CS1110

Lecture 21: **Invariants, Quicksort, Partitioning**

Upcoming schedule

Friday Apr 18: A4 due

Monday Apr 21: deadline for lab 11 checkoff

Tuesday Apr 22

- Lecture = (optional) review session
- Lab sessions = (optional) drop-in office hours. No new lab.
- **Prelim: 7:30-9pm, 200 Baker Lab (= the auditorium)**
  (No S/U students, so it should be less crowded)

Wednesday Apr 23: No labs or office hours

Prelim grades announced by email from CMS, *hopefully* by Thursday morning.

Slides by D. Gries, L. Lee, S. Marschner, W. White

Prelim 2 coverage: Lectures 12-22 (this Thursday), associated labs and assignments: recursion, using classes effectively, loops and loop invariants, sequence algorithms.

Preparing for the exam

Past exams are posted on the Exams section of the website. Profs Lee and Marschner wrote the Spring 2013 one, and the topic coverage is equivalent.

Review all lectures up to and including April 17. Be able to do A3, A4, labs 8-11 from scratch, cold. Be able to do the worked invariants exercises yourself.

As always, come to our many office hours/consulting hours for in-person help; see the Staff section of the webpage.

As always, watch Piazza for announcements, for helpful answers to other people's questions, etc.

At most one line needs to be fixed

```
1.  def apl(s):
2.      """Returns: num of adjacent equal pairs in string s"""
3.      x = 0;  k = 0
4.      # k: next possible ending place of adjacent pair, i.e.
5.      #inv: x = # adjacent equal pairs in s[0..k-1]
6.      while k < len(s):
7.          x += (s[k-1] == s[k])
8.          k += 1
9.      return x
```

(A) Error in line 3 (B) line 6 (C) line 7 (D) line 8
(E) no error

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6.     while k < len(s):
7.         x += (s[k-1] == s[k])
8.         k += 1
9.     return x
```

(A)Error in line 3: the one-line fix is to change "k=0" to "k=1"

At most one line needs to be fixed

```
1. def ap2(s):
2.     """Returns: num of adjacent equal pairs in string s"""
3.     x = 0;  j = 0
4.     # j: next possible start place of adjacent pair, i.e.
5.     #inv: x = # adjacent equal pairs in s[0..j]
6.     while j < len(s)-1:
           j += 1
7.         x += (s[j] == s[j+1])
8. return x
```

(A) Error in line 3 (B) line 6 (C) line 7 (D) line 8
(E) no error

At most one line needs to be fixed

```
1.  def ap2(s):
2.      """Returns: num of adjacent equal pairs in string s"""
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5.      #inv: x = # adjacent equal pairs in s[0..j]
6.     while j < len(s)-1:
           j += 1
7.         x += (s[j] == s[j+1])
8.  return x
```

(C) line 7: one-line fix is "s[j] == s[j+1]" to "s[j-1]==s[j]"

Warning about Python indexing errors

Since negative indices are not a syntactic error in Python, it can be *especially* hard debug indexing errors.

Example: conceptually, starting with k=0 and checking s[k] vs s[k-1] is an error because you're referring to a non-existent position "before the start of the string".

However, Python interprets s[-1] as meaning the item at the end of the string. Those aren't what the previous slides meant to compare!

Sorting: A Key Algorithmic Family

Q: Given a list of items, how can we arrange for them to be sorted in increasing order, in a time- and space-efficient manner?

Applications: making items easier to find.[1]

```
def sort(b, h, k):
    """Sort  b[h..k] in place.  Pre: b: list of ints; k>=h-1"""
    # Start with b[h], and organize the rest according to it?? No...
    # Note: we have h & k explicit to simplify recursive
    # structure.
```

[1]Also, computing poker-hand probabilities.

Motivation: A Famous Sorting Algorithm

```
def qsort(b, h, k):
    """Make b[h..k] sorted.
    Pre: b: list of ints; k>=h-1"""
```

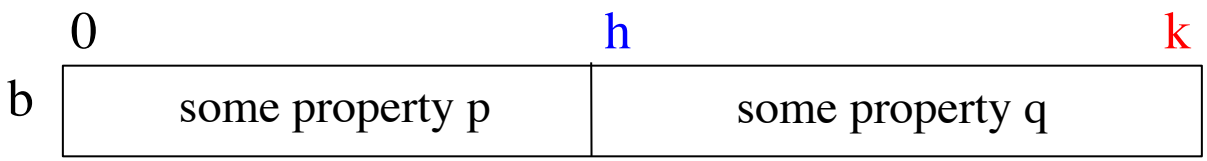
 # base case

```
i = partition(b, h, k)
```

Clicker Q1: recursive case

```
def partition(b, h, k):
    """Let x = b[h] be the pivot
value.  Rearrange b[h..k] so
that there is an i where
b[h..i-1] <= x, b[i]=x; b[i+1..k]
>=x.  Return i.
Pre: k>=h"""
# Can you do this without
# creating extra lists?
```

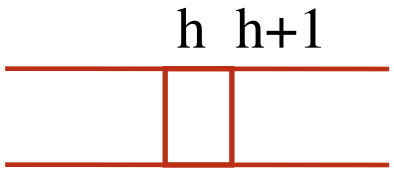
Pictorial Notation for Sequence Assertions



Equivalent to:

Property p holds on all items in b[0..h-1], and property q holds on all items in b[h..k].

(The precise location of the "vertical bars" matters.)



Can also indicate single items.

((h +1) – h = 1; it's all consistent, hurrah.)

Partition Algorithm

- Given a sequence b[h..k] with some *value* x in b[h]:

		h		k
pre:	b	***x***	?	

- Swap elements of b[h..k] and store in i to truthify postcondition:

		h		i	i+1		k
post:	b	<= ***x***		***x***		>= ***x***	

		h								k
change:	b	**3**	5	4	1	6	2	3	8	1

		h			i					k
into	b	1	2	1	**3**	5	4	6	3	8

		h				i				k
or	b	1	2	3	1	**3**	4	5	6	8

or…

- *x* is called the pivot value
  - *x* is not a program variable, but a standin for a number: the value *initially* in b[h]

Motivation: A Famous Sorting Algorithm

