Lecture 27

Sorting

Announcements for This Lecture

Finishing Up

- **Submit a course evaluation**
  - Will get an e-mail for this
  - Part of “participation grade”
- **Final: Dec 10th 2:00-4:30pm**
  - Study guide is posted
  - Announce reviews on Tues.
- **Conflict with Final time?**
  - Submit to conflict to CMS **by next Tuesday!**

Assignment 7

- Should be on bolt *collisions*
- Use weekend for final touches
  - Multiple lives
  - Winning or losing the game
- Also work the extensions
  - Add anything you want
  - Need at least two
  - Ask on Piazza if unsure
  - All else is **extra credit**

Linear Search

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- **Better**: Store an integer in i to truthify result condition post:

post: 1. v is not in b[h..i-1]

2. i = k OR v = b[i]

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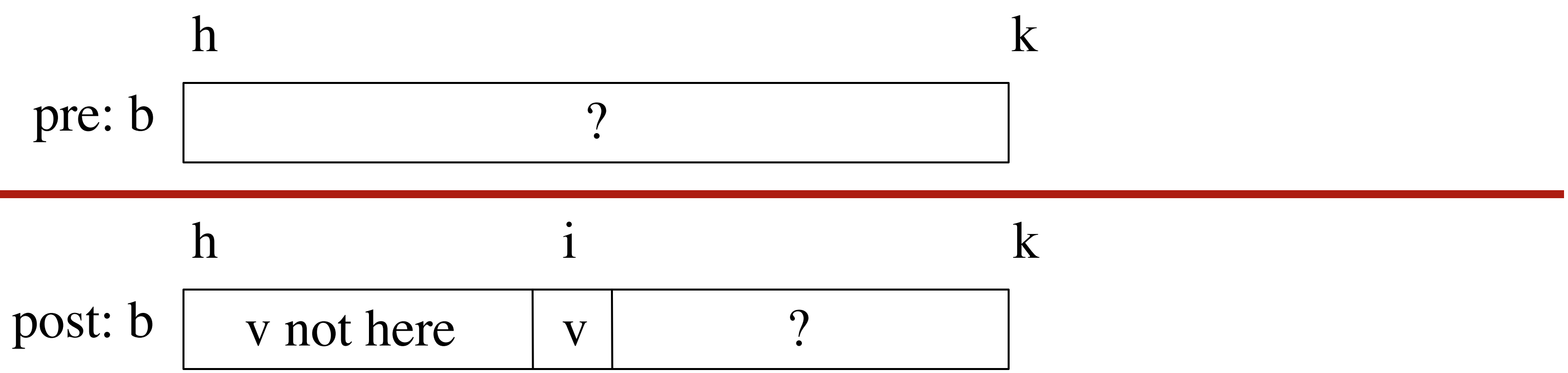

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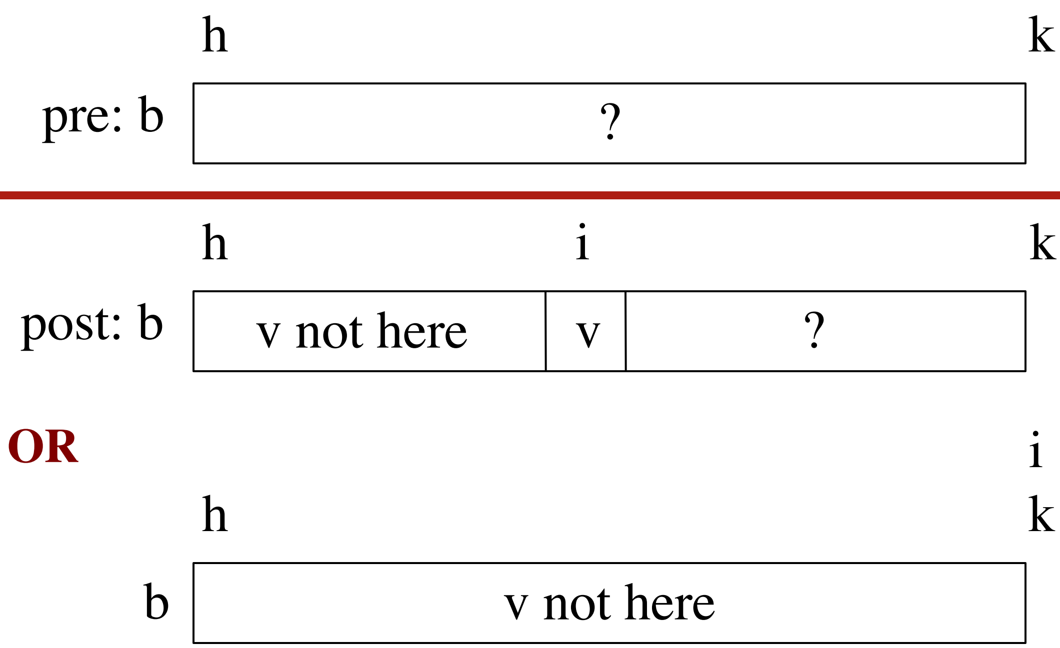

