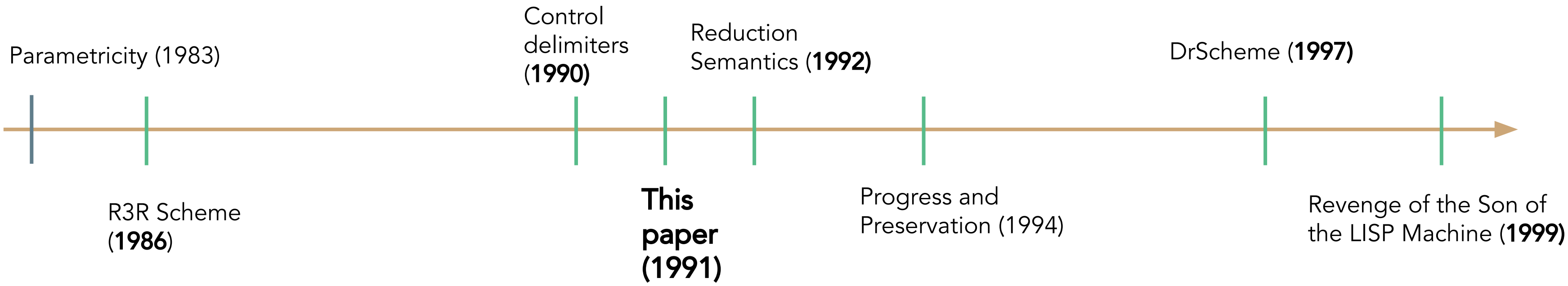
On the Expressive Power

of Programming Languages

Historical Context



Parametricity (1983)

R3R Scheme (**1986**)

Control delimiters (**1990**)

This paper (1991)

Reduction Semantics (**1992**)

Progress and Preservation (1994)

DrScheme (**1997**)

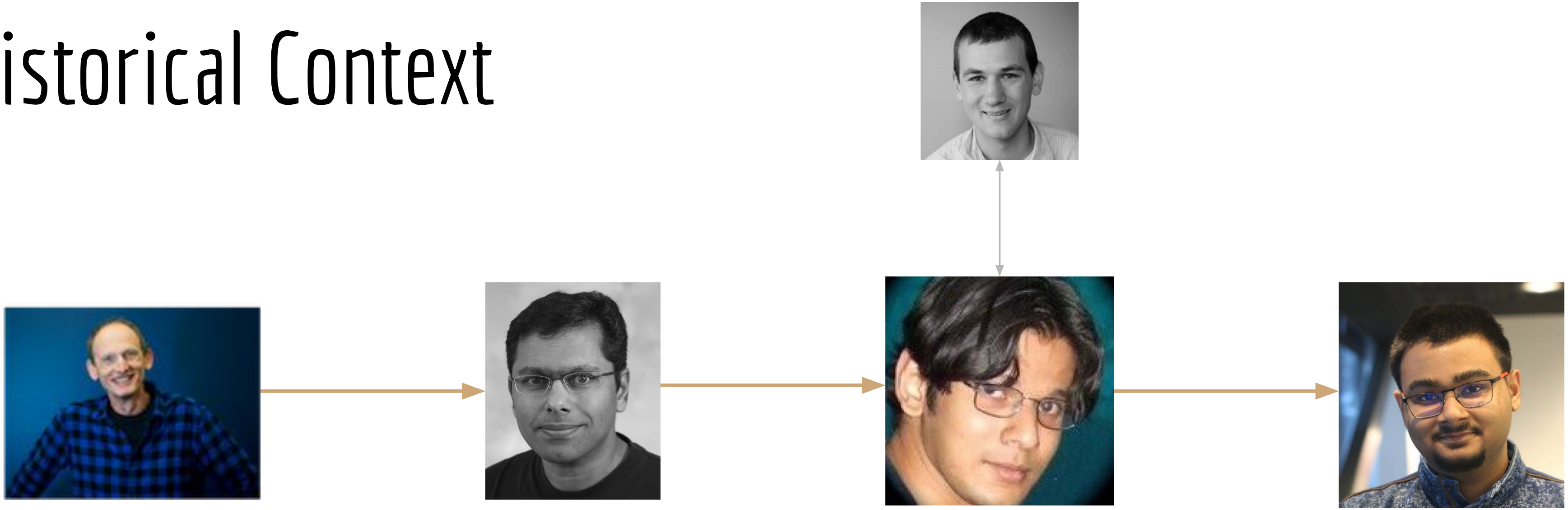
Revenge of the Son of the LISP Machine (**1999**)

