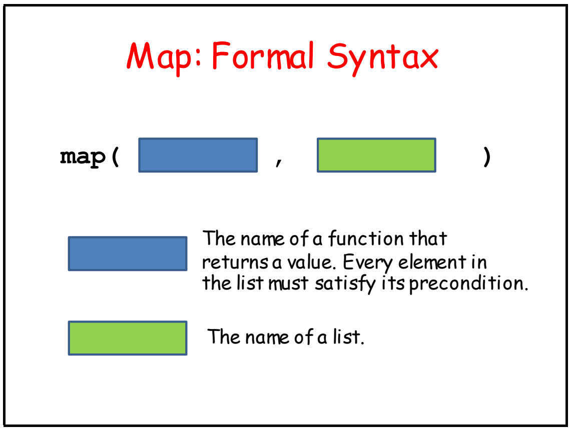
This:

```
y = []
for k in range(len(x)):
   y.append(math.sqrt(x([k]))
```

Is equivalent to this:

```
y = map(math.sqrt,x)
```

Assuming that x is an initialized list of nonnegative numbers



Map: Formal Syntax

map(🟦 , 🟩)

🟦 The name of a function that returns a value. Every element in the list must satisfy its precondition.

🟩 The name of a list.

Now, Back to ShowCloud

First: How Big a Window?

```
xMax = max(map(abs,x))
yMax = max(map(abs,y))
M = max(xMax,yMax)
MakeWindow(1.1*M,bgcolor=BLACK)
```

```
x = [-19,12,-4]
max(map(abs,x))
>>> 19
```

Next, Use DrawDisk For Each Point

```
r = M/50;
(d,i,j) = Diameter(x,y)
for k in range(len(x)):
   if k==i or k==j:
       DrawDisk(x[k],y[k],2*r,color=CYAN)
   DrawDisk(x[k],y[k],r,color=YELLOW)
```

i and j are the indices of the diameter points.

Before they are displayed, we paint a larger cyan dot.

Now, on to another example that highlights functions + lists

Example 2: Sorting a List of Numbers

Before:

x -->

50	40	10	80	20	60

After:

x -->

10	20	40	50	60	80

We Will Implement the Method of Selection Sort

At the Start:

x -->

50	40	10	80	20	60

High-Level:

```
for k in range(len(x)-1)
    Swap x[k] with the smallest
    value in x[k:]
```

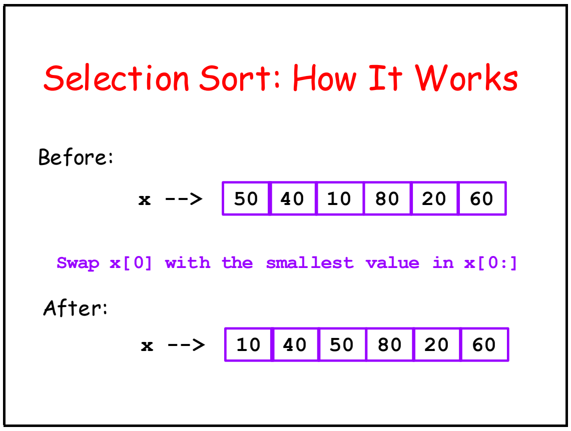
Selection Sort: How It Works

Before:

x -->

50	40	10	80	20	60

`Swap x[0] with the smallest value in x[0:]`



Selection Sort: How It Works

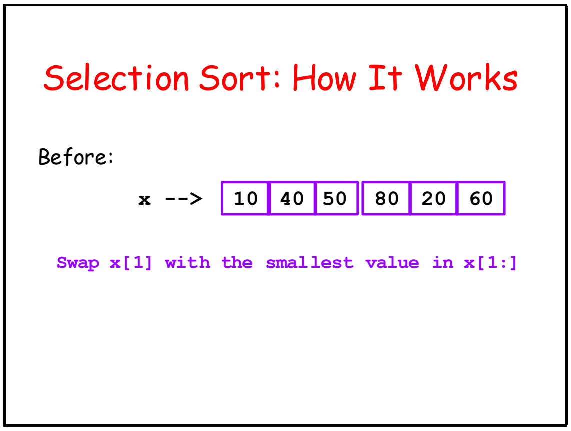
Before:

x -->	50	40	10	80	20	60

`Swap x[0] with the smallest value in x[0:]`

After:

