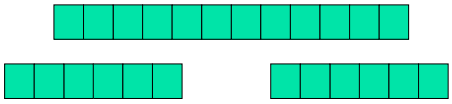
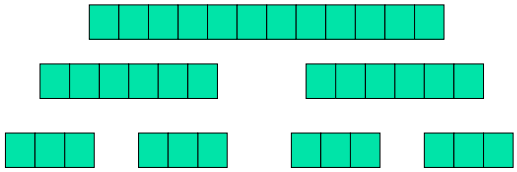
- Previous Lecture:
  - "Divide and conquer" strategies—recursion
    - Merge sort
    - Sierpinski Triangle, revisited
- Today's Lecture:
  - Insertion sort vs. merge sort
  - Timing with `tic toc`
  - Time efficiency vs. memory efficiency
- Announcements
  - Project 6 has been posted. Due 5/1, 6pm.
  - CS100M final will be 5/8 (Thurs) 9am. Tell us now if you have a final exam conflict. Email Kelly Patwell with your complete exam schedule (course #s and times)

Merge Sort

Merge Sort



Merge Sort



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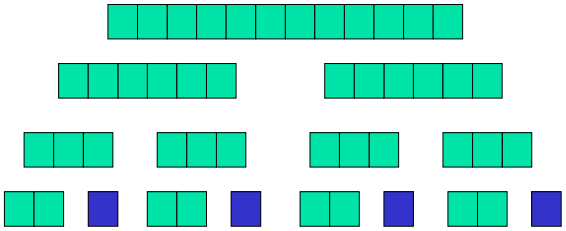
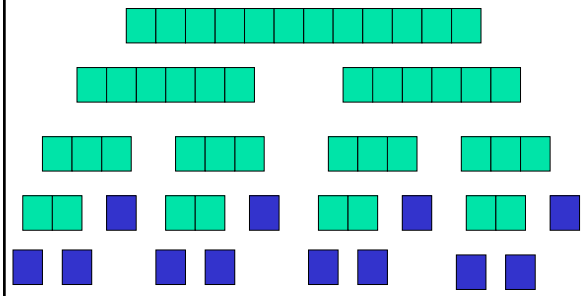
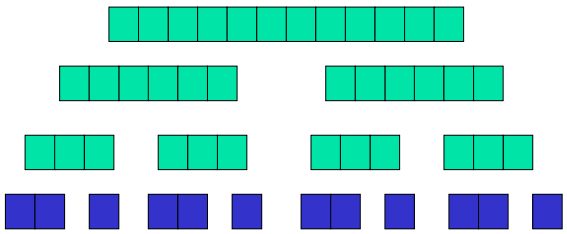


