

| How multi-dimensional arrays are stored: arrays of arrays int b[][]$=$ new int[][] $\{\{9,6,4\},\{5,7,7\}\}$; $\square$ |  |  |
| :---: | :---: | :---: |
|  |  |  |
|  | $9$ | 5 |
|  | $\square$ | 5 |
|  | 6 | $7$ |
| 1 r1 -... 2 | 4 | 7 |
| $b$ holds the name of a one-dimensional array object of b.length elements; its elements are 1D arrays. |  |  |
| b [i] holds the name of a 1D array of ints of length b[i].length. |  |  |
| java.util.Arrays.deepToString recursively creates an appropriate String. |  |  |
| 3 |  |  |

Review of two-dimensional arrays


Some mysteries: an odd asymmetry, and strange toString output (see demo).
Number of rows of d: d.length

Number of columns in row $r$ of $d: d[r]$.length

Ragged arrays: rows have different lengths
int[][] b; Declare variable $b$ of type int[][]
$\mathrm{b}=$ new int[2][] Create a 1-D array of length 2 and store its name in $b$. Its elements have type int[] (and start as null).
$\mathrm{b}[0]=$ new int[] $\{17,13,19\}$; Create int array, store its name in $\mathrm{b}[0]$.
$\mathrm{b}[1]=$ new int[] $\{28,95\}$; Create int array, store its name in $\mathrm{b}[1]$.


| Pascal's Triangle | 1 |  |  |  |  |  |  |  | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 1 |  | 1 |  |  |  | 1 |
|  |  | 1 |  | 2 |  | 1 |  |  | 2 |
|  | 1 |  | 3 |  | 3 |  | 1 |  | 3 |
| 1 |  | 4 |  | 6 |  | 4 |  | 1 | 4 |
| 1 | 5 |  | 10 |  | 10 |  | 5 |  | 5 |

Entry $p[i][j]$, entry $j$ of row $i$, is the number of ways $j$ elements can be chosen from a set of size i
$p[i][j]=" i$ choose $j "=\binom{i}{j}$
recursive formula (computed via dynamic programming): for $0<\mathrm{i}<\mathrm{j}, \quad \mathrm{p}[\mathrm{i}][\mathrm{j}]=\mathrm{p}[\mathrm{i}-1][\mathrm{j}-1]+\mathrm{p}[\mathrm{i}-1][\mathrm{j}]$

Application: representation of (irregular) sparse data
Large collections of association data abound, but often, many possible associations have the default value.

Netflix data: (movie, rater, score): $480 \mathrm{~K} \times 18 \mathrm{~K}=8.6 \mathrm{~B}$ possible scores to track, but there are only (!) 100 M actual scores.

GroupLens data (freely distributed by U. Minn): the small set has $943 \times 1682=1.5 \mathrm{M}$ possibilities, but only 100 K actual scores.

Main idea:
DON'T store an int array of length 1682 for each movie; store a rater-sorted array of score objects corresponding to just the raters who scored that movie (avg. length: 59!).

Another very useful technique (among many more substantive ones; take more CS courses!): map the movie/rater names to ints, $\mathrm{b} / \mathrm{c}$ they can be meaningful array indices.

Binomial theorem: Row $r$ gives the coefficients of $(x+y)^{r}$
$(x+y)^{2}=1 x^{2}+2 x y+1 y^{2}$
$(x+y)^{3}=1 x^{3}+3 x^{2} y+3 x y^{2}+1 y^{3}$
$(x+y)^{r}=\sum_{0 \leq k \leq r}(k$ choose $r) x^{k} y^{r-k}$