x -->	10	40	50	80	20	60

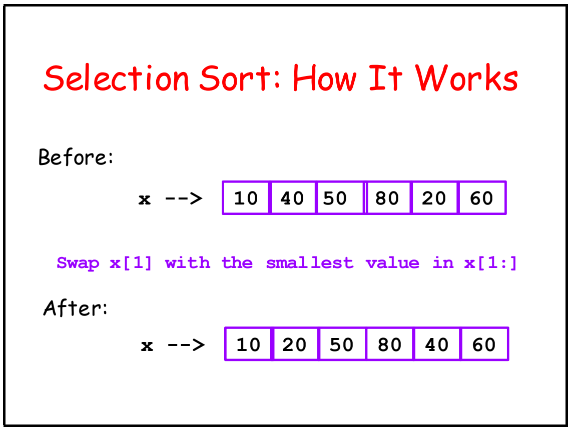


Selection Sort: How It Works

Before:

x -->	10	40	50	80	20	60

`Swap x[1] with the smallest value in x[1:]`



Selection Sort: How It Works

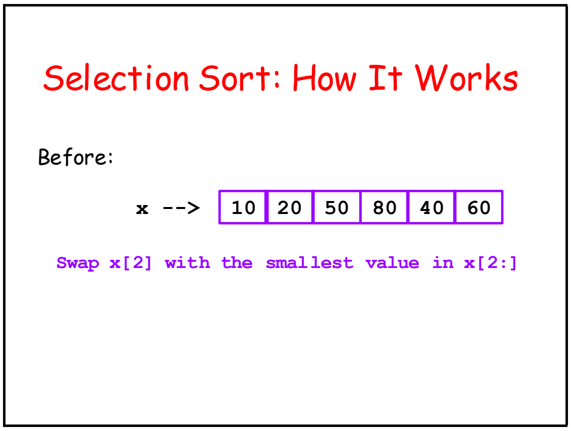
Before:

x -->	10	40	50	80	20	60

`Swap x[1] with the smallest value in x[1:]`

After:

x -->	10	20	50	80	40	60

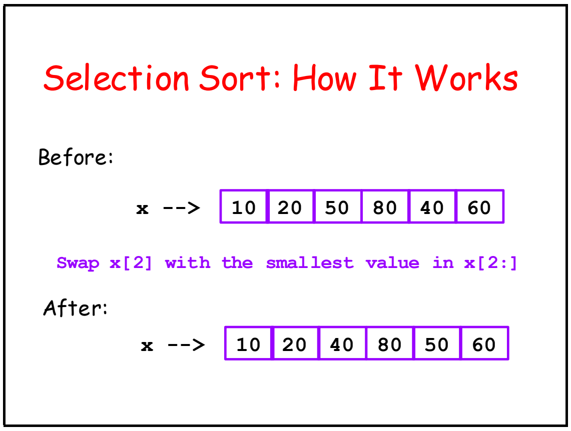


Selection Sort: How It Works

Before:

x -->	10	20	50	80	40	60

`Swap x[2] with the smallest value in x[2:]`



Selection Sort: How It Works

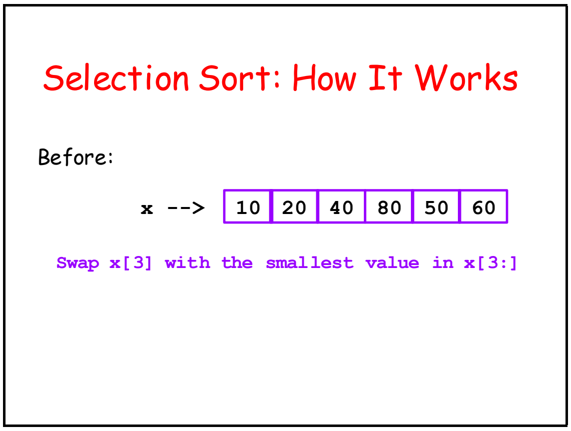
Before:

x -->	10	20	50	80	40	60

`Swap x[2] with the smallest value in x[2:]`

After:

x -->	10	20	40	80	50	60



Selection Sort: How It Works

Before:

x -->	10	20	40	80	50	60

`Swap x[3] with the smallest value in x[3:]`

Selection Sort: How It Works

Before:

x -->

10	20	40	80	50	60

`Swap x[3] with the smallest value in x[3:]`

After:

x -->

10	20	40	50	80	60

Selection Sort: How It Works

Before:

x -->

10	20	40	50	80	60

`Swap x[4] with the smallest value in x[4:]`

Selection Sort: How It Works

Before:

x -->

10	20	40	50	80	60

`Swap x[4] with the smallest value in x[4:]`

After:

x -->

10	20	40	50	60	80

Selection Sort: Recap

50	40	10	80	20	60
10	40	50	80	20	60
10	20	50	80	40	60
10	20	40	80	50	60
10	20	40	50	80	60
10	20	40	50	60	80
10	20	40	50	60	80

The Essential Helper Function: Select(x,i)

```
def Select(x,i):
    """ Swaps the smallest value in
    x[i:] with x[i]

    PreC: x is a list of integers  and
    i is an in that satisfies
    0<=i<len(x)"""
```

Does not return anything and it has a list argument

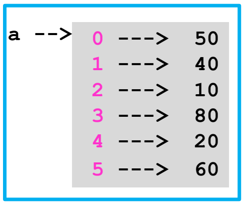
How Does it Work?

