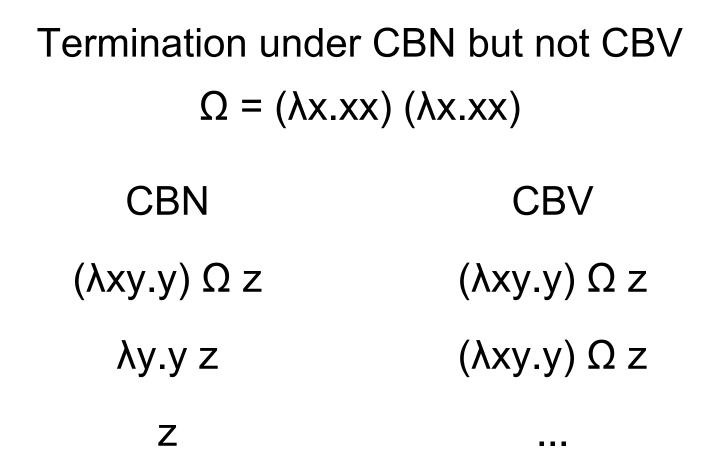
# Call-by-name, call-by-value, and the $\lambda$ -calculus

## G.D. Plotkin Presented by Dietrich Geisler

Call-by-name vs call-by-value

Define square =  $\lambda xy.x^*x$ Evaluate square(2+2, 2+3)

Call-by-name (CBN) Call-by-value (CBV) square(2+2, 2+3) square(2+2, 2+3)  $(2+2)^{*}(2+2)$  square(4, 2+3)  $4^{*}(2+2)$  square(4, 5)  $4^{*}4$   $4^{*}4$ 16 16



## Objective

- Transformation between CBV and CBN semantics
- A CBV evaluation of a program *P* should terminate if and only if the CBN evaluation of the translated *P* also terminates

**Theorem 2.** (Simulation).  $\Psi(\operatorname{Eval}_{\nu}(M)) = \operatorname{Eval}_{N}(\overline{M}(\lambda xx))$ , for any program M.

#### Some Context

#### Published in 1975

Previous Papers:

Recursive Functions (1960) Axiomatic Basis (1969)

Abstraction (1983) Expressive Power of PLs (1990) Higher-level languages:

C 1972 Scheme 1973 ML 1975 **Practicalities** 

# <S, E, C, D> machine

Machine for evaluating lambda expressions

Constapply(a, b)

Mechanism for syntactic sugar

# The $\lambda_v$ Calculus

$$\frac{e_1 \to e_1'}{e_1 \ e_2 \to e_1' \ e_2} \qquad \frac{e \to e'}{v \ e \to v \ e'}$$

$$(\lambda x.e) v \to e [v/x]$$

- I1.  $(\lambda x M) = (\lambda y [y/x] M) (y \notin FV(M)).$  ( $\alpha$ -rule)
  - 2.  $(\lambda x M) N = [N/x] M$  (if N is a value). ( $\beta$ -rule)
  - 3. (ab) = Constapply (a, b) (if this is defined). ( $\delta$ -rule)

#### Equality of terms

III. 
$$M = M$$
  
2.  $\frac{M}{M} = \frac{N}{N} = L$   
3.  $\frac{M}{N} = \frac{N}{M}$   
III.  $\frac{M}{(MZ)} = \frac{N}{(NZ)}$ ,  $\frac{M}{(ZM)} = \frac{M}{(ZN)}$   
2.  $\frac{M}{(\lambda x M)} = (\lambda x N)$ 

M=N iff M is equivalent to N

#### **Reduction of terms**

### M≥N iff M reduces to something equal to N

**Theorem 2.** (Church-Rosser theorem). If  $\lambda_v + M_1 \ge M_i$  (i = 2, 3) then for some  $M_4$ ,  $\lambda_v + M_i \ge M_4$  (i = 2, 3).

**Theorem 4.**  $Eval_V(M) = N$  iff  $M \stackrel{*}{\to}_V N$ , (for closed M and a value N).

# The $\lambda_n$ Calculus

$$\frac{e_1 \to e_1'}{e_1 \ e_2 \to e_1' \ e_2}$$

$$(\lambda x.e_1) \ e_2 \rightarrow e_1 \ [e_2/x]$$

II.  $(\lambda x M) = (\lambda y [y/x] M) (y \notin FV(M)) (\alpha$ -reduction). 2.  $(\lambda x M) N = [N/x] M (\beta$ -reduction).

No requirement that N is a value

#### Translating from CBN to CBV

# $(\lambda xy.y) \Omega z$

$$\underline{x} = x$$

$$\underline{\lambda x.M} = \lambda \alpha.\alpha(\lambda x.\underline{M})$$

$$\underline{M N} = \lambda \alpha.\underline{M} (\lambda \beta.\beta \underline{N} \alpha)$$

#### Translating from CBN to CBV

# $(\lambda xy.y \Omega) z$

$$\underline{x} = x$$
  

$$(\underline{\lambda x M}) = \lambda \varkappa \varkappa (\lambda x \underline{M})$$
  

$$(\underline{MN}) = \lambda \varkappa M (\lambda \alpha \alpha N \varkappa).$$