- Previous Lecture:
  - Insertion sort vs. merge sort
  - Timing with `tic toc`
  - Time efficiency vs. memory efficiency
- Today's Lecture:
  - Models and data
    - Congressional apportionment
    - Sensitivity analysis
- Announcements
  - Section in computer lab
  - Project 6 due 5/1, 6pm.
  - Survey on "clicker" use—see announcement on the web
  - CS100M final will be 5/8 (Thurs) 9am. Tell us now if you have a final exam conflict. Email Kelly Patwell with your complete exam schedule (course #s and times)

Proportional representation in the spirit of "one person, one vote"

Article I Section 2 of the US Constitution:

"Representatives... shall be apportioned among the several States, which may be included within this Union, according to their respective numbers..."

How do you quantify fairness?
There are different models of fairness.
(Were some models advanced for political reasons?)

The ratio of population to delegation size as a measurement of fairness

Distribute 435 Congressional seats among the 50 states so that the ratio of population to delegation size is roughly the same from state to state.

Sounds specific, but even with this "definition" of fairness there're different models that can be used as demonstrated throughout history… and in this lecture.

Related questions

How "close" is a state to losing a Congressional district because of population changes?

If Puerto Rico and/or Washington DC become states and the number of Congressional seats remain the same, which states would lose a seat?

- Reasoning about change is very important!
- How does the "answer" change if the data change or if the assumptions that underlie the computation change?
  → **Sensitivity analysis**

What are the "errors" in calculating the surface area of the Earth?

```
myPi= 3.14;
r= 3961.11345;
earthArea= 4*myPi*r*r;
```

A. Math error in myPi

B. Measurement error in r

C. Rounding error in arithmetic

D. *All of the above*

E. The spherical model of the Earth!

2000 US center of population

So who's gonna get this?

Sensitivity analysis

How far would the "center" of US population move if one more person moves to NY14850?

Order of…

- A. kilometers
- B. meters
- C. millimeters
- D. Micrometers→ no change

The apportionment problem

Distribute 435 Congressional seats among the 50 states so that the ratio of population to delegation size is roughly the same from state to state.

Subtext:
These examples provide distinct opportunities to review 100M programming techniques.

Ideal: Equal Representation

Number of states:	**n**
State populations:	**p(1),…,p(n)**
Total Population:	**P**
State delegation size:	**d(1),…,d(n)**
Number of seats:	**D**

$$\frac{P}{D} = \frac{p(1)}{d(1)} = \ldots = \frac{p(n)}{d(n)}$$

i.e.,

$$\boxed{d(i) = \frac{p(i)}{P} D}$$

But delegation size must be a whole number!!!

And so for NY in 2000..

$$\boxed{d(NewYork) = \frac{19004973}{281424177} 435 = 29.376}$$

Realistic situation

$$\frac{P}{D} \approx \frac{p(1)}{d(1)} \approx \ldots \approx \frac{p(n)}{d(n)}$$

Number of states:	**n**
State populations:	**p(1),…,p(n)**
Total Population:	**P**
State delegation size:	**d(1),…,d(n)**
Number of seats:	**D**

Definition

An Apportionment Method determines delegation sizes d(1),…,d(n) that are whole numbers so that representation is approximately equal:

$$\frac{p(1)}{d(1)} \approx \ldots \approx \frac{p(n)}{d(n)}$$

Jefferson Method 1790-1830

Decide on a "common ratio," the ideal number of constituents per district.

In 1790: `r = 33000`

Delegation size for the i-th state is

```
d(i) = floor( p(i)/r )
```


State	Pop	Reps	Pop/Reps
Connecticut	236841	7	33834
Delaware	55540	1	55540
Georgia	70835	2	35417
Kentucky	68705	2	34352
Maryland	278514	8	34814
Massachusetts	475327	14	33951
New Hampshire	141822	4	35455
New Jersey	179570	5	35914
New York	331589	10	33158
North Carolina	353523	10	35352
Pennsylvania	432879	13	33298
Rhode Island	68446	2	34223
South Carolina	206236	6	34372
Vermont	85533	2	42766
Virginia	630560	19	33187

Jefferson Method 1790-1830

Population and the chosen common ratio determine the size of Congress:

Year	P	r	D
1790	3615920	33000	105
1800	4889823	33000	141
1810	6584255	35000	181
1820	8969878	40000	213
1830	11931000	47700	240

Webster Method 1840

```
d(i) = round( p(i) / 70680 )
```

instead of **floor**

Common Ratio

Size of Congress also determined by common ratio

Hamilton Method (1850-1900)

This method fixes the size of Congress.

