

- Previous Lecture:
  - Branching
  - Logical operators
- Today's Lecture:
  - Logical operators and values
  - More branching—*nesting*
  - The idea of repetition
- Announcement:
  - Project 1 (P1) due today at 6pm
  - Please register your clicker by Monday 2/4

"Truth table"

X, Y represent boolean expressions.  
E.g.,  $d > 3.14$

X	Y	X && Y "and"	X    Y "or"	~y "not"
T	T	T	T	F
T	F	F	T	T
F	T	F	T	F
F	F	F	F	T

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Logical operators

**&&** logical and: Are both conditions true?  
E.g., we ask "is  $L \leq x_c$  and  $x_c \leq R$ ?"  
In our code: `L <= xc && xc <= R`

**||** logical or: Is at least one condition true?  
E.g., we can ask if  $x_c$  is outside of  $[L, R]$ ,  
i.e., "is  $x_c \leq L$  or  $R \leq x_c$ ?"  
In code: `xc <= L || R <= xc`

**~** logical not: Negation  
E.g., we can ask if  $x_c$  is **not outside**  $[L, R]$ .  
In code: `~(xc <= L || R <= xc)`

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- Logical operators will short-circuit
- "It's a good thing."
  - Consider the compound condition  
`L <= xc && xc <= R`
  - If **L** is greater than **xc**, then the 1<sup>st</sup> condition  $\Rightarrow$  *false*. Then the entire compound condition must give *false* as well, no matter what **xc** and **R** are.
  - A **&&** condition short-circuits to false if the left operand evaluates to *false*
  - A **||** condition short-circuits to \_\_\_\_\_ if \_\_\_\_\_.
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Always use logical operators to connect simple boolean expressions

Why is it wrong to use the expression  
`L <= xc <= R`  
for checking if  $x_c$  is in  $[L, R]$ ?

Example: Suppose **L** is 5, **R** is 8, and **xc** is 10. We know that 10 is not in  $[5, 8]$ , but the expression  
`L <= xc <= R` gives...

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"Truth table"

Matlab uses 0 to represent false,  
1 to represent true

X	Y	X && Y "and"	X    Y "or"	~X "not"
1	1	1	1	0
1	0	0	1	
0	1	0	1	1
0	0	0	0	

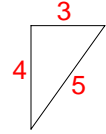
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Variables  $a$ ,  $b$ , and  $c$  have whole number values. True or false: This fragment prints "Yes" if there is a right triangle with side lengths  $a$ ,  $b$ , and  $c$  and prints "No" otherwise.

```
if a^2 + b^2 == c^2
    disp('Yes')
else
    disp('No')
end
```

A: true  
B: false

```
a = 5;
b = 3;
c = 4;
if (a^2+b^2==c^2)
    disp('Yes')
else
    disp('No')
end
```

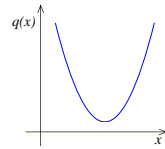


This fragment prints "No" even though we have a right triangle!

```
a = 5;
b = 3;
c = 4;
if ((a^2+b^2==c^2) || (a^2+c^2==b^2) || (b^2+c^2==a^2))
    disp('Yes')
else
    disp('No')
end
```

Consider the quadratic function

$$q(x) = x^2 + bx + c$$



on the interval  $[L, R]$ :

- Is the function strictly increasing in  $[L, R]$ ?
- Which is smaller,  $q(L)$  or  $q(R)$ ?
- What is the minimum value of  $q(x)$  in  $[L, R]$ ?

