- Previous Lecture:
  - "Divide and conquer" strategies
    - Binary search
    - Merge sort
- Today's Lecture:
  - "Divide and conquer" strategies—recursion
    - Merge sort
    - Sierpinski Triangle, revisited
- Announcements
  - Section in classrooms this week
  - CS100M final will be 5/8 (Thurs) 9am. Tell us now if you have a final exam conflict. Email Kelly Patwell with your complete exam schedule (course #s and times)

CS101J:

On-line course material; in-person scheduled exams

Two (or three) options:

- Register and work thru in FALL 08. You can take ENGRD/CS 211 in Spring 09.
- Learn the material on your own in Summer 08, then in Fall 08 register + submit assignments + take the tests within the first two weeks. You can take CS211 FA08.
- Possibility: CS offers CS101J in the summer (5/28-6/19). You must be on campus to take the tests during this period.

Quiz 2

- 3 questions
- A quiz counts as an exercise and you can miss "several" without lowering your score. E.g., if there will be 15 exercises in total you can miss 3 (about 20%).
- Answer the quiz using your registered clicker.
- Honor system: Use only your clicker and don't consult your neighbors or notes in any way.


```
function y = mergeSort(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
      y = x;
else
      m  = floor(n/2);
      y1 = mergeSort(x(1:m));
      y2 = mergeSort(x(m+1:n));
      y  = merge(y1,y2);
end
```

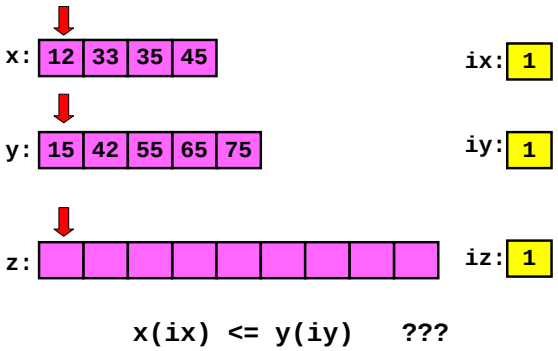

An important sub-problem is the merging of two sorted arrays into one single sorted array

