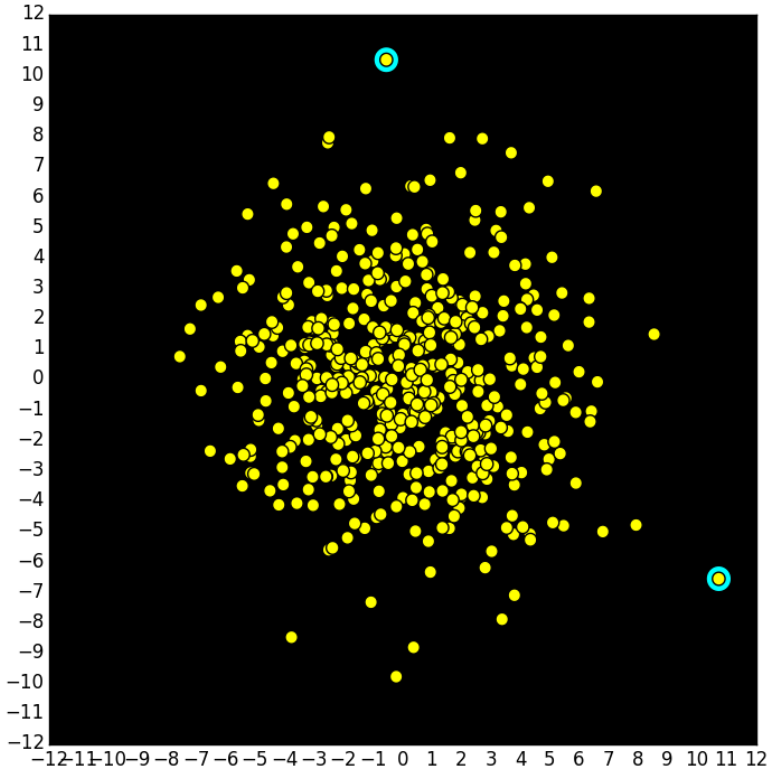
15. Functions and Lists

Topics:

- Subscripting
- Map
- Searching a list
- Example 1: Clouds of points
- Example 2: Selection Sort

Example 1: Computing the Diameter of a Cloud of Points



500 Points. Which two are furthest apart and what is their separation?

Same Problem: What's the Biggest Number in This Table?

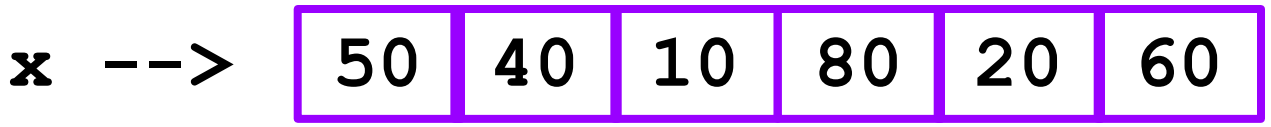
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1		Amsterdam	Berlin	Bordeaux	Brussels	Copenhagen	Dublin	Lisbon	London	Madrid	Milan	Munich	Paris	Rome	Zurich
2	Amsterdam	0	650.594	1084.367	204.7	766.456	946.404	2254.519	476.014	1783.664	1071.746	820.188	503.852	1657.55	818.784
3	Berlin	651.304	0	1634.132	764.787	379.95	1506.491	2804.284	1036.101	2333.429	1033.586	582.566	1053.617	1513.741	844.044
4	Bordeaux	1084.547	1630.51	0	890.135	1785.177	1444.887	1174.092	975.717	703.237	1018.437	1284.774	582.938	1508.036	1021.859
5	Brussels	207.37	767.381	891.025	0	908.03	775.414	2061.177	306.244	1590.322	881.246	784.539	310.51	1467.05	628.274
6	Copenhagen	768.376	381.155	1785.864	906.197	0	1646.681	2956.016	1177.511	2485.161	1414.722	1080.551	1205.349	2011.726	1185.589
7	Dublin	939.78	1499.75	1439.475	769.049	1640.41	0	2609.627	453.606	2138.772	1641.326	1554.938	863.552	2227.14	1388.364
8	Lisbon	2251.111	2797.07	1171.514	2056.699	2951.741	2611.451	0	2142.281	626.064	2150.158	2448.668	1749.502	2535.253	2185.753
9	London	478.973	1038.94	978.668	308.242	1179.603	455.078	2148.82	0	1677.965	1180.519	1094.131	402.745	1766.323	927.557
10	Madrid	1782.485	2328.44	702.888	1588.073	2483.115	2144.045	625.192	1673.655	0	1581.588	1978.157	1280.876	1966.683	1669.123
11	Milan	1074.297	1035.63	1019.438	905.951	1415.052	1672.432	2152.653	1202.042	1580.336	0	492.726	847.819	584.634	279.263
12	Munich	822.285	582.946	1282.395	783.498	1078.905	1559.472	2450.087	1090.302	1976.382	490.983	0	828.256	929.685	314.143
13	Paris	502.799	1048.75	583.225	308.387	1203.429	869.622	1753.377	400.452	1282.522	848.469	830.414	0	1418.908	653.608
14	Rome	1660.357	1514.24	1509.825	1492.011	1976.829	2257.272	2540.524	1788.102	1968.207	586.94	930.682	1431.299	0	865.323
15	Zurich	821.854	845.704	1021.829	653.218	1186.023	1419.699	2189.521	949.309	1668.309	279.652	315.164	653.299	865.456	0
16															

Distances | Times

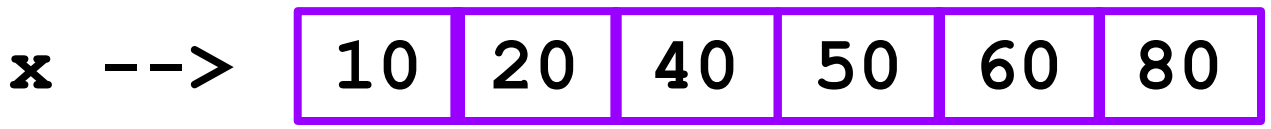
Which two cities are furthest apart and what is their separation?

Example 2: Sorting a List of Numbers

Before:



After:



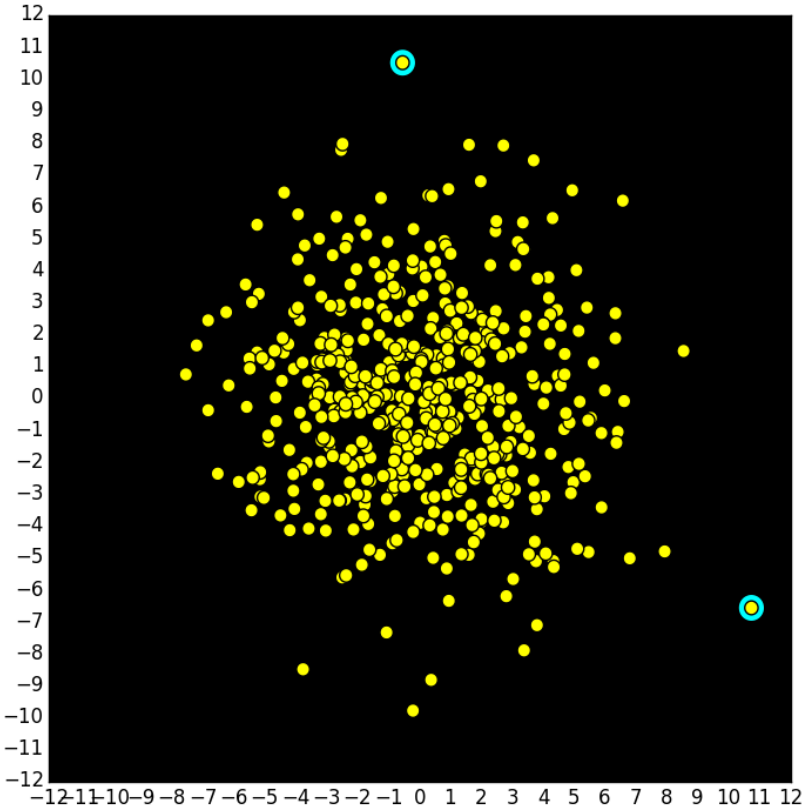
Sorting Algorithms

There are many sorting algorithms:

Selection Sort
Insertion Sort
Bubble Sort
Merge Sort
Quick Sort

A great venue for practicing list-based computing and for studying such things as efficiency and recursion (which we will do later).