Allocations are based on the "ideal ratio":

Total Population / Total Number of Seats

The 1850 Case (31 States)

P = 21840083
D = 234
Ideal ratio, r = P/D ≈ 93334

```
% Round 1 allocation…
for i=1:31
    d(i)= floor( p(i)/r )
end
```

At this point, all but 14 of the 234 seats have been given out.

p(i)/r where r is the ideal ratio

AL	6.798	KY	9.622	NC	8.074
AR	2.047	LA	4.498	OH	21.218
CA	1.768	ME	6.248	PA	24.769
CT	3.973	MD	5.859	RI	1.581
DE	0.971	MA	10.655	SC	5.513
FL	0.768	MI	4.261	TN	9.717
GA	8.073	MS	5.171	TX	2.028
IL	9.123	MO	6.933	VT	3.366
IN	10.590	NH	3.407	VI	13.207
IA	2.059	NJ	5.244	WI	3.272
		NY	33.186		

floor(p(i)/r)

AL	6.798	KY	9.622	NC	8.074
AR	2.047	LA	4.498	OH	21.218
CA	1.768	ME	6.248	PA	24.769
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These 14 states most deserve an extra seat

AL	6.798	KY	9.622	NC	8.074
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Alabama paradox:
AL would lose 1 seat if Congress increases by 1 seat (1880 census)

Method of Equal Proportions

This method has been in use since 1940.

For the 2000 apportionment:

```
n = 50
D = 435
```

Determine the delegation sizes `d(1:50)`

Given the state populations `p(1:50)`


```
% Every state gets a district
d= ones(50,1);
% "Deal out" remaining districts
% by ranking the states and each time giving a
% district to the "most deserving state"
```



```
% Every state gets a district
d= ones(50,1);
% "Deal out" remaining districts
for k= 51:435
    % Let i be the index of the state that
    % most deserves an additional district
    d(i)= d(i) + 1;
end
```

How to quantify "most deserving"?

The Method of Small Divisors

At this point in the "card game" deal a district to the state having the largest quotient

```
p(i)/d(i)
```

Tends to favor small states.

How to quantify "most deserving"?

The Method of Large Divisors

At this point in the "card game" deal a district to the state having the largest quotient

```
p(i)/(d(i) + 1)
```

Tends to favor large states

How to quantify "most deserving"?

The Method of Major Fractions

At this point in the "card game" deal a district to the state having the largest value of

```
( p(i)/d(i) + p(i)/(d(i)+1) )/2
```

Compromise via the arithmetic mean

How to quantify "most deserving"?

The Method of Equal Proportions

At this point in the "card game" deal a district to the state having the largest value of

```
sqrt( p(i)/d(i) * p(i)/(d(i)+1) )
```

Compromise via the geometric mean

Allocate using the method of equal proportions

```
% Every state gets a district
d= ones(50,1);
% "Deal out" remaining districts
for k= 51:435
   [z,i]= max((p./d).*(p./(d+1)))
   d(i)= d(i) + 1;
end
```

See posted version for detail

A Sensitivity Analysis

- The 435th district was awarded to North Carolina.
- Was that a "close call"? Was there another state that "almost" won this last district? Quantify the close call.
  - Look at the "most deserving" ranks for the last district handed out. Which state was second? (Utah) Was this 2nd highest rank "close" to the max?
  - How many people will need to move from NC to UT in order for the last district to go to UT (instead of NC)?

Move from NC to UT

NC: 645931
UT: 645684

Equal Proportion ranking when dealing out the last district

North Carolina just beat out Utah for the last congressional seat based on 2000 census.

Can show that if 670 people move from NC to UT, then NC loses a seat and UT gains one.

Other questions

If Puerto Rico and/or Washington DC become states and the total number of representatives remains at 435, then what states lose a congressional seat?

If the population of New York remains fixed and all other states grow by 5% during the2000-10 decade, then how many seats will NY lose?

A useful structure array

```
C = CensusData
```

assigns to the structure array C the apportionments and census results for the census years 1890 through 2000.

C(k) houses information pertaining to the k-th census/apportionment.

C has these fields…

```
year    The year of the census. (1790,
        1800,...,2000).

states  k-by-16 char array that names
        existing states during the census.

pop     k-by-1 real array that
        specifies the state  populations.

reps    k-by-1 real array that specifies
        the state apportionments.
```


Example

```
C = CensusData;
Pop  = C(10).pop;
Reps = C(10).reps;
P = 0; D = 0;
for i=1:length(pop)
   P = P + Pop(i);
   D = D + Reps(i);
end
r = P/D;  % r is the ideal ratio based
          % on the 10th census
```


A somewhat related problem

Gerrymandering: the art of drawing district boundaries so as to favor incumbents