12	33	35	45

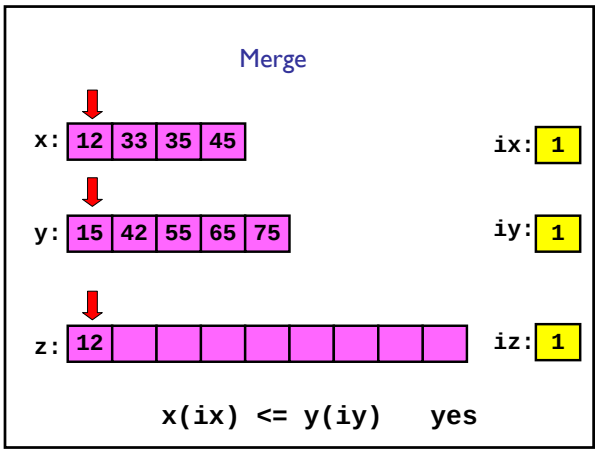
15	42	55	65	75

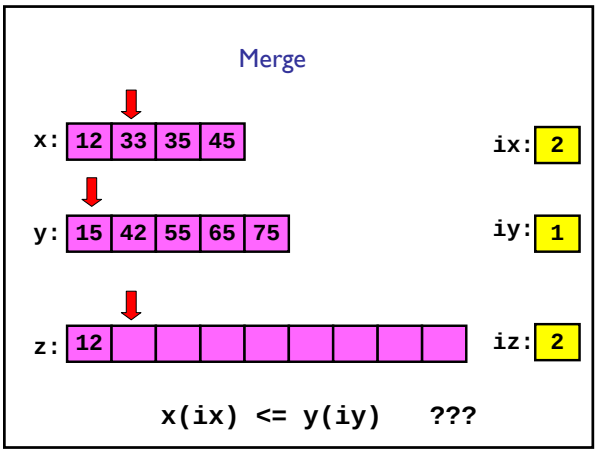
12	15	33	35	42	45	55	65	75

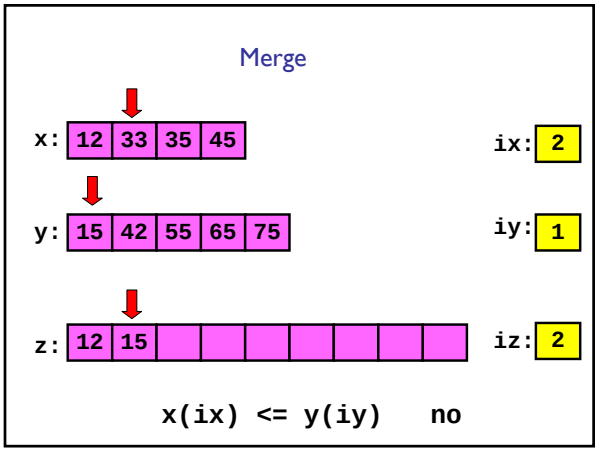
Merge

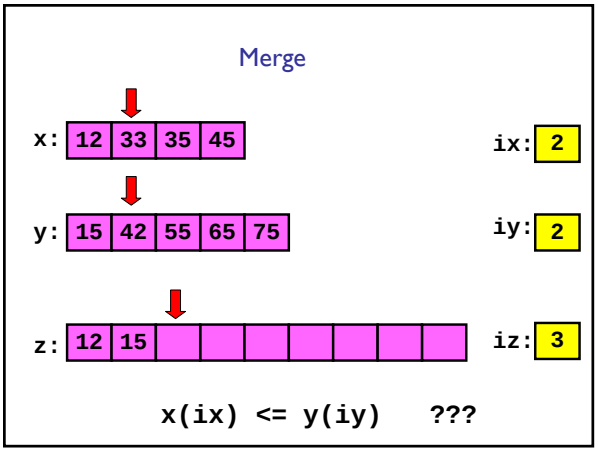


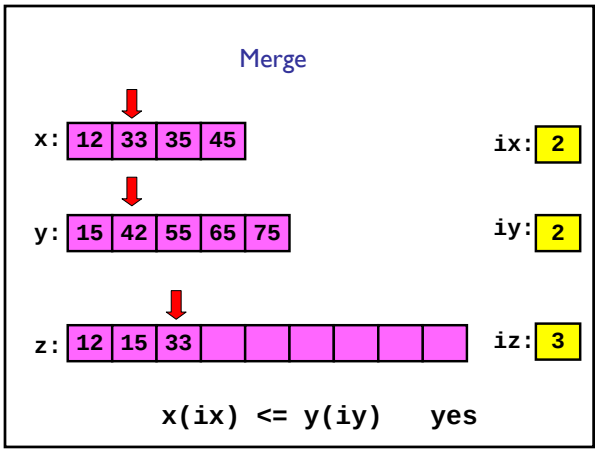
`x(ix) <= y(iy)  ???`

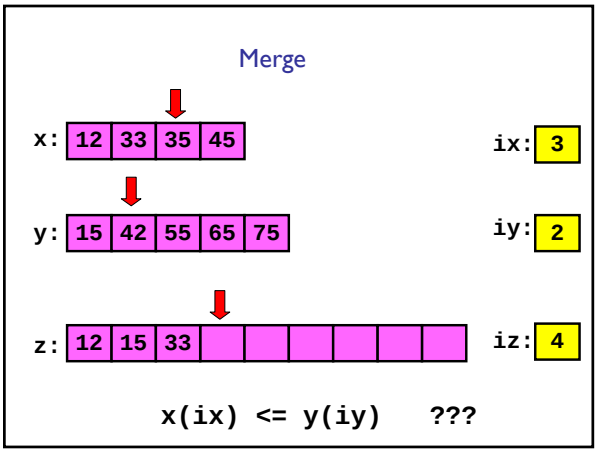

Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12
ix: 1
iy: 1
iz: 1
x(ix) <= y(iy) yes

