Lecture 23

Sorting

Announcements for This Lecture

Assignments

- A5 is now graded
  - Mean: 95, Median: 97
  - Average Time: 4-5 hours
  - Longer than I expected…
- A6 is due **Friday**
  - Just activated in CMS
  - Should be on stenography
- A7 due Monday, Dec. 3
  - Week **after** classes

Next Two Weeks

- Reading
  - **Chapter 19**: Tkinter
  - Alternative to Kivy
  - But similar concepts
- Next Tue is important!
  - Will need it for A7
- No lab next week
  - This week is "last lab"
  - Lab final week is optional

Announcements for This Lecture

Assignments

- A6 is due **Tomorrow**
  - Hopefully you are close
  - Trying to add consultants
  - Keep reading Piazza
- A7 due Monday, Dec. 3
  - Week **after** classes
  - Online Saturday
  - Do not need lecture until the paddle task

Next Two Weeks

- Reading
  - **Chapter 19**: Tkinter
  - Alternative to Kivy
  - But similar concepts
- Next Tue is important!
  - Will need it for A7
  - Unifies attribute invariants and loop invariants
  - Last major topic of course

Binary Search

- Look for value v in **sorted** segment b[h..k]

	h			k
pre: b	?			

	h	i		k
post: b	< v	>= v		

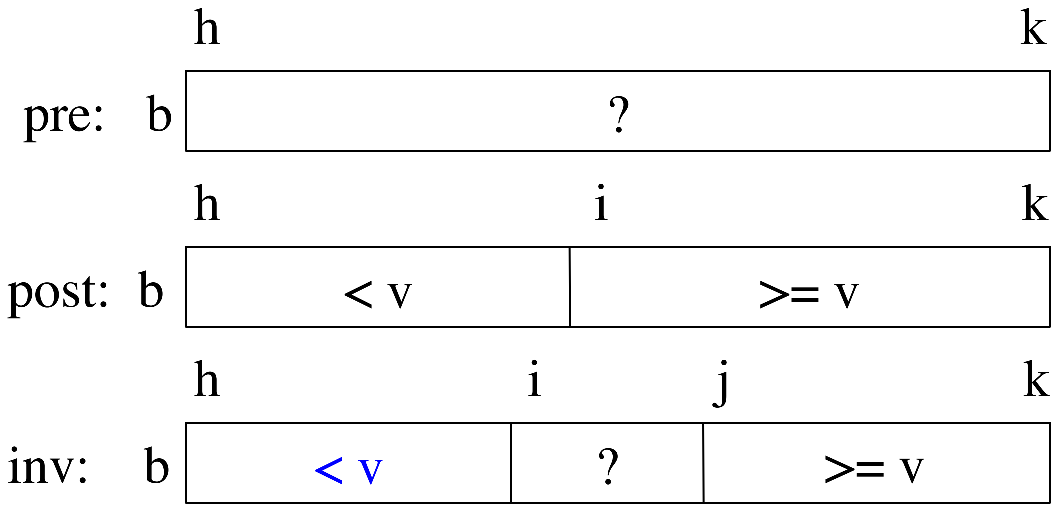
	h	i	j	k
inv: b	< v	?	>= v	

New statement of the invariant guarantees that we get leftmost position of v if found

	h									k
	0	1	2	3	4	5	6	7	8	9
Example b	3	3	3	3	3	4	4	6	7	7

- if v is 3, set i to 0
- if v is 4, set i to 5
- if v is 5, set i to 7
- if v is 8, set i to 10

Binary Search



New statement of the invariant guarantees that we get leftmost position of v if found

```
i = h; j = k+1;
while i != j:
```

Looking at b[i] gives linear search from left.

Looking at b[j-1] gives linear search from right.

Looking at middle: b[(i+j)/2] gives binary search.

Sorting: Arranging in Ascending Order

pre: b [0 … n] ?

post: b [0 … n] sorted

Insertion Sort:

inv: b [0 … i] sorted [i … n] ?

```
i = 0
while i < n:
    # Push b[i] down into its
    # sorted position in b[0..i]
    i = i+1
```

0						i
2	4	4	6	6	7	5

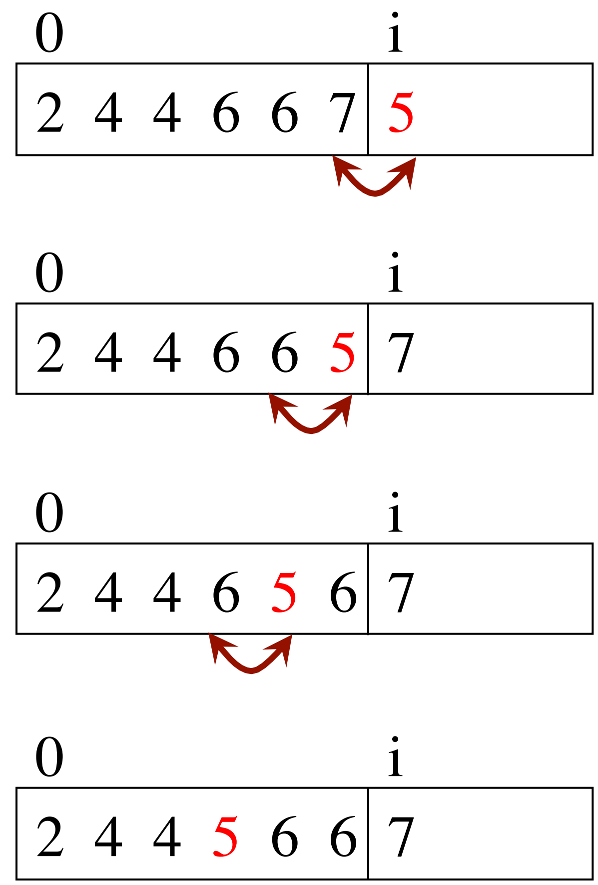
0						i
2	4	4	5	6	6	7

Insertion Sort: Moving into Position

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

swap shown in the lecture about lists



The Importance of Helper Functions

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

VS

Can you understand all this code below?

```
i = 0
while i < n:
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            temp = b[j]
            b[j] = b[j-1]
            b[j-1] = temp
        j = j -1
    i = i +1
```