Example 1: Computing the Diameter of a Cloud of Points



We will develop a module PointCloud

It Will Have Three Functions

`MakeCloud(n,sigma)`

This generates two lists x and y that define the coordinates of the points in the cloud.

`Diameter(x,y)`

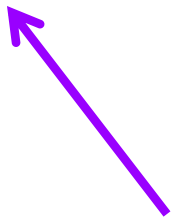
This will compute the diameter of the cloud using the (x,y) coordinates of its points.

`ShowCloud(x,y)`

This will use simpleGraphicsE to display the cloud and highlight the "diameter points".

The Function MakeCloud

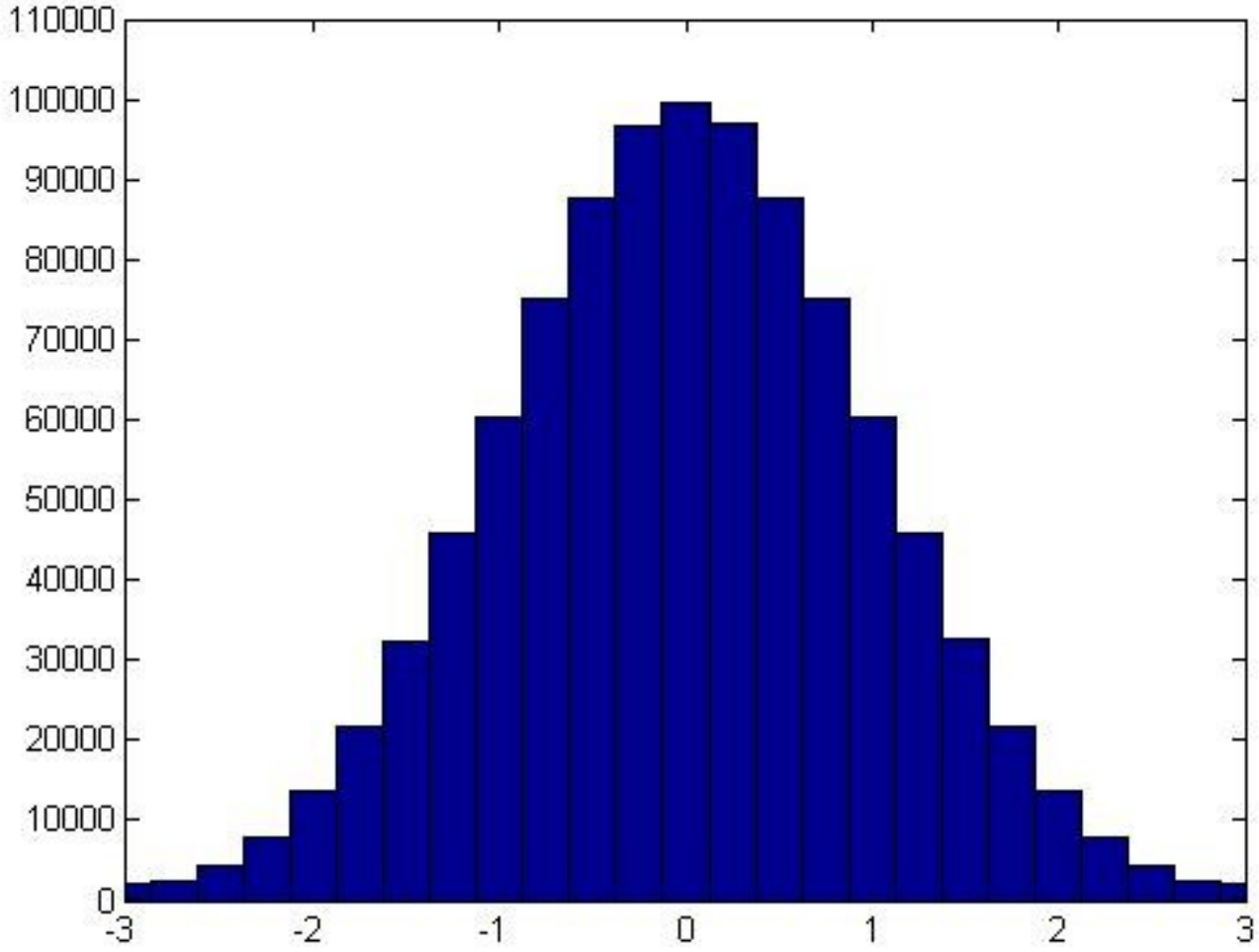
```
from random import normalvariate as randn

def MakeCloud(n,sigma):
    x=[]
    y=[]
    for k in range(n):
        r = randn(0,sigma)
        x.append(r)
        r = randn(0,sigma)
        y.append(r)
    return (x,y)
```



New Feature

The normal distribution

Generating floats from the Normal Distribution



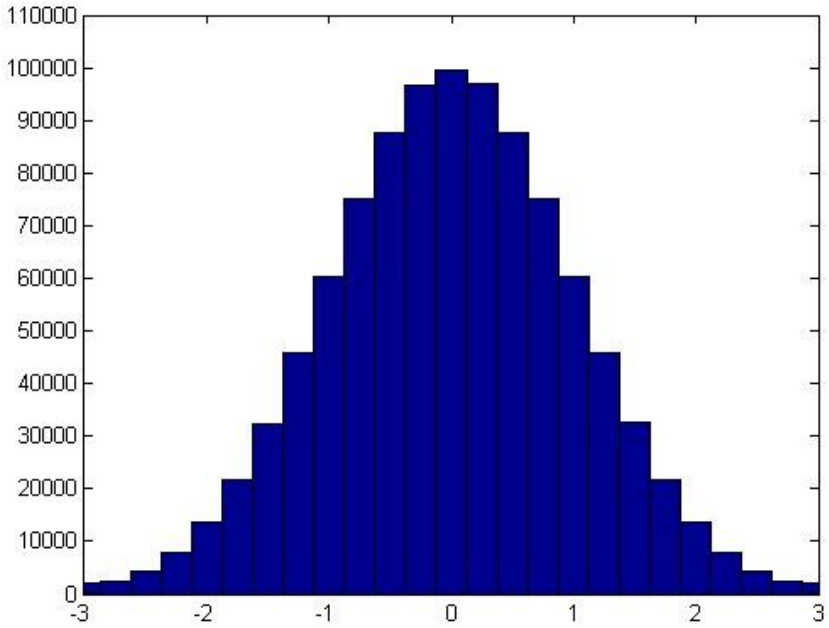
Generating floats from the Normal Distribution

If `mu` and `sigma` (positive) are floats, then

```
x = random.normalvariate(mu,sigma)
```

assigns to `x` a "random" float sampled from the normal distribution with mean mu and standard deviation sigma

Generating floats from the Normal Distribution



This is a histogram of the numbers this generates:

```
for k in range(10**6):
    r = randn(0,1)
```

Mean = 0
Standard deviation = 1

MakeCloud Returns Two Lists

```
from random import normalvariate as randn

def MakeCloud(n,sigma):
    x=[]
    y=[]
    for k in range(n):
        r = randn(0,sigma)
        x.append(r)
        r = randn(0,sigma)
        y.append(r)
    return (x,y)
```

New Feature

A function that returns more than one thing.