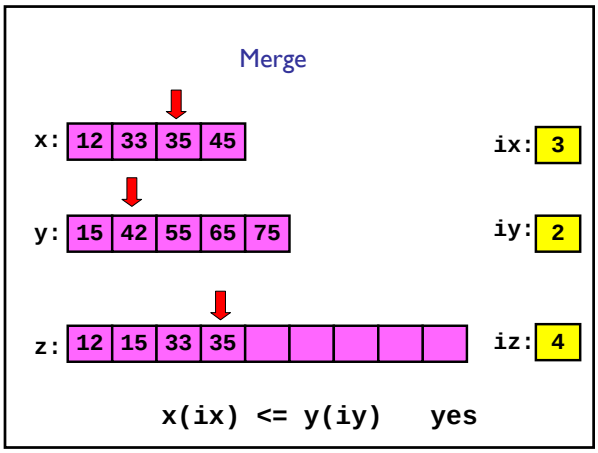

Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12
ix: 2
iy: 1
iz: 2
x(ix) <= y(iy) ???

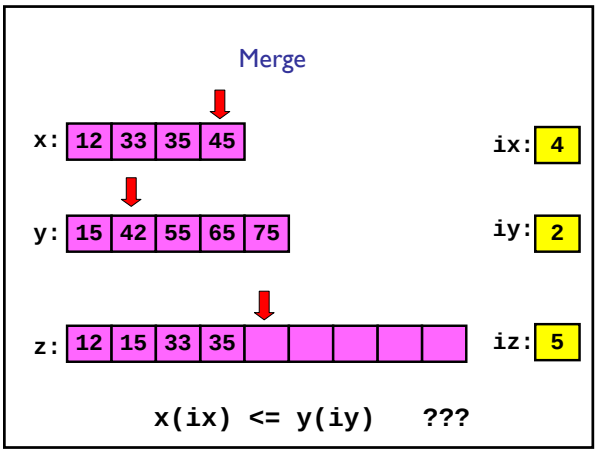

Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12 15
ix: 2
iy: 1
iz: 2
x(ix) <= y(iy) no

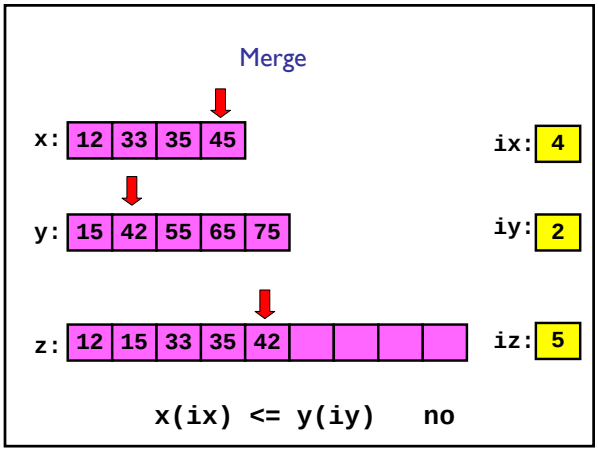

Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12 15
ix: 2
iy: 2
iz: 3
x(ix) <= y(iy) ???

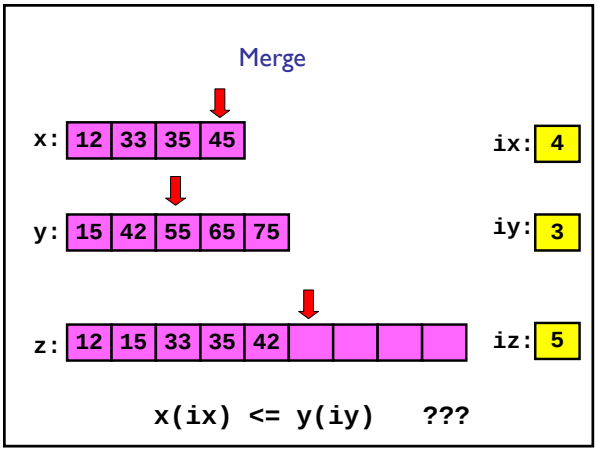

Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12 15 33
ix: 2
iy: 2
iz: 3
x(ix) <= y(iy) yes

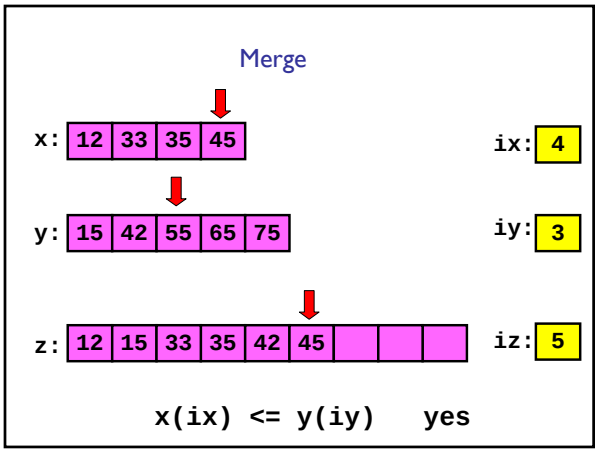

Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12 15 33
ix: 3
iy: 2
iz: 4
x(ix) <= y(iy) ???

