

This 90-minute exam has 6 questions (numbered 0..5) worth a total of 100 points. Spend a few minutes looking at all the questions before beginning. Use the back of the pages if you need more space.

**Question 0 (2 points).** Fill in the information, legibly, at the top of *each* page. (Hint: do it now.)

**Question 1 (13 points). Arrays and methods.**

Below to the left is a square array. Write a method that turns such an array into its transpose, which is shown to the right. The transpose of a square array arises from switching rows and columns. If the array is the pixels in an image, then transposing the image turns it on its side.

You do not have to write loop invariants, although the practice of writing an invariant for a loop and stating in a comment at the beginning of the repetend what the repetend does increases programming effectiveness. And if you want to swap  $b[h]$  and  $b[k]$ , just say that instead of writing it in Java.

A B C D E		A F K P U
F G H I J		B G L Q V
K L M N O	transpose:	C H M R W
P Q R S T		D I N S X
U V W X Y		E J O T Y

`/** = Change m into its transpose.`

`Precondition: m is not null and is square (the number of rows = the number of columns) */`  
`public static void transpose(int[][] m) {`

`}`

**Question 2 (27 points). Miscellaneous**

(a) Write a single statement that declares and initializes a two-dimensional `int` array `b` to look like the table that appears to the right.

1	3	6	10
2	5	9	13
4	8	12	15
7	11	14	16

(b) To the right is a class definition. Below are 5 statements. Draw the variables that are declared in the statements. Then, execute the statements one by one, in order, drawing any objects that are created during execution and storing values in appropriate variables when an assignment is executed. When storing a value in a variable, cross out, do not erase, the old value.

```
public class C {
    public int x;
    public int y;
    public C(int a, int b) {
        x= a; y= b;
    }
}
```

```
C h= new C(3, 1);
C k= new C(5, 6);
C j= h;
j.x=j.x + 1;
j.y= h.x + h.y;
```

(c) Any positive integer can be reduced to a single digit by repeatedly adding its digits together until the result is < 10. For example, change 257 to 25 + 7 and then to 7 + 7 and then to 14 and finally to 5. Or do it this way: change 257 to 2 + 57 and then to 2 + 12 and then to 2 + 3 and then to 5. Or, try 257 -> 25 + 7 -> 32 -> 5. There are other ways to do it. Write the following recursive function. Do not use a loop, and do not introduce Strings.

```
/** = n reduced to a single digit (by repeatedly adding its digits. Precondition: n > 0 */
public static int addUp(int n) {
```

```
}
```

**Question 3 (18 points)** Below is a class `Rational`. Complete the bodies of the constructor, and functions `toString` and `equals`. Read the whole class, all of its methods, before doing anything.

```
/** An instance is a rational number */
public class Rational {
    /** Class invariant: the rational number is num / den
        Restrictions on fields: 1. den > 0
                               2. if num = 0, then den = 1
                               3. num / den is in lowest possible terms.
        E.g. instead of 20/8 or -5/10, these rational numbers are stored as 5/2 and -1/2. */

    private int num;
    private int den;

    /** Constructor: an instance with rational number n / d.
        Precondition: d != 0 */
    public Rational(int n, int d) {

    }

    /** = the numerator */
    public int getNumerator() { return num; }

    /** = the denominator */
    public int getDenominator() { return den; }

    /** = a representation of this rational number, in the form num/den. */
    public String toString() {

    }

    /** = "r is of class Rational and has the same value as this Rational number". */
    public boolean equals(Object r) {

    }

    /** Change num and den so that this rational number is in the lowest possible terms,
        e.g. 8/24 becomes 1/3. */
    private void reduce(){
        You do not have to write this procedure.
    }
}
```

**Question 4.** (25 points). Vector  $v$  may contain instances of many different classes, e.g. Rational (see question 3), Integer, Double, and others. There may be duplicates in  $v$ —for example, two instances of Rational in class  $v$  may contain the same rational number. We want to write procedures that will remove duplicates of class Rational from  $v$ . By a duplicate, we do *not mean* the same object; rather, we mean an object that contains the same rational number.

Below, two procedures are specified. Write their bodies. Do not use recursion. You *must* use the invariants P1, and P2, which are given both in English and as pictures, to write loops (with initialization). Remember our principle to rely on previously written functions, where possible.

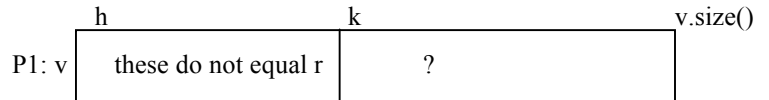
Use `v.remove(k)` to remove element `k` from `v`.

`/** Remove all Rational elements from v[h..] that have the same value as r.`

`Precondition: 0 <= h < v.size(). */`

`public static void removeRationalEquals(Vector v, int h, Rational r) {`

`// inv P1. No Rational element in  
v[h..k-1] has the same value as r */`



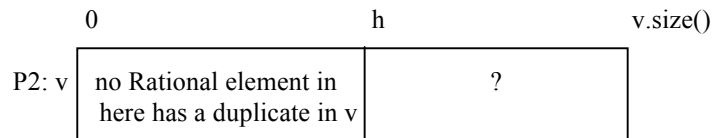
`// Postcondition: no Rational element in v[h..] has the same value as r`

`}`

`/** Remove all duplicate Rational numbers from v (so that each appears in it only once). */`

`public static void removeRationalDups(Vector v) {`

`// inv P2: No Rational instance in v[0..h-1]  
has a duplicate in v.*/`



`Postcondition: No Rational instance in v has a duplicate in v.`

`}`

**Question 5 (15 points). Known algorithms.**

(a) (10 pts) Write a specification and header for a binary search function that searches sorted array segment  $b[p..q-1]$ . Your specification may be in terms of pictures, English, mathematical notation, or a mixture of all of them. But it must be a specification of binary search *as we have presented the algorithm in class*. Note carefully which array segment is to be searched.

0	_____	out of 02
1	_____	out of 13
2	_____	out of 27
3	_____	out of 18
4	_____	out of 25
5	_____	out of 15
Total		_____ out of 100

(b) (10 pts) Develop the body of the function specified above. When developing the body, write the invariant for the loop first. Then develop the loop and initialization using the four loopy questions. An answer that does not have a suitable loop invariant will receive no credit. Note carefully that the sorted array segment to be searched is  $b[p..q-1]$ .