Note the parentheses

MakeCloud Returns Two Lists

```
>>> (x,y) = MakeCloud(3,1)
>>> print x
>>> print y
```

```
[-2.328, -0.044, -0.241]

[ 2.737,  2.078, -1.272]
```

Note the parentheses

MakeCloud

```
from random import normalvariate as randn

def MakeCloud(n,sigma):
    x=[]
    y=[]
    for k in range(n):
        r = randn(0,sigma)
        x.append(r)
        r = randn(0,sigma)
        y.append(r)
    return x,y
```

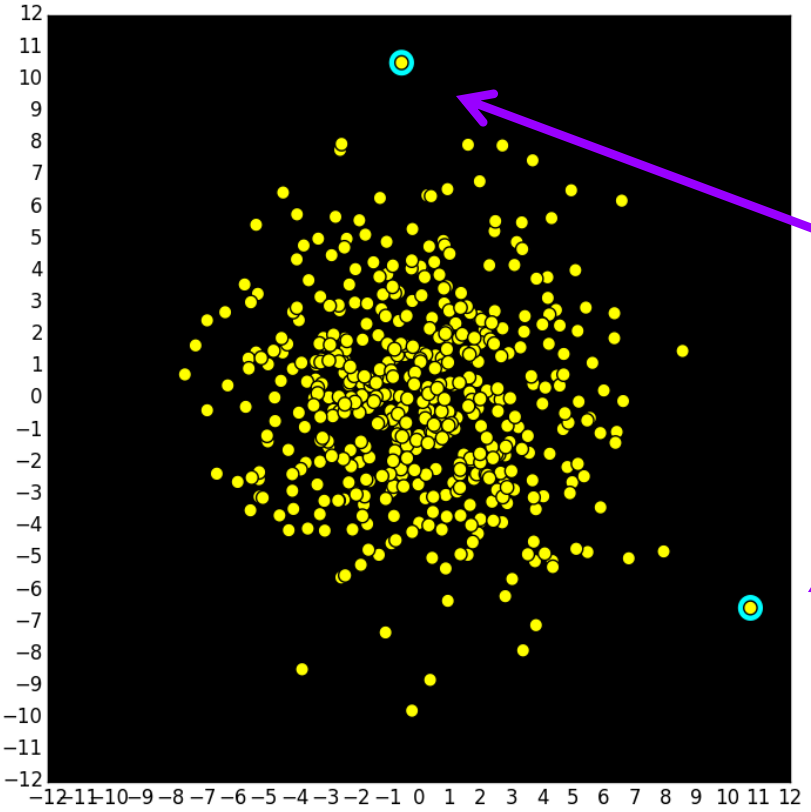
Old Stuff

x and y start out as empty lists.

Repeatedly generate a random number and append to x

Ditto for y

The Diameter Function: What It Computes



The "diameter points" and the distance between them

Input: lists x and y that define the yellow dots

Diameter: Formal Specs

```
def Diameter(x,y):
    """ Returns (d,imax,jmax) where d is a
    float that is the diameter of a cloud of
    points defined by lists x and y. imax and
    jmax are ints that are the indices of the
    diameter points.

    The diameter of a cloud of points is the
    maximum distance between any two points in
    the cloud. The two points for which this
    occurs are called diameter points.

    PreC: x and y are lists of floats with the
    same length.
```

Diameter: The Implementation

```
def Diameter(x,y):
    d = 0
    n = len(x)
    for i in range(n):
        for j in range(n):
            dx = x[i]-x[j]
            dy = y[i]-y[j]
            dij = sqrt(dx**2+dy**2)
            if dij>d:
                d = dij
                imax = i
                jmax = j
    return (d,imax,jmax)
```

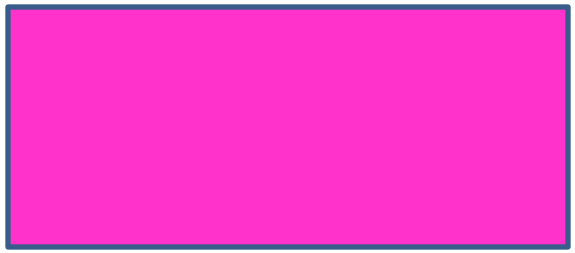
New Feature

Nested Loops

Nested Loops

In this situation we have a loop whose body contains a loop

```
for blahblahblah
```



and  contains a loop.

Nested Loops: A Simple Example

```
for i in range(2):
    for j in range(3):
        print i,j
    print 'Inner'
print 'Outer'
```

Nested Loops: A Simple Example

```
for i in range(2):
    for j in range(3):
        print i,j
    print 'Inner'

print 'Outer'
```

Execute the loop body with i=0

Nested Loops: A Simple Example

```
for i in range(2):
    for j in range(3):
        print i,j
    print 'Inner'

print 'Outer'
```

```
0   0
0   1
0   2
Inner
```

Execute the loop body with i=0

Nested Loops: A Simple Example

```
for i in range(2):
    for j in range(3):
        print i,j
    print 'Inner'

print 'Outer'
```

```
0   0
0   1
0   2
Inner
```

Execute the loop body with i=1

Nested Loops: A Simple Example

```
for i in range(2):
    for j in range(3):
        print i,j
    print 'Inner'
print 'Outer'
```

```
0 0
0 1
0 2
Inner
1 0
1 1
1 2
Inner
```

Execute the loop body with i=1

Nested Loops: A Simple Example

```
for i in range(2):
    for j in range(3):
        print i,j
    print 'Inner'
print 'Outer'
```

```
0 0
0 1
0 2
Inner
1 0
1 1
1 2
Inner
```

Go to the next statement after the loop body.

Nested Loops: A Simple Example

```
for i in range(2):
    for j in range(3):
        print i,j
    print 'Inner'
print 'Outer'
```

```
0 0
0 1
0 2
Inner
1 0
1 1
1 2
Inner
Outer
```

Go to the next statement after the loop body.

Back to Diameter

When developing nested-loop solutions, it is essential to apply the methodology of step-wise refinement, perhaps preceded by a small example

Aspects of our problem

- Must check all possible pairs of points.
- Look at their separation distance
- What's the largest among these distances?

Suppose There Are 3 points

From	To	Dist
(x[0],[y[0])	(x[0],y[0])	0
(x[0],[y[0])	(x[1],y[1])	7
(x[0],[y[0])	(x[2],y[2])	9
(x[1],[y[1])	(x[0],y[0])	7
(x[1],[y[1])	(x[1],y[1])	0
(x[1],[y[1])	(x[2],y[2])	10
(x[2],[y[2])	(x[0],y[0])	9
(x[2],[y[2])	(x[1],y[1])	10
(x[2],[y[2])	(x[2],y[2])	0

Number of possibilities.: 9 = 3x3

Suppose There Are 3 points

From	To	Dist
(x[0],[y[0])	(x[0],y[0])	0
(x[0],[y[0])	(x[1],y[1])	7
(x[0],[y[0])	(x[2],y[2])	9
(x[1],[y[1])	(x[0],y[0])	7
(x[1],[y[1])	(x[1],y[1])	0
(x[1],[y[1])	(x[2],y[2])	10
(x[2],[y[2])	(x[0],y[0])	9
(x[2],[y[2])	(x[1],y[1])	10
(x[2],[y[2])	(x[2],y[2])	0

Number of possibilities.: 9 = 3x3

And now, stepwise refinement in action….