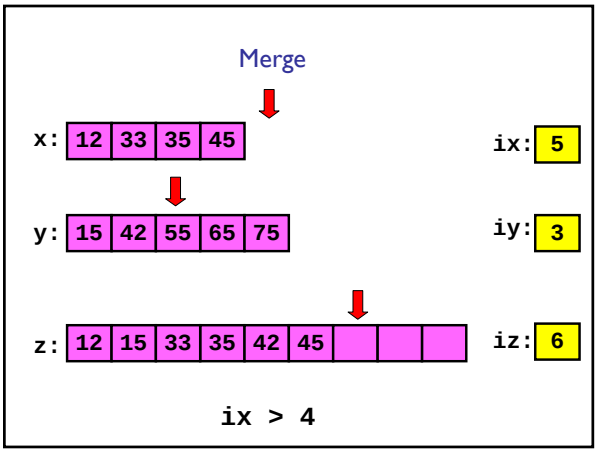

Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12 15 33 35
ix: 3
iy: 2
iz: 4
x(ix) <= y(iy) yes

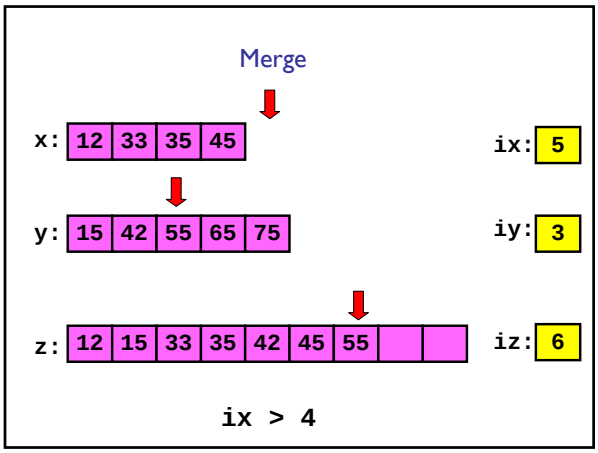

Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12 15 33 35
ix: 4
iy: 2
iz: 5
x(ix) <= y(iy) ???

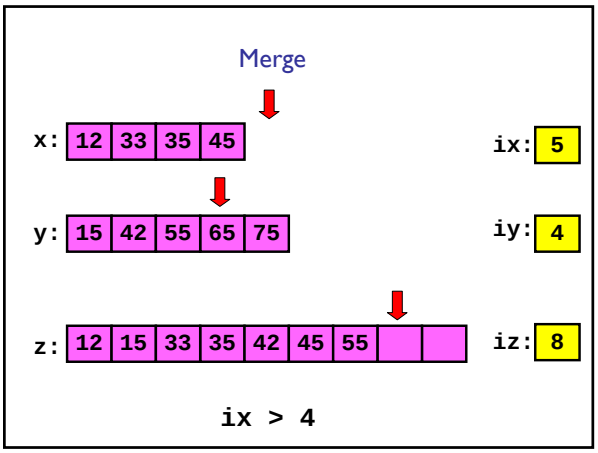

Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12 15 33 35 42
ix: 4
iy: 2
iz: 5
x(ix) <= y(iy) no