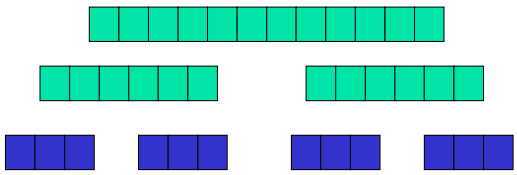
Merge Sort



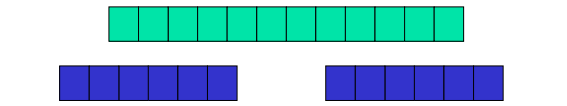
Merge Sort



Merge Sort



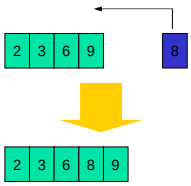
Merge Sort

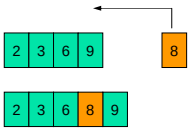


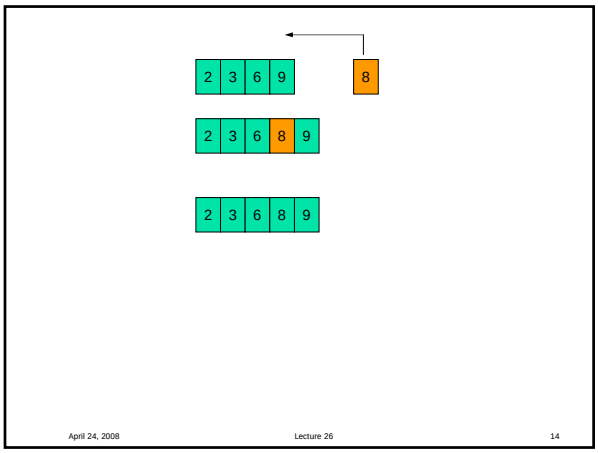
Merge Sort

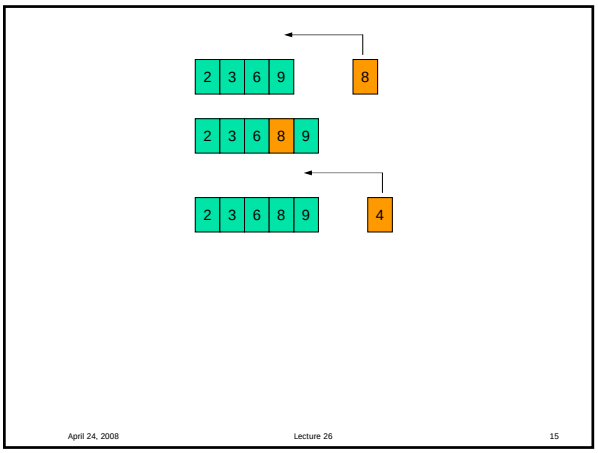
Insertion Sort

- Given a sorted array x, insert a number y such that the result is sorted






2 3 6 9 8
2 3 6 8 9
2 3 6 8 9
April 24, 2008 Lecture 26 14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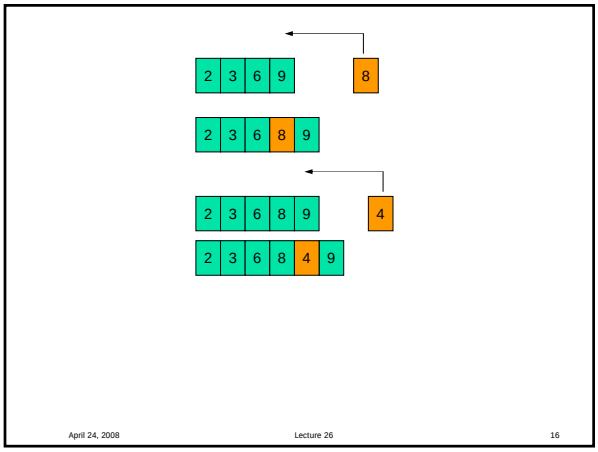
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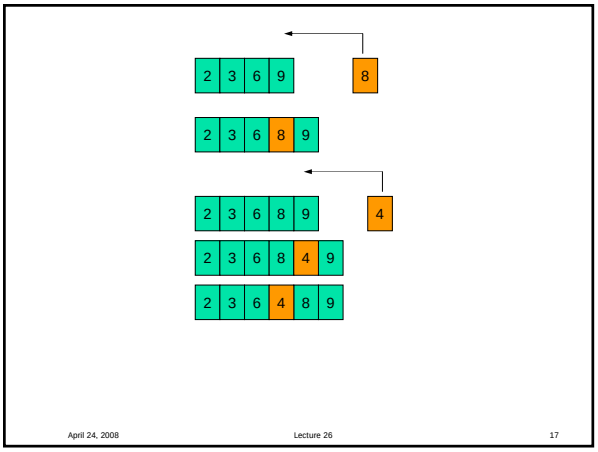
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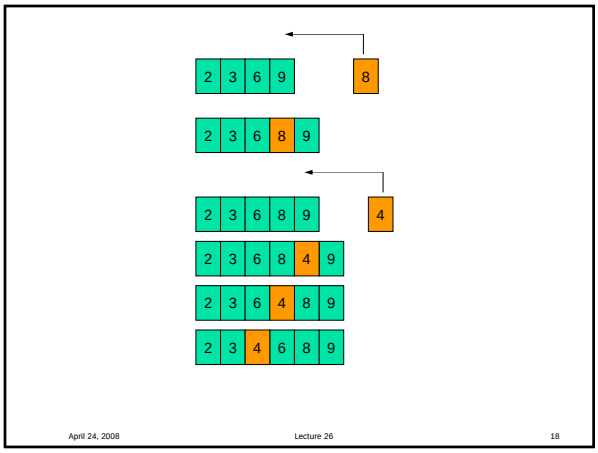
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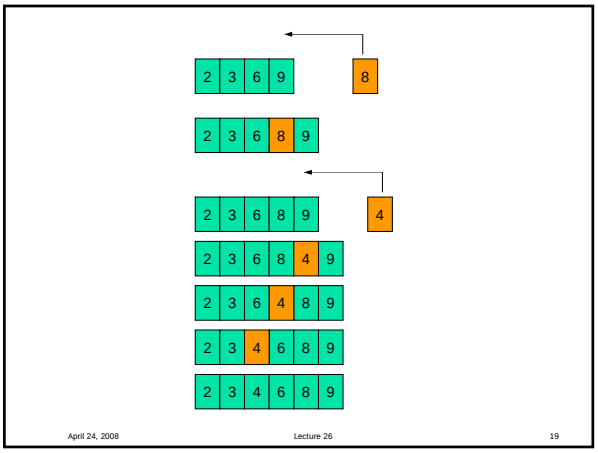
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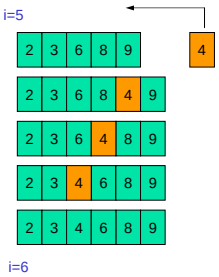
April 24, 2008 Lecture 26 19