First Solution

```
d = 0
n = len(x)
for i in range(n):
    # Examine the distance from
    # (x[i],y[i]) to every other point
```

Second Solution

```
d = 0
n = len(x)
for i in range(n):
    for j in range(n):
        # Examine the distance from
        # (x[i],y[i]) to (x[j],y[j])
```

Third Solution

```
d = 0
n = len(x)
for i in range(n):
     for j in range(n):
          dx = x[i]-x[j]
          dy = y[i]-y[j]
          dij = sqrt(dx**2+dy**2)
           # Compare dij to d revising
           # the latter if necessary
```

Fourth Solution

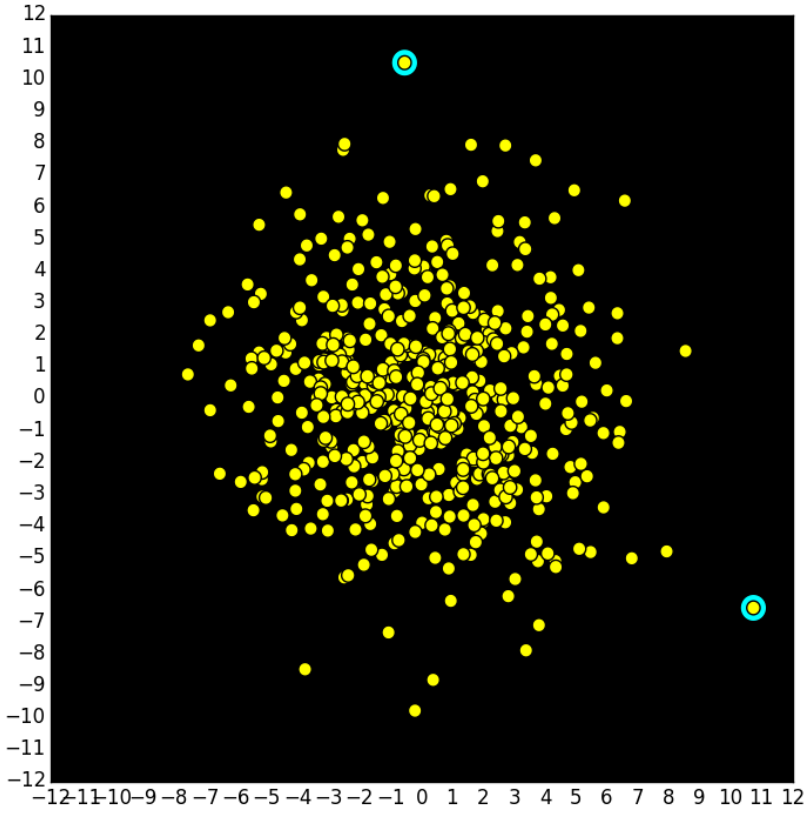
```
d = 0
n = len(x)
for i in range(n):
     for j in range(n):
          dx = x[i]-x[j]
          dy = y[i]-y[j]
          dij = sqrt(dx**2+dy**2)
           if dij>d:
                d = dij
                imax = i
                jmax = j
return (d,imax,jmax)
```

Fourth Solution

```
d = 0
n = len(x)
for i in range(n):
     for j in range(n):
          dx = x[i]-x[j]
          dy = y[i]-y[j]
          dij = sqrt(dx**2+dy**2)
          if dij>d:
               d = dij
               imax = i
               jmax = j
return (d,imax,jmax)
```

We have to "remember" where the max separation occurs.

Next Up: ShowCloud



ShowCloud: Specs

```
def ShowCloud(x,y):
    """ Displays a point cloud
    defined by x and y and highlights
    the two points  that define
    its diameter.

    PreC: x and y are lists of
    floats with the same length.
    """
```

First: How Big a Window?

New Feature: map

```
xMax = max(map(abs,x))
yMax = max(map(abs,y))
M = max(xMax,yMax)
MakeWindow(1.1*M,bgcolor=BLACK)
```

Idea: look at the x and y coordinates of the points and see how big they can be.

Map: Apply a Function to Each Element in a List

Example. Apply the absolute value function to every list element

```
>>> x = [10,-20,-40]
>>> x = map(abs,x)
>>> print x
[10,20,40]
```

Map: Apply a Function to Each Element in a List

Example. Apply the floor function to every list element:

```
>>> x = [11.3, 12.4, 15.0]
>>> x = map(math.floor,x)
>>> print x
[11.0,12.0,15.0]
```

Map: Apply a Function to Each Element in a List

This:

```
y = []
for k in range(len(x)):
    y.append(math.sqrt(x([k]))
```

Is equivalent to this:

```
y = map(math.sqrt,x)
```

Assuming that x is an initialized list of nonnegative numbers

Map: Formal Syntax

`map(`  `,`  `)`

The name of a function that returns a value. Every element in the list must satisfy its precondition.

The name of a list.

Now, Back to ShowCloud

First: How Big a Window?

```
xMax = max(map(abs,x))
yMax = max(map(abs,y))
M = max(xMax,yMax)
MakeWindow(1.1*M,bgcolor=BLACK)
```

```
x = [-19,12,-4]
max(map(abs,x))
>>> 19
```

Next, Use DrawDisk For Each Point

```
r = M/50;
(d,i,j) = Diameter(x,y)
for k in range(len(x)):
    if k==i or k==j:
        DrawDisk(x[k],y[k],2*r,color=CYAN)
    DrawDisk(x[k],y[k],r,color=YELLOW)
```

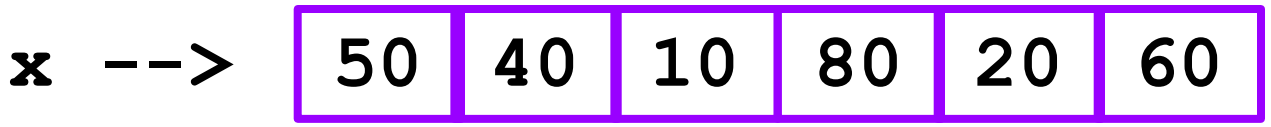
i and j are the indices of the diameter points.

Before they are displayed, we paint a larger cyan dot.

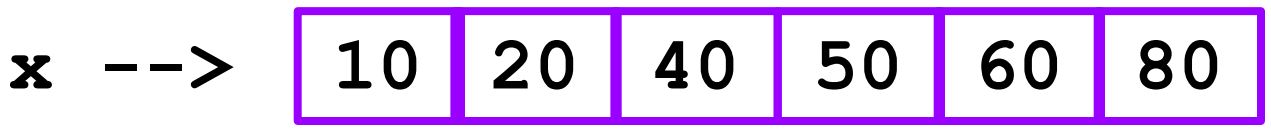
Now, on to another example that highlights functions + lists

Example 2: Sorting a List of Numbers

Before:

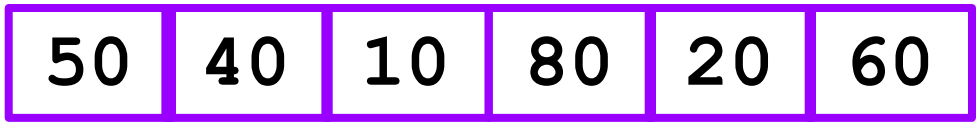


After:



We Will Implement the Method of Selection Sort

At the Start:

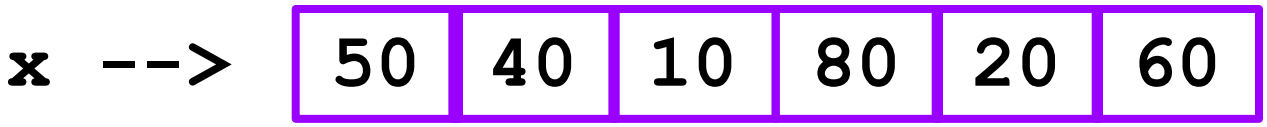


High-Level:

```
for k in range(len(x)-1)
    Swap x[k] with the smallest
    value in x[k:]
```