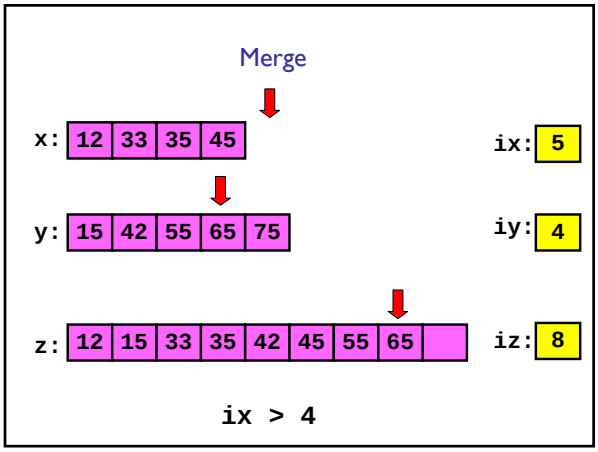

Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12 15 33 35 42
ix: 4
iy: 3
iz: 5
x(ix) <= y(iy) ???

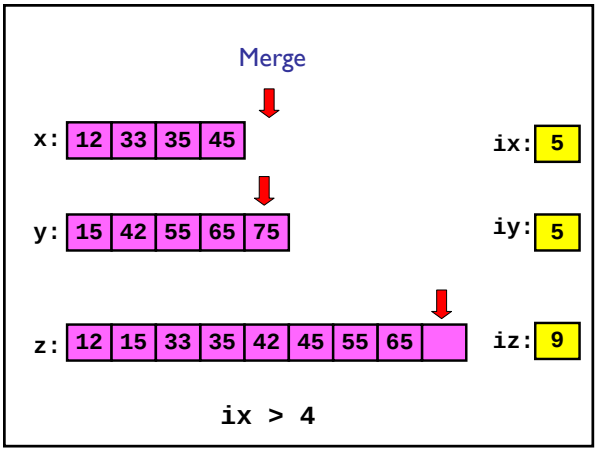

Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12 15 33 35 42 45
ix: 4
iy: 3
iz: 5
x(ix) <= y(iy) yes

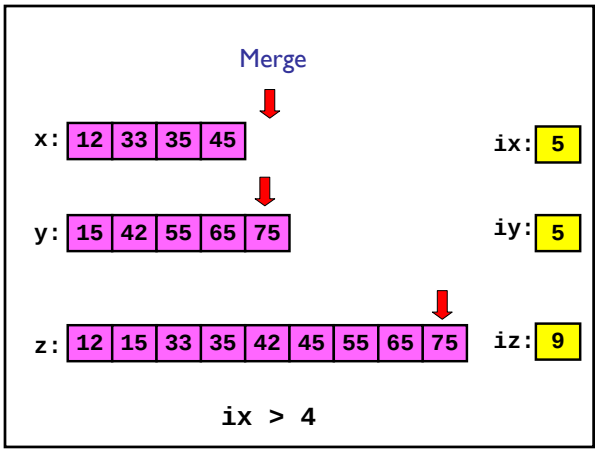

Merge
x: 12 33 35 45
y: 15 42 55 65 75
z: 12 15 33 35 42 45
ix: 5
iy: 3
iz: 6
ix > 4







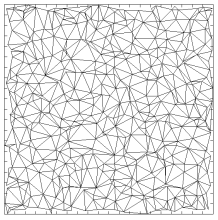





```
function z = merge(x,y)
n = length(x); m = length(y);
z = zeros(1,n+m);
ix = 1; iy = 1;
for iz=1:(n+m)
    if  ix > n
        z(iz)= y(iy);  iy = iy+1;
    elseif  iy>m
        z(iz)= x(ix);  ix = ix + 1;
    elseif  x(ix) <= y(iy)
        z(iz)= x(ix);  ix = ix + 1;
    else
        z(iz)= y(iy);  iy = iy + 1;
    end
end
```

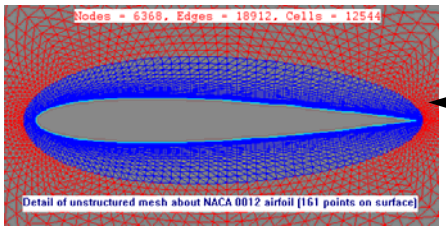
Divide-and-conquer methods also show up in geometric situations

Chop a region up into triangles with smaller triangles in "areas of interest"



Recursive mesh generation

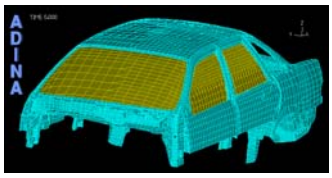
Mesh Generation



An area of interest

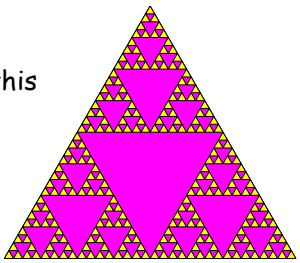
Step one in simulating flow around an airfoil is to generate a mesh and (say) estimate velocity at each mesh point.