Linear Search

h k

pre: b | ? |

h i k

post: b | v not here | v | ? |

OR

i

h k

b | v not here |

h i k

inv: b | v not here | ? |

Linear Search

```
def linear_search(b,v,h,k):
    """Returns: first occurrence of v in b[h..k-1]"""
    # Store in i index of the first v in b[h..k-1]
    i = h

    # invariant: v is not in b[0..i-1]
    while i < k and b[i] != v:
        i = i + 1

    # post: v is not in b[h..i-1]
    #       i >= k or b[i] == v
    return i if i < k else -1
```

Analyzing the Loop

1. Does the initialization make **inv** true?
2. Is **post** true when **inv** is true and **condition** is false?
3. Does the repetend make progress?
4. Does the repetend keep the invariant **inv** true?

Binary Search

- **Vague:** Look for v in **sorted** sequence segment b[h..k].

Binary Search

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- **Better:**
  - Precondition: b[h..k-1] is sorted (in ascending order).
  - Postcondition: b[h..i-1] < v and v <= b[i..k]
- Below, the array is in non-descending order:

	h		k
pre: b	? sorted		

	h	i	k
post: b	< v	>= v	

Binary Search

- Look for value v in **sorted** segment b[h..k]

	h			k
pre: b	?			

	h	i		k
post: b	< v	>= v		

	h	i	j	k
inv: b	< v	?	>= v	

New statement of the invariant guarantees that we get leftmost position of v if found

	h									k
	0	1	2	3	4	5	6	7	8	9
Example b	3	3	3	3	3	4	4	6	7	7

- if v is 3, set i to 0
- if v is 4, set i to 5
- if v is 5, set i to 7
- if v is 8, set i to 10

Binary Search

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  - Postcondition: b[h..i-1] < v and v <= b[i..k]
- Below, the array is in non-descending order:

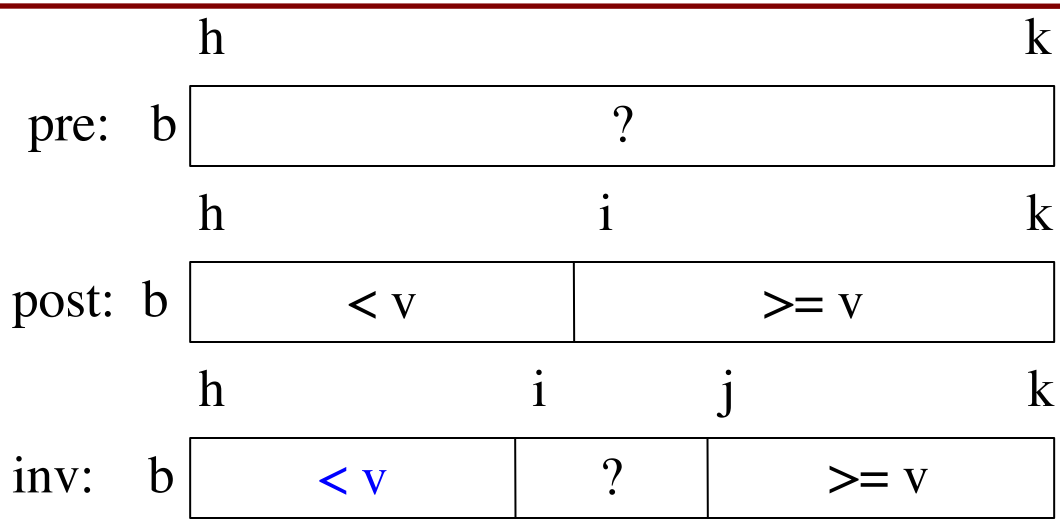
	h		k
pre: b	?		