Selection Sort: How It Works

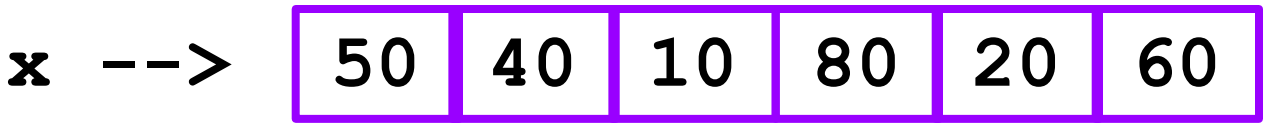
Before:



Swap x[0] with the smallest value in x[0:]

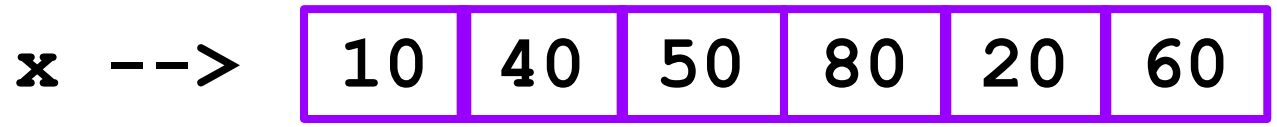
Selection Sort: How It Works

Before:



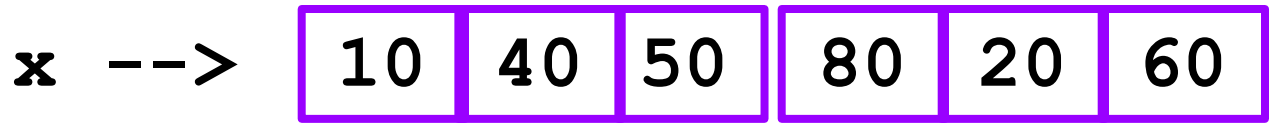
Swap x[0] with the smallest value in x[0:]

After:



Selection Sort: How It Works

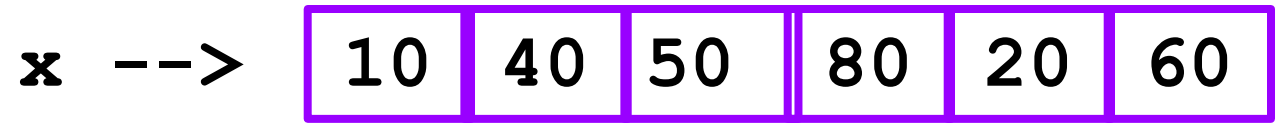
Before:




```
Swap x[1] with the smallest value in x[1:]
```

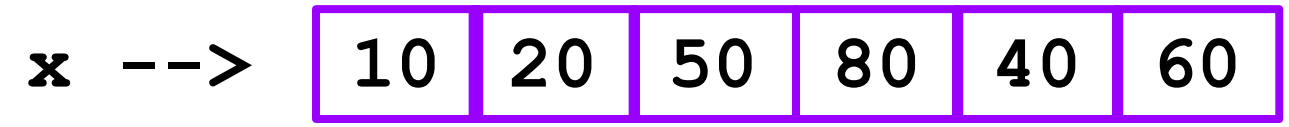
Selection Sort: How It Works

Before:



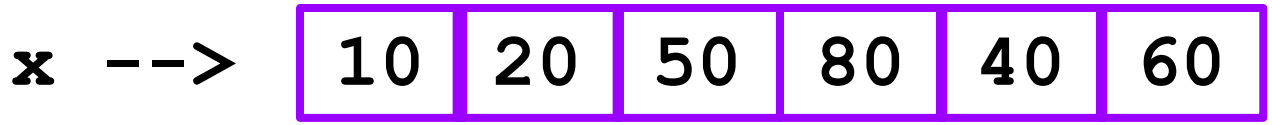
Swap x[1] with the smallest value in x[1:]

After:



Selection Sort: How It Works

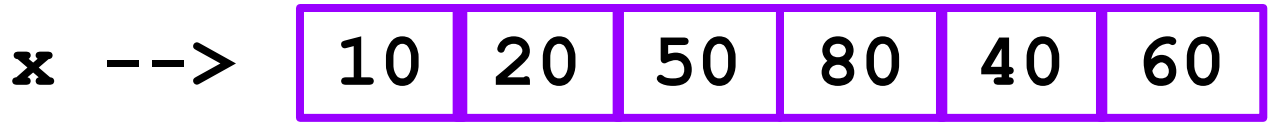
Before:




```
Swap x[2] with the smallest value in x[2:]
```

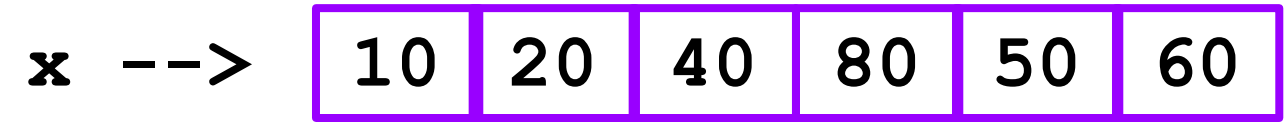
Selection Sort: How It Works

Before:



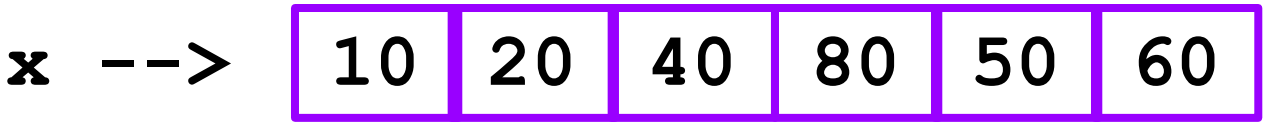

```
Swap x[2] with the smallest value in x[2:]
```

After:



Selection Sort: How It Works

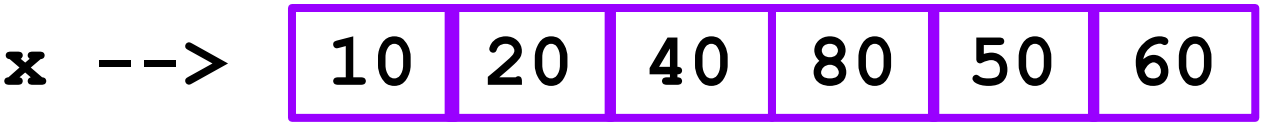
Before:



Swap x[3] with the smallest value in x[3:]

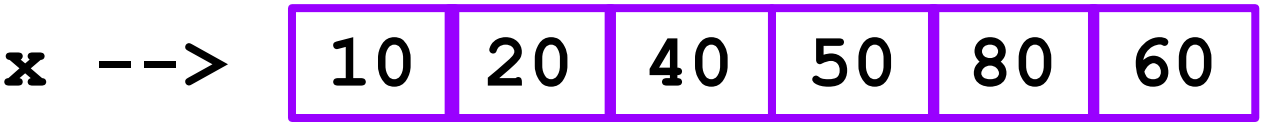
Selection Sort: How It Works

Before:



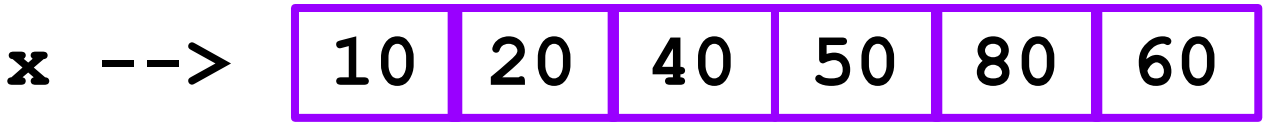

```
Swap x[3] with the smallest value in x[3:]
```

After:



Selection Sort: How It Works

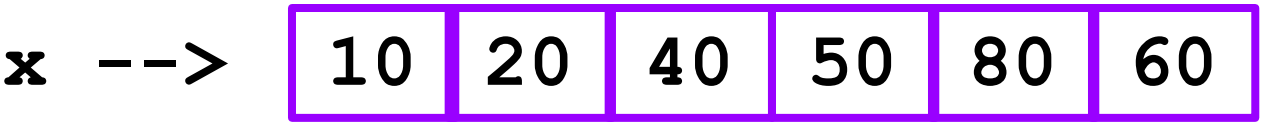
Before:




```
Swap x[4] with the smallest value in x[4:]
```

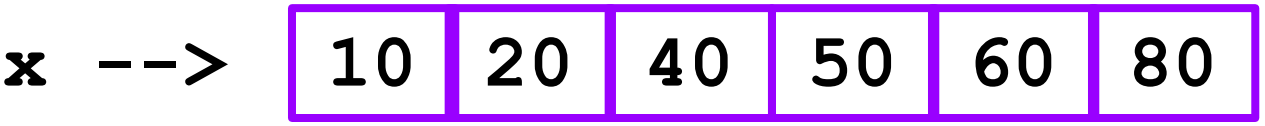
Selection Sort: How It Works

Before:

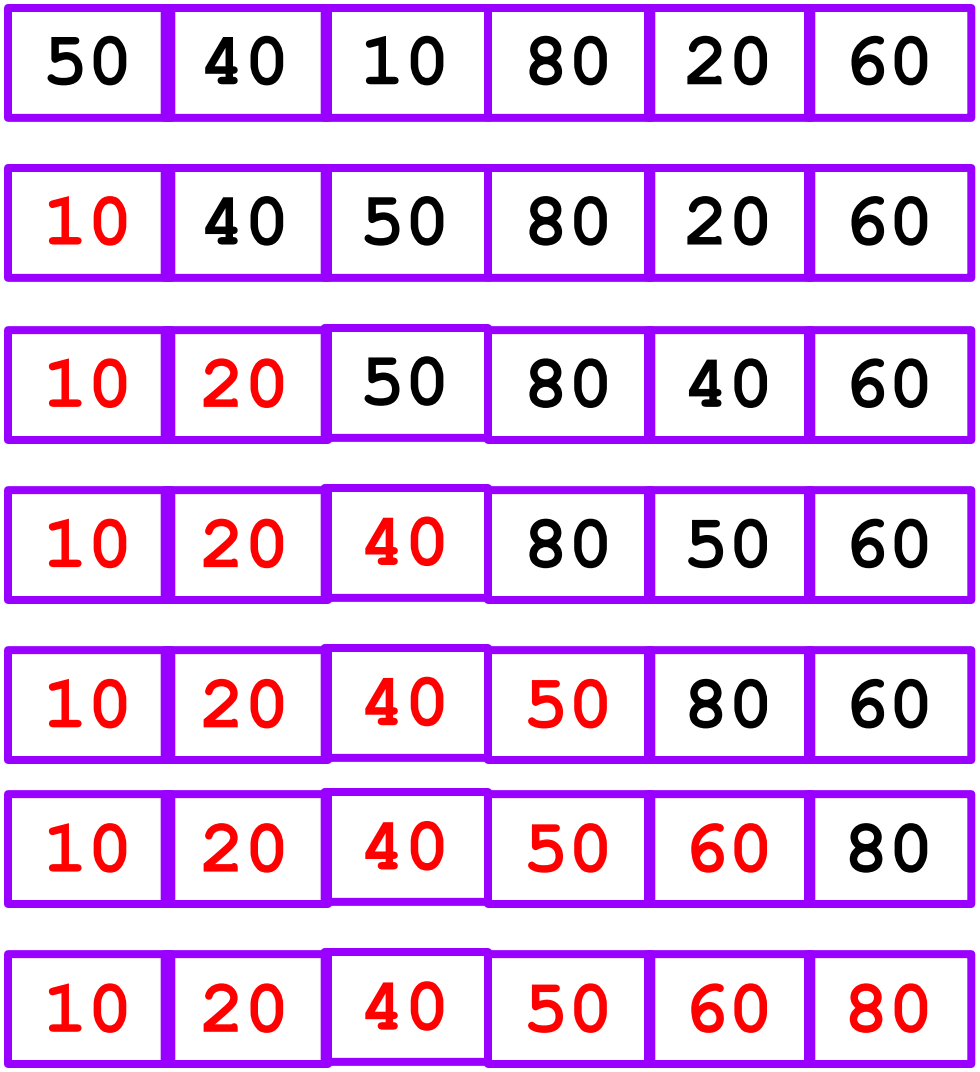


Swap x[4] with the smallest value in x[4:]

After:



Selection Sort: Recap



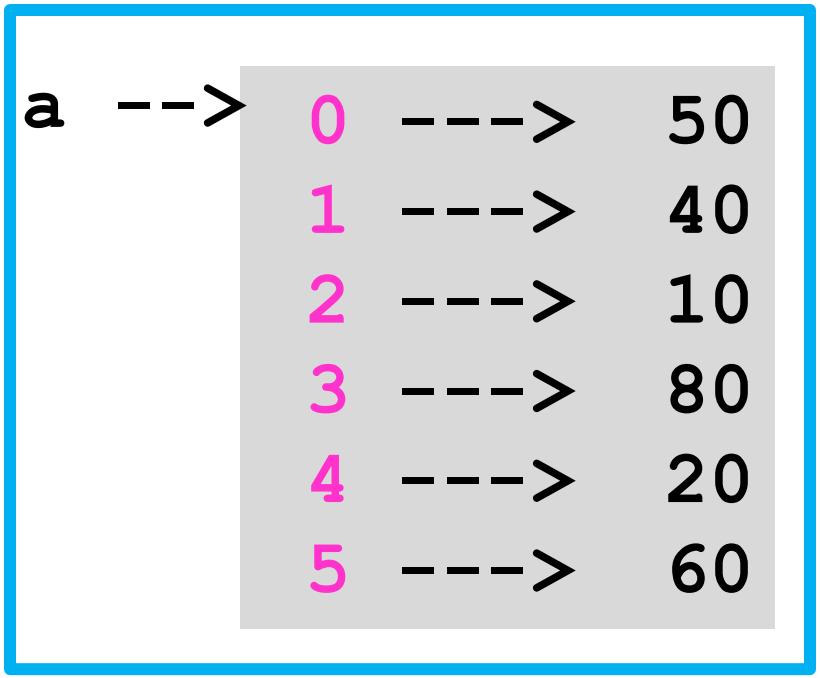
The Essential Helper Function: Select(x,i)

```
def Select(x,i):
    """ Swaps the smallest value in
    x[i:] with x[i]

    PreC: x is a list of integers and
    i is an in that satisfies
    0<=i<len(x)"""
```