Mesh Generation in 3D



Why is mesh generation a divide & conquer process?

Let's draw this graphic



The basic operation

if *the triangle is big enough*

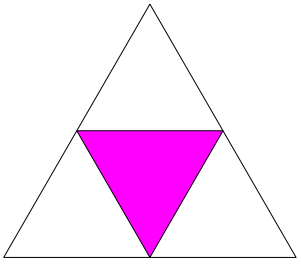
Connect the midpoints.
Color the interior triangle mauve.

else

Color the whole triangle yellow.

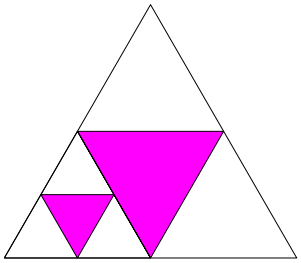
end

At the Start…

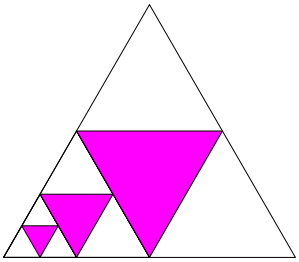


Recur on this idea:

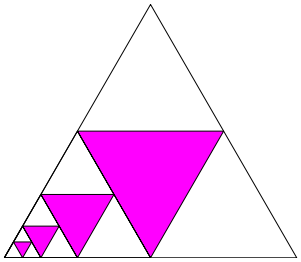
Apply same idea to the lower left triangle



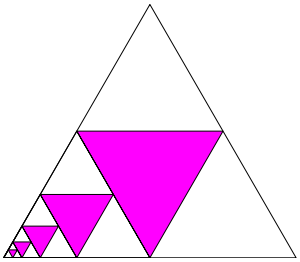
Recur again



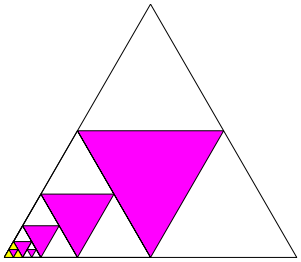
… and again



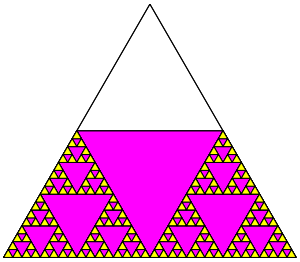
… and again



Now, climb your way out.



…, etc.



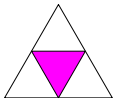
```
function drawTriangle(x,y,level)
% Draw recursively colored triangles.
% x,y are 3-vectors that define the vertices of a triangle.

if level==5
  % Recursion limit (depth) reached
  % Color whole triangle yellow
else
  % Draw the triangle...

  % Draw and color the interior triangle mauve

  % Apply the process to the three "corner" tri

end
```




```
function drawTriangle(x,y,level)
% Draw recursively colored triangles.
% x,y are 3-vectors that define the vertices of a triangle.

if level==5
  % Recursion limit (depth) reached
    fill(x,y,'y')  % Color whole triangle yellow
else
  % Draw the triangle...
    plot([x x(1)],[y y(1)],'k')
  % Draw and color the interior triangle mauve.
    a = [(x(1)+x(2))/2 (x(2)+x(3))/2 (x(3)+x(1)
    b = [(y(1)+y(2))/2 (y(2)+y(3))/2 (y(3)+y(1)
    pause
    fill(a,b,'m')
    pause
  % Apply the process to the three "corner" tri
    drawTriangle([x(1) a(1) a(3)],[y(1) b(1) b(3)],level+1)
    drawTriangle([x(2) a(2) a(1)],[y(2) b(2) b(1)],level+1)
    drawTriangle([x(3) a(3) a(2)],[y(3) b(3) b(2)],level+1)
end
```