NOTE: you do not need to "memorize" the algorithms in this lab for the prelim on 8 Oct. But you will need to know them for the final. The practice this lab gives you should help you understand the use of loop invariants drawn as array diagrams.

This lab gives you practice with developing loops from invariants and with the array algorithms that you have to know. Your lab instructor and any consultants that are present will help you.

In our sample answers, we don't draw all the pictures that are required. If an invariant is not drawn as a picture, you should draw it as a picture before continuing to work on the problem. Need help? Ask your TA or consultant.

When you are finished, show your TA what you have done. If you finish at least exercises $1-4$, your lab instructor will mark you down as having completed the assignment. If you don't finish that much, finish exercises 1-4 by the next week and show them to your TA at the beginning of the lab next week. In any case, try to do all the exercises because that will help you understand and gain skill in solving such problems.

Note: if you wish to test your programs in DrJava, you may find it useful to print out the contents of an array. Method java.util.Arrays.toString (Object[]) prints out the contents of its array parameter.

Note: you may write something like "swap $\mathrm{b}[\mathrm{i}]$ and $\mathrm{b}[\mathrm{j}]$ " instead of the three assignments that perform the swap.

1. Give a formula for the number of values in the segment $b[x . . y-1]$ :
b

| x | y |  |
| :--- | :--- | :--- |
|  |  |  |

Not sure what the formula is? Then ask, and memorize it. Also, learn to figure it out by looking at segments of size 1 and 2. The formula is a useful tool. Knowing it will help you develop loops that deal with ranges of integers.
2. Draw an array $b$ that satisfies these conditions: $b[0 . i]>=5, b[i+1 . . j]=5, b[j+1 .]<.=5$.
3. Below is a precondition and postcondition for the algorithm to find the minimum of an array $b[h . k]$. Below that are four different loop invariants for the problem. Write a loop (with initialization) for each one.

Precondition: $\mathrm{h}<=\mathrm{k}<$ b.length
Postcondition R: $\mathrm{b}[\mathrm{x}]$ is the minimum of $\mathrm{b}[\mathrm{h} . . \mathrm{k}]$
(a) invariant $\mathrm{P} 1: \mathrm{b}[\mathrm{x}]$ is the minimum of $\mathrm{b}[\mathrm{h} . \mathrm{t}]$
(b) invariant $\mathrm{P} 2: \mathrm{b}[\mathrm{x}]$ is the minimum of $\mathrm{b}[\mathrm{h} . . \mathrm{s}-1]$
(c) invariant $\mathrm{P} 3: \mathrm{b}[\mathrm{x}]$ is the minimum of $\mathrm{b}[\mathrm{r} . \mathrm{k}]$
(d) invariant P4: $\mathrm{b}[\mathrm{x}]$ is the minimum of $\mathrm{b}[\mathrm{w}+1 . . \mathrm{k}]$

Below are the postcondition and invariants drawn as pictures. Use either the text versions or the pictures.

## $\mathrm{R}: \quad \mathrm{h}$

k
b


P1: h
t $k$
b

4. The purpose of the algorithms in this question is to swap the values of array $b$ and to store a value in $k$ so that the postcondition given below is true. Array $b$ is not necessarily sorted initially. Besides the postcondition, we give three different invariants; write a loop (with initialization) for each one. Before you begin, write the precondition, postcondition, and invariant as pictures.

Precondition $\mathrm{Q}: \mathrm{b}[0 .]=$. ? - i.e. nothing is known about the values in b .
Postcondition $\mathrm{R}: \mathrm{b}[0 . . \mathrm{k}] \leq 6$ and $\mathrm{b}[\mathrm{k}+1 .]>$.
(a) invariant P : $: \mathrm{b}[0 . . \mathrm{h}] \leq 6$ and $\mathrm{b}[\mathrm{k}+1 .]>$.
(b) invariant $\mathrm{P} 2: \mathrm{b}[0 . . \mathrm{k}] \leq 6$ and $\mathrm{b}[\mathrm{t} . \mathrm{}]>$.
(c) invariant $\mathrm{P} 3: \mathrm{b}[0 . . \mathrm{s}-1] \leq 6$ and $\mathrm{b}[\mathrm{k}+1 .]>$.
5. Below is the precondition and postcondition for the partition algorithm. Below that are three different invariants. Develop the partition algorithm (which uses only swap operations) using each of the three invariants. Before you start, write all assertions as pictures. This algorithm manipulates an array segment $\mathrm{b}[\mathrm{h} . \mathrm{k}]$.

Precondition: $\mathrm{b}[\mathrm{h}]=\mathrm{x}$ for some x AND (this is just so we can talk about what is in $\mathrm{b}[\mathrm{h}]$ initially; $\mathrm{h} \leq \mathrm{k}<\mathrm{b}$.length $\quad \mathrm{x}$ is not a program variable.)
Postcondition: $\mathrm{b}[\mathrm{h} . \mathrm{j}-1] \leq \mathrm{x}=\mathrm{b}[\mathrm{j}]<=\mathrm{b}[\mathrm{j}+1 . . \mathrm{k}]$
(a) invariant P1: $\mathrm{b}[\mathrm{h} . \mathrm{j}-1] \leq \mathrm{x}=\mathrm{b}[\mathrm{j}]<=\mathrm{b}[\mathrm{t} . . \mathrm{k}]$
(b) invariant P2: $\mathrm{b}[\mathrm{h} . \mathrm{j}-1] \leq \mathrm{x}=\mathrm{b}[\mathrm{j}]<=\mathrm{b}[\mathrm{q}+1 . . \mathrm{k}]$
(c) invariant P3: $\mathrm{b}[\mathrm{h} . \mathrm{j}-1] \leq \mathrm{x}=\mathrm{b}[\mathrm{j}]<=\mathrm{b}[\mathrm{j}+1 . . \mathrm{n}-1]$
6. Write selection sort, to sort array $b$, where b.length $>=1$, in several ways, using the invariants provided below. Before you do each one, write the invariant as a picture. Then write the statement(s) you use to maintain the invariant in the repetend in English - state WHAT it is to do, not HOW it is to do it. We do NOT want to see a nested loop (a loop within the repetend of your first loop).
postcondition: $\mathrm{b}[0 . . \mathrm{b}$. length -1$]$ is sorted (in ascending order)
(a) invariant $\mathrm{P} 1: \mathrm{b}[0 . . \mathrm{k}-1]$ is sorted and $\mathrm{b}[0 . . \mathrm{k}-1] \leq \mathrm{b}[\mathrm{k} .$.
(b) invariant $\mathrm{P} 2: \mathrm{b}[0 . . \mathrm{h}]$ is sorted and $\mathrm{b}[0 . . \mathrm{h}] \leq \mathrm{b}[\mathrm{h}+1 .$.
(c) invariant P3: $\mathrm{b}[\mathrm{s}+1$..b.length -1$]$ is sorted and $\mathrm{b}[0 . . \mathrm{s}] \leq \mathrm{b}[\mathrm{s}+1 .$.