Does not return anything and it has a list argument

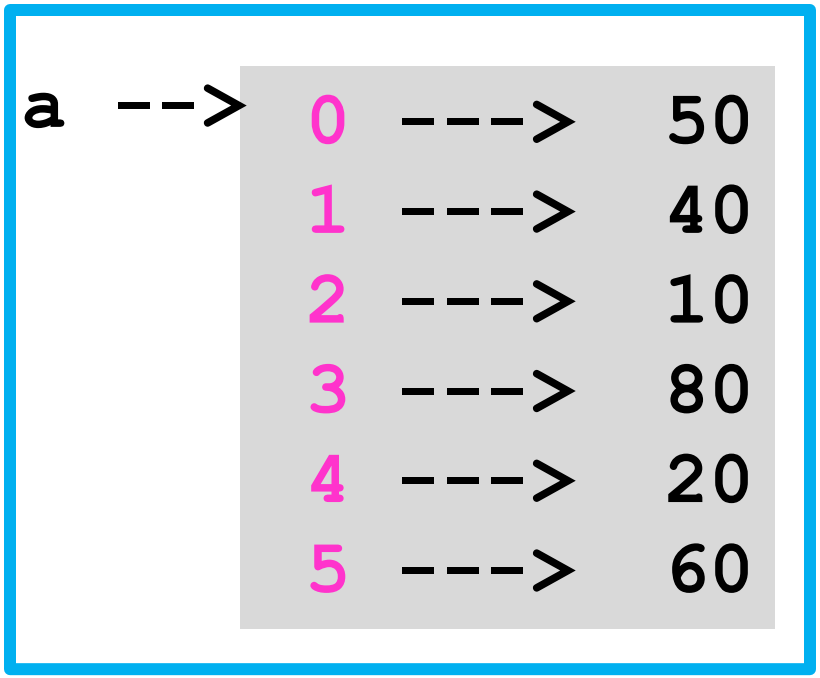
How Does it Work?

The calling program has a list. E.g.,



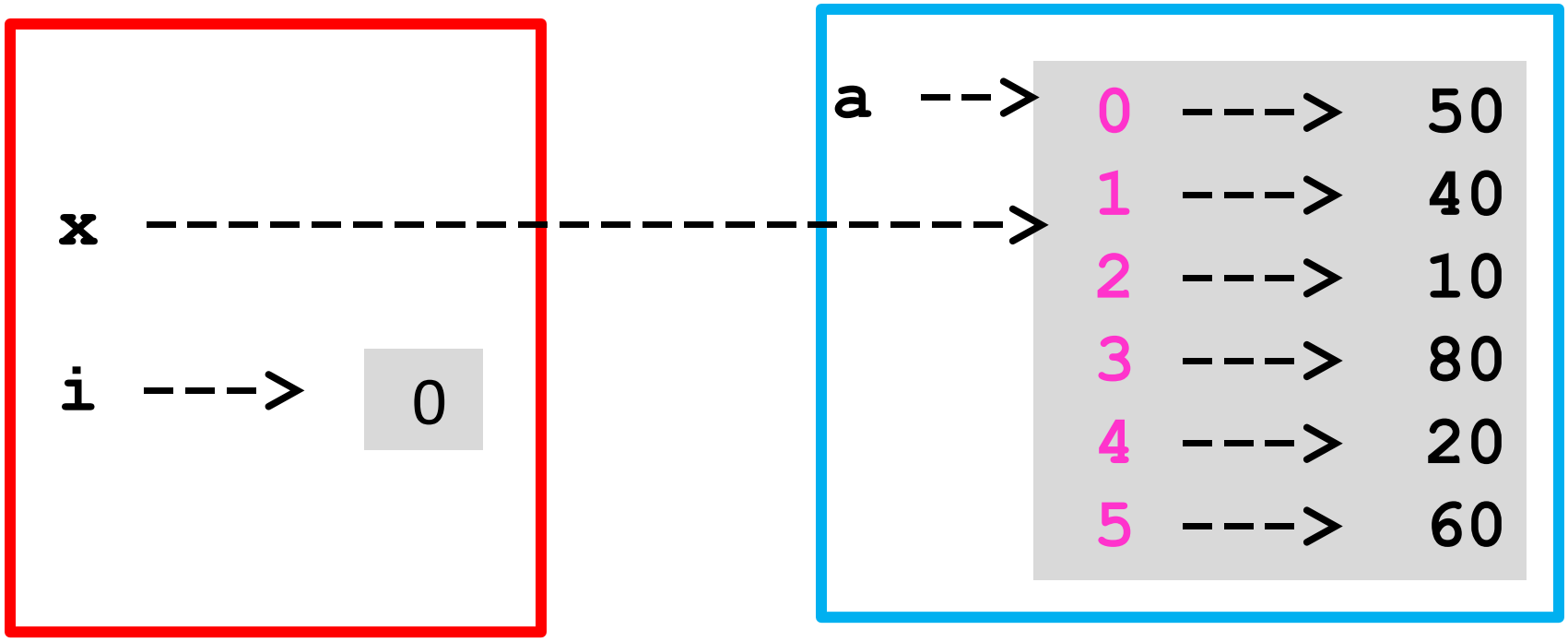
How Does it Work?

The calling program executes `Select(a,0)` and control passes to `Select`



How Does Select Work?

- Nothing new about the assignment of 0 to i.
- But there is no assignment of the list a to x.
- Instead x now refers to the same list as a.



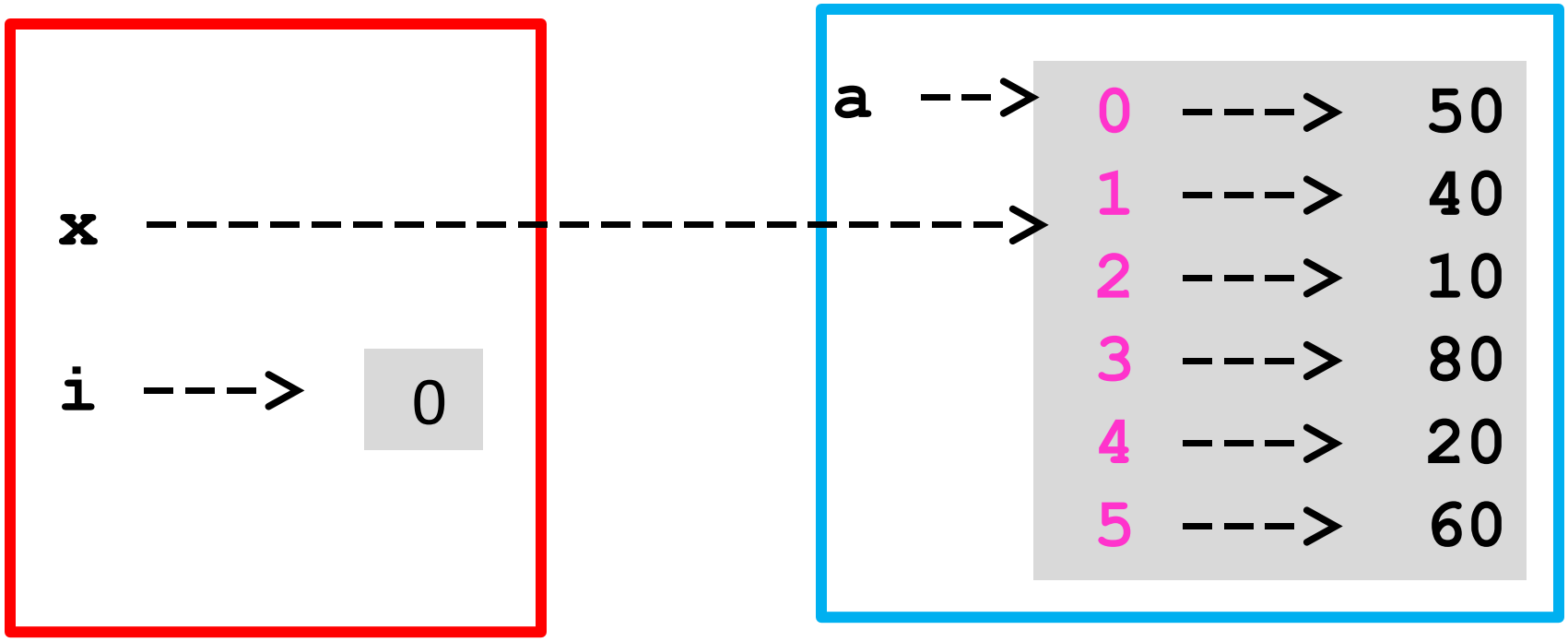
How Does Select Work?

