

1. /** = n reduced to a single digit by repeatedly adding its digits together.

Precondition: $n > 0$. */

```
public static int reduce(int n) {
    if (n < 10)
        return n;
    return reduce(n/10 + n%10);
}
```

2. // double arrays a and b have the same length.

```
double[] c= new double[a.length];
```

```
int k= c.length;
```

```
// Inv: c[k..c.length-1] contains elementwise
```

```
// averages of a[k..] and b[k..].
```

```
while (k != 0) {
    c[k-1]= (a[k-1] + b[k-1]) / 2.0;
    k= k - 1;
}
```

```
// Post: c[0..] contains elementwise averages of
```

```
// a[0..] and b[0..].
```

3. /** = a two-dimensional array that contains $n+1$ rows, where each row k contains in its first $k+1$ elements row k of Pascal's triangle. */

```
public static int[][] pascal(int n){
```

```
    int[][]p= new int[n+1][n+1];
```

```
    // inv: p[0..r-1] contains the first r rows
    // of Pascal's triangle
```

```
    for (int r= 0; r <= n; r= r+1) {
        //Store row r of Pascal's triangle in row p[r].
        p[r][0]= 1;
        for (int k= 1; k < r; k= k+1) {
            p[r][k]= p[r-1][k-1] + p[r-1][k];
        }
        p[r][r]= 1;
    }
    return p;
}
```

- 4a. /** An instance is an integer in mod "modulus" arithmetic. */

```
public class Mod {
```

```
    private int m;        // The modulus.  $m > 1$ 
```

```
    private int k;        // The integer.  $0 \leq k < m$ 
```

```
    /** Constructor: integer k in mod m arithmetic.
```

```
    Precondition:  $m > 1$  and  $k \geq 0$ .*/
```

```
    public Mod(int k, int m) {
        this.m= m;
        this.k= k % m;
    }
}
```

```
/** If this object and r do not have the same
    modulus, return null; otherwise, return an object
    that contain the sum of the two mod-m integers
    represented by this object and r. */
```

```
public Mod add(Mod r) {
    if (m != r.m)
        return null;
    return new Mod(k + r.k, m);
}
```

```
/** = "ob is a non-null Mod object with the same
    modulus and value as this one". */
```

```
public boolean equals(Object ob) {
    if (!(ob instanceof Mod))
        return false;
    Mod mob= (Mod)ob;
    return k == mob.k && m == mob.m;
}
```

- 4b. Override a method $m(\dots)$ inherited from a superclass by redefining it in the subclass. Call the overridden method using **super.m(...)**.

4c. Instance variable or field: in a class. Class variable or static variable: in a class. Parameter: in the header of a method definition. Local variable: in a method body.

- 4d. Yes. Because of the principal that inherited fields should be initialized first. If such a call is missing, the call **super()**; is used.

5. /** = the integer k that satisfies

$d[p..k] \leq w < d[k+1..q-1]$.

Precondition: b is in dictionary order. */

```
public static int bSearch(String[] d,
    int p, int q, String w) {
    int k= p-1;
    int t= q;
    // inv:  $d[p..k] \leq w < d[t..q-1]$ 
    while (k+1 < t) {
        int e= (k + t) / 2;
        if (comesBefore(d[e], w) <= 0) k= e;
        else t= e;
    }
    return k;
}
```