Develop the insertion sort algorithm



- The sorted segment grows one element at a time—need to keep track of the length of the sorted segment, say, index i
- What is the simpliest (shortest) case? A list of length 1, so start with i=1
- Inserting the (i+1)th element requires a series of swaps: swap until the element to be inserted is at the correct place
  ➡ a while-loop

i=1: insert x(2) into x(1:1)

9	6	3	2	8	4

i=1: insert x(2) into x(1:1)

9	6	3	2	8	4
6	9	3	2	8	4

i=1: insert x(2) into x(1:1)

9	6	3	2	8	4

i=2: insert x(3) into x(1:2)

6	9	3	2	8	4

i=1: insert x(2) into x(1:1)

9	6	3	2	8	4
6	9	3	2	8	4

i=2: insert x(3) into x(1:2)

6	3	9	2	8	4

i=1: insert x(2) into x(1:1)

9	6	3	2	8	4
6	9	3	2	8	4
6	3	9	2	8	4

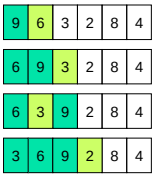
i=2: insert x(3) into x(1:2)

3	6	9	2	8	4

i=1: insert x(2) into x(1:1)

i=2: insert x(3) into x(1:2)

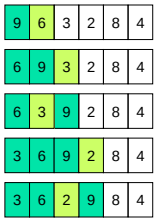
i=3: insert x(4) into x(1:3)



i=1: insert x(2) into x(1:1)

i=2: insert x(3) into x(1:2)

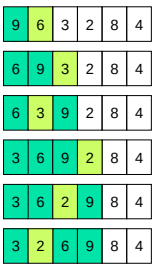
i=3: insert x(4) into x(1:3)



i=1: insert x(2) into x(1:1)

i=2: insert x(3) into x(1:2)

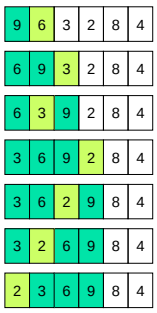
i=3: insert x(4) into x(1:3)



i=1: insert x(2) into x(1:1)

i=2: insert x(3) into x(1:2)

i=3: insert x(4) into x(1:3)