```
def qsort(b, h, k):
    """Make b[h..k] sorted.
    Pre: b: list of ints; k>=h-1"""
    # base case
    if k < h:  # empty is sorted
        return
    i = partition(b, h, k)
```

Clicker Q1: recursive case

(A) qsort(b,h,i-1)
    qsort (b, i+1, k)

(B) qsort(b,h,i-1)
    qsort(b,h,i+1,k)

(C) qsort (b,h,k)

(D) qsort(b,h,i+1)
    qsort(b,i+2,k)

(E) more than one works

Motivation: A Famous Sorting Algorithm

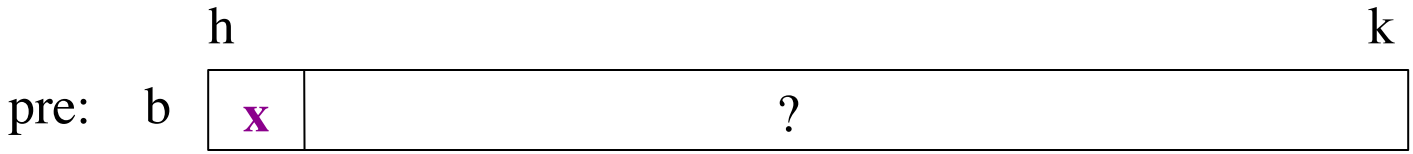
```
def qsort(b, h, k):
    """Make b[h..k] sorted.
    Pre: b: list of ints; k>=h-1"""
    # base case
    if k < h:  # empty is sorted
        return
    i = partition(b, h, k)
```

Clicker Q1: recursive case

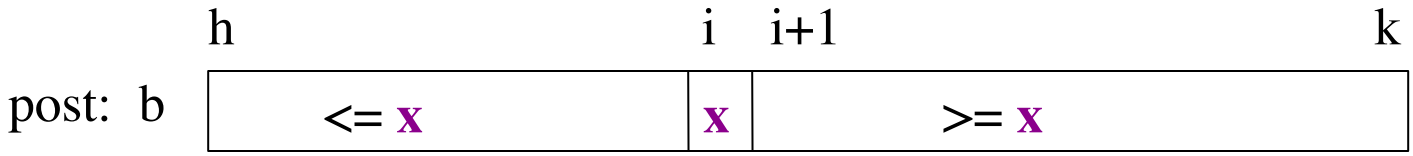
```
qsort(b,h,i-1)
qsort(b,i+1,k)
```

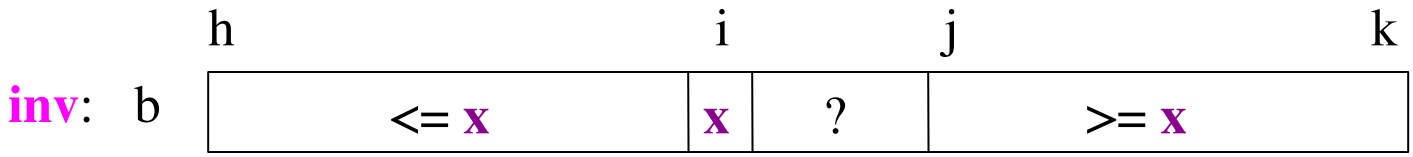
An Invariant to Guide Our Thinking

- Given a sequence b[h..k] with some value x in b[h]:



- Swap elements of b[h..k] and store in i to truthify post:





- Agrees with precondition when i = h, j = k+1
- Agrees with postcondition when j = i+1

Partition Algorithm Implementation

```
def partition(b, h, k):
    """Partition list b[h..k] around a pivot x = b[h]; return index of pivot point
   Pre: k>=h"""
   what goes here?
   # invariant: b[h..i-1] < x, b[i] = x, b[j..k] >= x, b[i+1..j-1] unknown
    while what goes here?
          if b[i+1] >= x:
           # Move to end of block.
           b[i+1], b[j-1] = b[j-1], b[i+1]
           j = j - 1
        else:   # b[i+1] < x
          CLICKER Q3
    # post: b[h..i-1] < x, b[i] is x, and b[i+1..k] >= x
    return i
```

(A) b[i-1], b[j-1] = b[j-1], b[i-1]

j = j -1

(B) b[i+1], b[j-1] = b[j-1], b[i-1]

(C) b[i+1], b[i] = b[i], b[i+1]

(D) b[i+1], b[i] = b[i], b[i+1]

i += 1

Partition Algorithm Implementation

```
def partition(b, h, k):
    """Partition list b[h..k] around a pivot x = b[h]"""
    i = h; j = k+1; x = b[h]
    # invariant: b[h..i-1] < x, b[i] = x, b[j..k] >= x
    while i < j-1:
        if b[i+1] >= x:
            # Move to end of block.
            b[i+1], b[j-1] = b[j-1], b[i+1]
            j = j - 1
        else:   # b[i+1] < x
            b[i], b[i+1] = b[i+1], b[i]
            i = i + 1
    # post: b[h..i-1] < x, b[i] is x, and b[i+1..k] >= x
    return i
```