	h	i	k
post: b	< v	>= v	

	h	i	j	k
inv: b	< v	?	> v	

Called binary search because each iteration of the loop cuts the array segment still to be processed in half

Binary Search



New statement of the invariant guarantees that we get leftmost position of v if found

```
i = h; j = k+1;
while i != j:
```

Looking at b[i] gives linear search from left.

Looking at b[j-1] gives linear search from right.

Looking at middle: b[(i+j)/2] gives binary search.

Sorting: Arranging in Ascending Order

	0	n
pre: b	?	

	0	n
post: b	sorted	

Insertion Sort:

	0	i	n
inv: b	sorted	?	

```
i = 0
while i < n:
    # Push b[i] down into its
    # sorted position in b[0..i]
    i = i+1
```

0						i
2	4	4	6	6	7	5

←

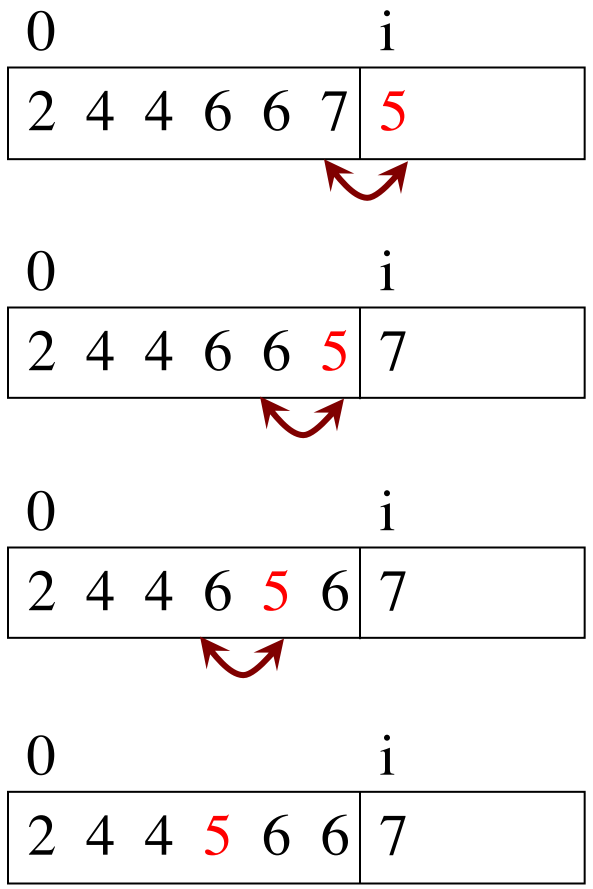
0						i
2	4	4	5	6	6	7

Insertion Sort: Moving into Position

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

swap shown in the lecture about lists



The Importance of Helper Functions

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

VS

Can you understand all this code below?

```
i = 0
while i < n:
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            temp = b[j]
            b[j] = b[j-1]
            b[j-1] = temp
        j = j -1
    i = i +1
```

Insertion Sort: Performance

```
def push_down(b, i):
    """Push value at position i into
    sorted position in b[0..i-1]"""
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

- b[0..i-1]: i elements
- Worst case:
  - i = 0: 0 swaps
  - i = 1: 1 swap
  - i = 2: 2 swaps
- Pushdown is in a loop
  - Called for i in 0..n
  - i swaps each time

Insertion sort is an n^2 algorithm

Total Swaps: 0 + 1 + 2 + 3 + … (n-1) = (n-1)*n/2

Algorithm "Complexity"

- **Given**: a list of length n and a problem to solve
- **Complexity**: *rough* number of steps to solve worst case
- Suppose we can compute 1000 operations a second:

Complexity	n=10	n=100	n=1000
n	0.01 s	0.1 s	1 s
n log n	0.016 s	0.32 s	4.79 s
n^2	0.1 s	10 s	16.7 m
n^3	1 s	16.7 m	11.6 d
2^n	1 s	4×10^{19} y	3×10^{290} y

Major Topic in 2110: Beyond scope of this course

Sorting: Changing the Invariant

pre: b [0 … n]: ?

post: b [0 … n]: sorted

Selection Sort:

inv: b [0 … i]: sorted, ≤ b[i..] | [i … n]: ≥ b[0..i-1]

First segment always contains smaller values