The calling program has a list. E.g.,

```
a -->   0 --->  50
        1 --->  40
        2 --->  10
        3 --->  80
        4 --->  20
        5 --->  60
```

How Does it Work?

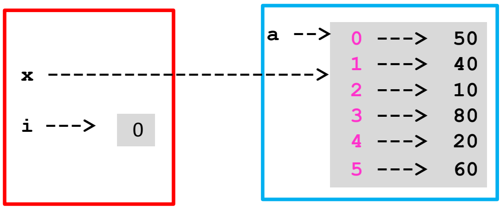
The calling program executes `Select(a,0)` and control passes to `Select`



```
a -->  0 --->  50
       1 --->  40
       2 --->  10
       3 --->  80
       4 --->  20
       5 --->  60
```

How Does Select Work?

- Nothing new about the assignment of 0 to i.
- But there is no assignment of the list a to x.
- Instead x now refers to the same list as a.



```
                                a -->  0 --->  50
x ---------------------------------->  1 --->  40
                                       2 --->  10
i --->  0                              3 --->  80
                                       4 --->  20
                                       5 --->  60
```

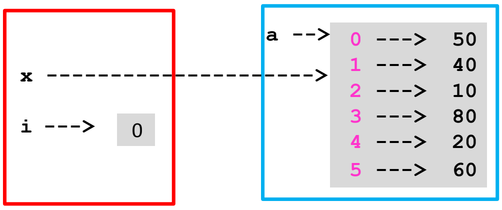
How Does Select Work?

If inside Select we have

`t = x[0]; x[0] = x[2]; x[2] = t`

it is as if we said

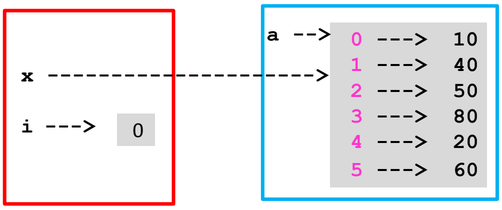
`t = a[0]; a[0] = a[2]; a[2] = t`



```
                                a -->  0 --->  50
x ---------------------------------->  1 --->  40
                                       2 --->  10
i --->  0                              3 --->  80
                                       4 --->  20
                                       5 --->  60
```

How Does Select Work?

It changes the list a in the calling program. We say x and a are aliased. They refer to the same list



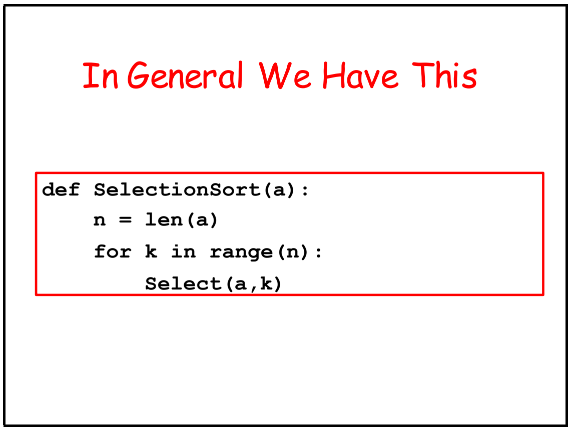
```
                                a -->  0 --->  10
x ---------------------------------->  1 --->  40
                                       2 --->  50
i --->  0                              3 --->  80
                                       4 --->  20
                                       5 --->  60
```

Let's Assume This Is Implemented

```
def Select(x,i):
    """ Swaps the smallest value in
    x[i:] with x[i]

    PreC: x is a list of integers  and
    i is an in that satisfies
    0<=i<len(x)"""
```

After this:	The list a looks like this					
Initialization	50	40	10	80	20	60
`Select(a,0)`	10	40	50	80	20	60
`Select(a,1)`	10	20	50	80	40	60
`Select(a,2)`	10	20	40	80	50	60
`Select(a,3)`	10	20	40	50	80	60
`Select(a,4)`	10	20	40	50	60	80
`Select(a,5)`	10	20	40	50	60	80



In General We Have This

```
def SelectionSort(a):
    n = len(a)
    for k in range(n):
        Select(a,k)
```