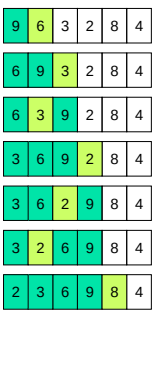


i=1: insert x(2) into x(1:1)

i=2: insert x(3) into x(1:2)

i=3: insert x(4) into x(1:3)

i=4: insert x(5) into x(1:4)




```
function x = insertSort(x)
% Sort vector x in ascending order with insertion sort

n = length(x);
for i= 1:n-1
      % Sort x(1:i+1) given that x(1:i) is sorted









end
```

```
function x = insertSort(x)
% Sort vector x in ascending order with insertion sort

n = length(x);
for i= 1:n-1
      % Sort x(1:i+1) given that x(1:i) is sorted
      j= i;
      need2swap= x(j+1) < x(j);
      while  need2swap

          % swap x(j+1) and x(j)




          j= j-1;
          need2swap= j>0 && x(j+1)<x(j);
      end
end
```



```
function x = insertSort(x)
% Sort vector x in ascending
  order with insertion sort

n = length(x);
for i= 1:n-1
      % Sort x(1:i+1) given that
  x(1:i) is sorted
      j= i;
      need2swap= x(j+1) < x(j);
      while  need2swap
```


How do merge sort and insertion sort compare?

- Find out by timing (benchmarking) the two functions
- In Matlab
  - `tic` – (re)sets timer
  - `toc` – returns the time elapsed since tic

Which method is more efficient?

A. Merge Sort B. Insertion Sort

Use tic toc to perform timing operation

```
x= rand(1000,1);
% Time InsertSort
  tic
  y= insertSort(x);
  t= toc;  % #seconds since tic
```


How do merge sort and insertion sort compare?

- Merge sort: N logN
- Insertion sort: (worst case) takes j operations to insert an element in a sorted array of j elements. In total
  1+2+…+(N-1) = N(N-1)/2, say N^2 for big N
- Insertion sort is done *in-place*; merge sort requires much more memory

How to choose??

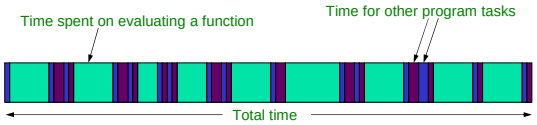
- Depends on application
- Merge sort is especially good for sorting large data set (but watch out for memory usage)
- Insertion sort is "order N^2" at worst case, but what about an average case? If the application requires that you maintain a sorted array, insertion sort may be a good choice

Why not just use Matlab's sort function?

- Flexibility
- E.g., to maintain a sorted list, just write the code for insertion sort
- E.g., sort strings or other complicated structures
- Sort according to some criterion set out in a function file
  - Observe that we have the comparison `x(j+1)<x(j)`
  - The comparison can be a function that returns a boolean value

Expensive function evaluations

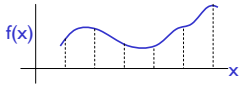
- Consider the execution of a program that is dominated by multiple calls to an expensive-to-evaluate function (e.g., climate simulation models)



- Can try to improve efficiency by dealing with the expensive function evaluations

Dealing with expensive function evaluations

- Can the function code be improved?
- Can we do fewer function evaluations?
- Can we pre-compute and store specific function values so that during the main program execution the program can just look up the values?
  - Consider function f(x). If there are many function calls and few distinct values of x, can get substantial speedup
  - Only speeds up main program execution—it still takes time to do the pre-computation



What are some issues and potential problems with the "table look-up" strategy?

- Accuracy—need a "dense grid" to get high accuracy → significant memory usage
- If an exact x-value is not found, need some kind of approximation
- Incur searching cost if the x-values are not simple indices
- Feasible in high dimensions (multiple dependent variables)?