```
i = 0
while i < n:
    # Find minimum in b[i..]
    # Move it to position i
    i = i+1
```

b[0..i-1]	b[i..n]
2 4 4 6 6	8 9 9 7 8 9
2 4 4 6 6	7 9 9 8 8 9
2 4 4 6 6 7	9 9 8 8 9

Sorting: Changing the Invariant

	0	n
pre: b	?	

	0	n
post: b	sorted	

Selection Sort:

	0	i	n
inv: b	sorted, ≤ b[i..]	≥ b[0..i-1]	

First segment always contains smaller values

```
i = 0
while i < n:
    j = index of min of b[i..n-1]
    swap(b,i,j)
    i = i+1
```

					i						n
2	4	4	6	6	8	9	9	7	8	9	

					i						n
2	4	4	6	6	7	9	9	8	8	9	

Selection sort also is an n^2 algorithm

Partition Algorithm

- Given a list segment b[h..k] with some value x in b[h]:

	h		k
pre: b	x	?	

- Swap elements of b[h..k] and store in j to truthify post:

	h	i	i+1	k
post: b	<= x	x	>= x	

		h								k
change:	b	**3**	5	4	1	6	2	3	8	1
		h			i					k
into	b	1	2	1	**3**	5	4	6	3	8
		h				i				k
or	b	1	2	3	1	**3**	4	5	6	8

- x is called the pivot value
  - x is not a program variable
  - denotes value initially in b[h]

Sorting with Partitions

- Given a list segment b[h..k] with some value x in b[h]:

		h		k
pre:	b	**x**	?	

- Swap elements of b[h..k] and store in j to truthify post:

		h			i	i+1	k
post:	b	<= **y**	**y**	>= **y**	**x**	>= **x**	

Partition Recursively

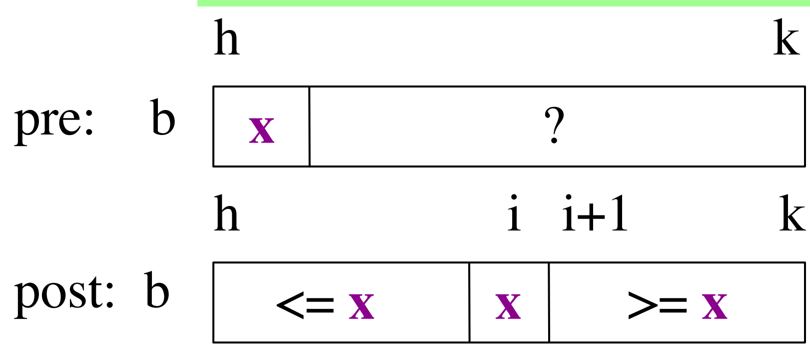
Recursive partitions = sorting

- Called **QuickSort** (why???)
- Popular, fast sorting technique

QuickSort

```
def quick_sort(b, h, k):
    """Sort the array fragment b[h..k]"""
    if  b[h..k] has fewer than 2 elements:
        return
    j = partition(b, h, k)
    # b[h..j–1] <= b[j] <= b[j+1..k]
    # Sort b[h..j–1]  and  b[j+1..k]
    quick_sort (b, h, j–1)
    quick_sort (b, j+1, k)
```

- **Worst Case:**
  array already sorted
  - Or almost sorted
  - n^2 in that case
- **Average Case:**
  array is scrambled
  - n log n in that case
  - Best sorting time!



Final Word About Algorithms

- **Algorithm:**
  - Step-by-step way to do something
  - Not tied to specific language
- **Implementation:**
  - An algorithm in a specific language
  - Many times, not the "hard part"
- Higher Level Computer Science courses:
  - We teach advanced algorithms (pictures)
  - Implementation you learn on your own

List Diagrams

Demo Code