### Conclusion

If  $x_c$  is between  $L$  and  $R$

Then min is at  $x_c$

Otherwise

Min value is at one of the endpoints

### Start with pseudocode

If  $x_c$  is between  $L$  and  $R$

Min is at  $x_c$

Otherwise

Min is at one of the endpoints

We have decomposed the problem into three pieces! Can choose to work with any piece next: the if-else construct/condition, min at  $x_c$ , or min at an endpoint

Set up structure first: if-else, condition

```

if L<=xc && xc<=R
    Then min is at xc
else
    Min is at one of the endpoints
end
    
```

Now refine our solution-in-progress. I'll choose to work on the if-branch next

Refinement: filled in detail for task "min at xc"

```

if L<=xc && xc<=R
    % min is at xc
    qMin= xc^2 + b*xc + c;
else
    Min is at one of the endpoints
end
    
```

Continue with refining the solution... else-branch next

Refinement: detail for task "min at an endpoint"

```

if L<=xc && xc<=R
    % min is at xc
    qMin= xc^2 + b*xc + c;
else
    % min is at one of the endpoints
    if %xc left of bracket
        %min is at L
    else %xc right of bracket
        %min is at R
    end
end
    
```

Continue with the refinement, i.e., replace comments with code

Refinement: detail for task "min at an endpoint"

```

if L<=xc && xc<=R
    % min is at xc
    qMin= xc^2 + b*xc + c;
else
    % min is at one of the endpoints
    if xc < L
        qMin= L^2 + b*L + c;
    else
        qMin= R^2 + b*R + c;
    end
end
    
```

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Final solution (given b,c,L,R,xc)

```

if L<=xc && xc<=R
    % min is at xc
    qMin= xc^2 + b*xc + c;
else
    % min is at one of the endpoints
    if xc < L
        qMin= L^2 + b*L + c;
    else
        qMin= R^2 + b*R + c;
    end
end
    
```

An if-statement can appear within a branch—just like any other kind of statement!

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Notice that there are 3 alternatives→can use elseif!

```

if L<=xc && xc<=R
    % min is at xc
    qMin= xc^2 + b*xc + c;
elseif xc < L
    qMin= L^2 + b*L + c;
elseif xc > R
    qMin= R^2 + b*R + c;
end
    
```

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```

if L<=xc && xc<=R
    % min is at xc
    qMin= xc^2 + b*xc + c;
elseif xc < L
    qMin= L^2 + b*L + c;
else
    qMin= R^2 + b*R + c;
end
    
```

True or false: We don't need the `elseif` keyword at all (in the Matlab language).

A: true  
B: false

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### Top-Down Design

```

graph TD
    A[State problem] --> B[Define inputs & outputs]
    B --> C[Design algorithm]
    C --> D[Convert algorithm to program]
    D --> E[ ]
    C -- Decomposition --> C
    C -- Stepwise refinement --> C
    D -.-> C
    
```

An algorithm is an **idea**. To use an algorithm you must choose a programming language and **implement** the algorithm.

If  $xc$  is between  $L$  and  $R$   
Then min value is at  $xc$

Otherwise  
Min value is at one of the endpoints

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```

if L<=xc && xc<=R
    % min is at xc
else
    % min is at one of the endpoints
end
    
```

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```

if L<=xc && xc<=R
    % min is at xc
else
    % min is at one of the endpoints
end
    
```

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```

if L<=xc && xc<=R
    % min is at xc
    qMin= xc^2 + b*xc + c;
else
    % min is at one of the endpoints
end
    
```

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```

if L<=xc && xc<=R
    % min is at xc
    qMin= xc^2 + b*xc + c;
else
    % min is at one of the endpoints
end
    
```

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```

if L<=xc && xc<=R
    % min is at xc
    qMin= xc^2 + b*xc + c;
else
    % min is at one of the endpoints
    if xc < L
    else
    end
end
    
```

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```

if L<=xc && xc<=R
    % min is at xc
    qMin= xc^2 + b*xc + c;
else
    % min is at one of the endpoints
    if xc < L
        qMin= L^2 + b*L + c;
    else
        qMin= R^2 + b*R + c;
    end
end
    
```

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Question

A stick of unit length is split into two pieces. The breakpoint is randomly selected. On average, how long is the shorter piece?

Physical experiment?  
 Thought experiment? → analysis  
 Computational experiment! → simulation

*Need to repeat many trials!*

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