If inside Select we have

```
t = x[0]; x[0] = x[2]; x[2] = t
```

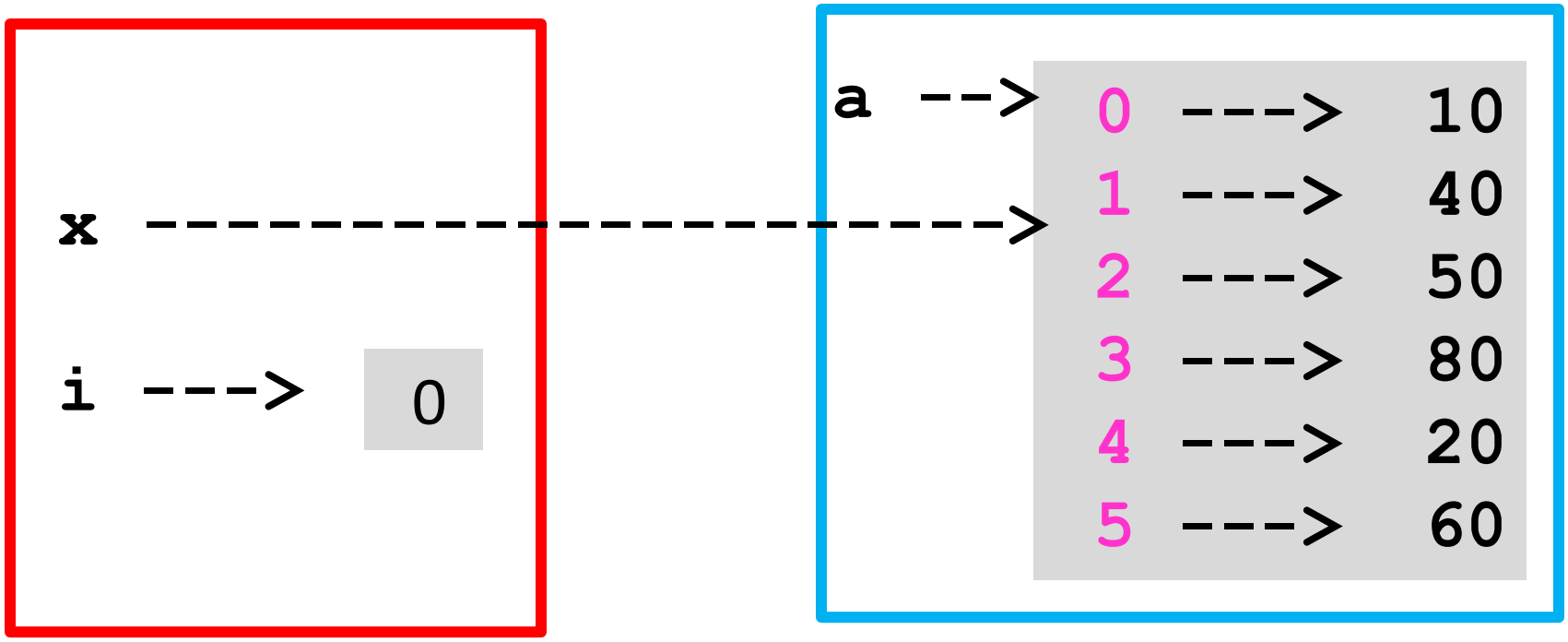
it is as if we said

```
t = a[0]; a[0] = a[2]; a[2] = t
```



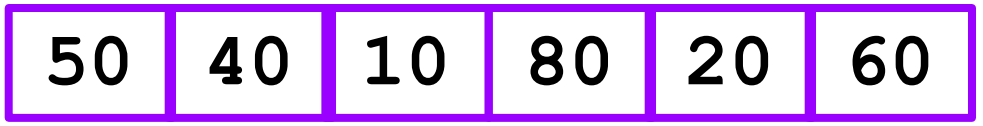
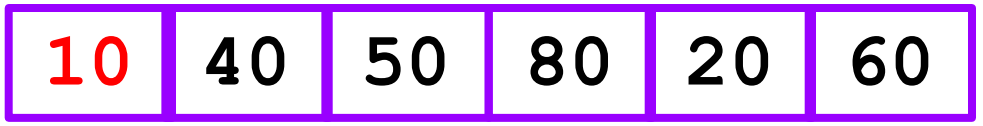
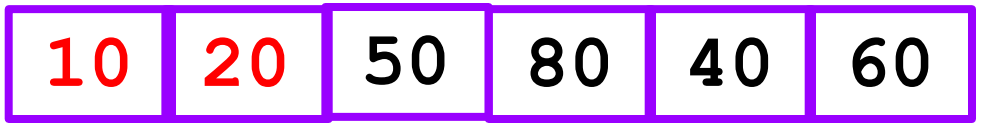
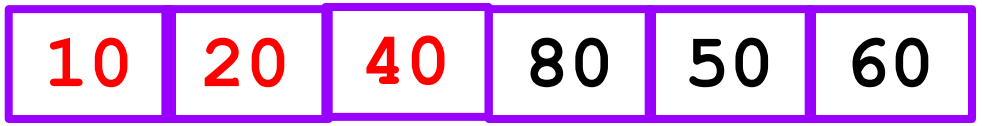
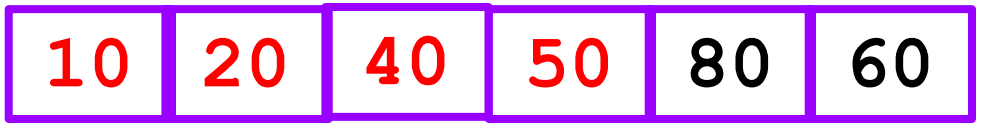
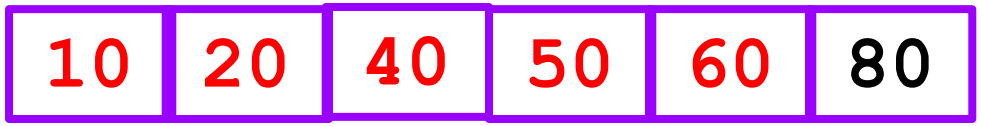
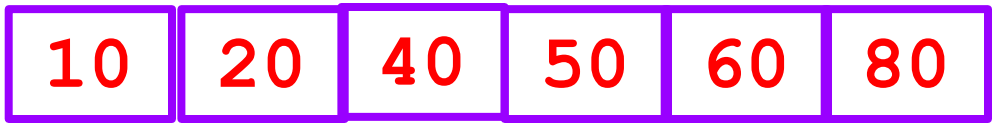
How Does Select Work?

It changes the list a in the calling program. We say x and a are aliased. They refer to the same list



Let's Assume This Is Implemented

```
def Select(x,i):
    """ Swaps the smallest value in
    x[i:] with x[i]

    PreC: x is a list of integers and
    i is an in that satisfies
    0<=i<len(x)"""
```

After this:	The list a looks like this
Initialization	
`Select(a,0)`	
`Select(a,1)`	
`Select(a,2)`	
`Select(a,3)`	
`Select(a,4)`	
`Select(a,5)`	

In General We Have This

```
def SelectionSort(a):
    n = len(a)
    for k in range(n):
        Select(a,k)
```