Insertion Sort: Performance

```
def push_down(b, i):
    """Push value at position i into
    sorted position in b[0..i-1]"""
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

- b[0..i-1]: i elements
- Worst case:
  - i = 0: 0 swaps
  - i = 1: 1 swap
  - i = 2: 2 swaps
- Pushdown is in a loop
  - Called for i in 0..n
  - i swaps each time

Insertion sort is an n^2 algorithm

Total Swaps: $0 + 1 + 2 + 3 + \ldots (n-1) = (n-1)*n/2$

Algorithm "Complexity"

- **Given**: a list of length n and a problem to solve
- **Complexity**: *rough* number of steps to solve worst case
- Suppose we can compute 1000 operations a second:

Complexity	n=10	n=100	n=1000
n	0.01 s	0.1 s	1 s
n log n	0.016 s	0.32 s	4.79 s
n^2	0.1 s	10 s	16.7 m
n^3	1 s	16.7 m	11.6 d
2^n	1 s	4×10^{19} y	3×10^{290} y

Major Topic in 2110: Beyond scope of this course

Sorting: Changing the Invariant

pre: b

0	n
?	

post: b

0	n
sorted	

Selection Sort:

inv: b

0	i	n
sorted, ≤ b[i..]	≥ b[0..i-1]	

First segment always contains smaller values

```
i = 0
while i < n:
    # Find minimum in b[i..]
    # Move it to position i
    i = i+1
```

					i					n
2	4	4	6	6	8	9	9	7	8	9

					i					n
2	4	4	6	6	7	9	9	8	8	9

						i				n
2	4	4	6	6	7	9	9	8	8	9

Sorting: Changing the Invariant

	0	n
pre: b	?	

	0	n
post: b	sorted	

Selection Sort:

	0	i	n
inv: b	sorted, ≤ b[i..]	≥ b[0..i-1]	

First segment always contains smaller values

```
i = 0
while i < n:
    j = index of min of b[i..n-1]
    swap(b,i,j)
    i = i+1
```

					i					n
2	4	4	6	6	8	9	9	7	8	9

					i					n
2	4	4	6	6	7	9	9	8	8	9

Selection sort also is an n^2 algorithm

Partition Algorithm

- Given a list segment b[h..k] with some value x in b[h]:

pre: b

h	k
x	?

- Swap elements of b[h..k] and store in j to truthify post:

post: b

h	i	i+1 k
<= x	x	>= x

change: b

h								k
3	5	4	1	6	2	3	8	1

into b

h			i					k
1	2	1	**3**	5	4	6	3	8

or b

h				i				k
1	2	3	1	**3**	4	5	6	8

- x is called the pivot value
  - x is not a program variable
  - denotes value initially in b[h]

Sorting with Partitions

- Given a list segment b[h..k] with some value x in b[h]:

pre: b

h	k
x	?

- Swap elements of b[h..k] and store in j to truthify post:

post: b

h			i	i+1 k
<= **y**	**y**	>= **y**	**x**	>= **x**

Partition Recursively

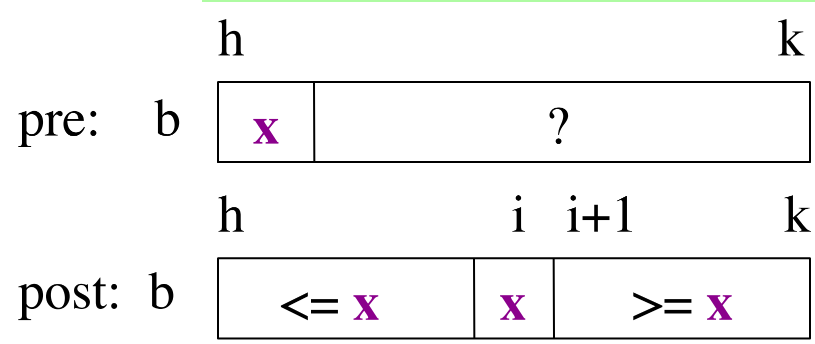
Recursive partitions = sorting

- Called **QuickSort** (why???)
- Popular, fast sorting technique

QuickSort

```
def quick_sort(b, h, k):
    """Sort the array fragment b[h..k]"""
    if  b[h..k] has fewer than 2 elements:
        return
    j = partition(b, h, k)
    # b[h..j–1] <= b[j] <= b[j+1..k]
    # Sort b[h..j–1]  and  b[j+1..k]
    quick_sort (b, h, j–1)
    quick_sort (b, j+1, k)
```

- **Worst Case:** array already sorted
  - Or almost sorted
  - n^2 in that case
- **Average Case:** array is scrambled
  - n log n in that case
  - Best sorting time!



Final Word About Algorithms

- **Algorithm:**
  - Step-by-step way to do something
  - Not tied to specific language
- **Implementation:**
  - An algorithm in a specific language
  - Many times, not the "hard part"
- Higher Level Computer Science courses:
  - We teach advanced algorithms (pictures)
  - Implementation you learn on your own

List Diagrams

Demo Code