Historical Context

1991: Writes this paper

1994: Shriram pivots from CompBio after reading it

Historical Context



1991: Writes this paper

1994: Shriram pivots from CompBio after reading it

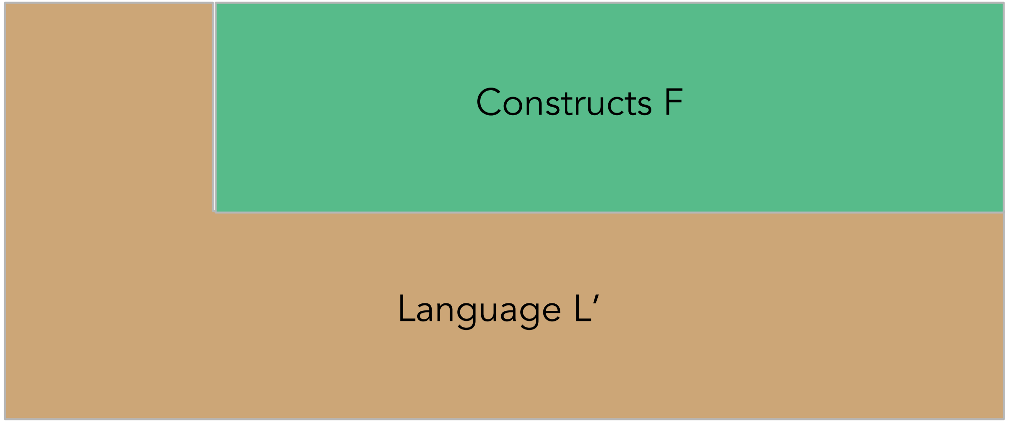
2010: Essence of JavaScript

2019: nothing of note.

Expressivity

Language L’

Expressivity

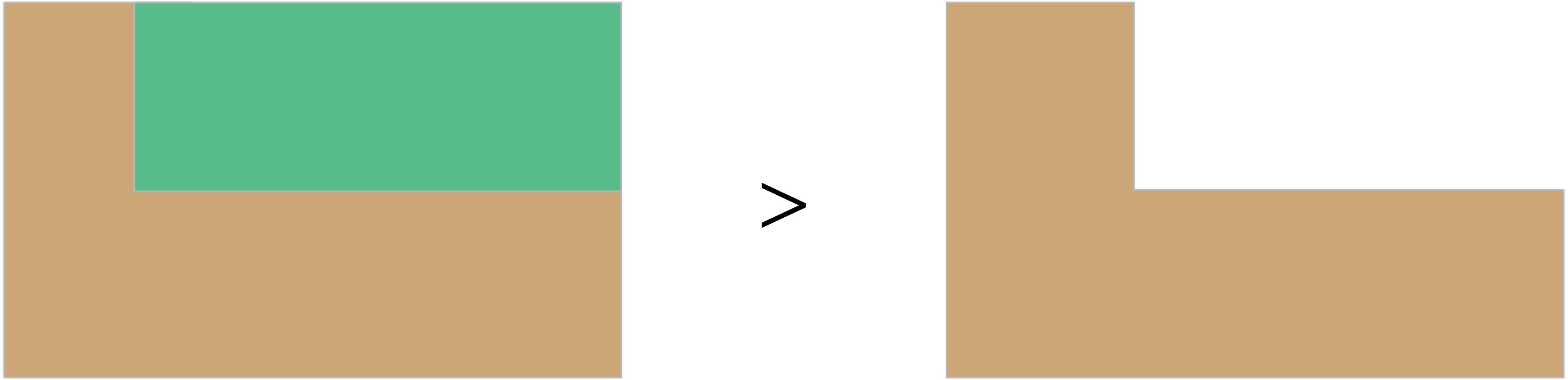


Constructs F

Language L’

Language L

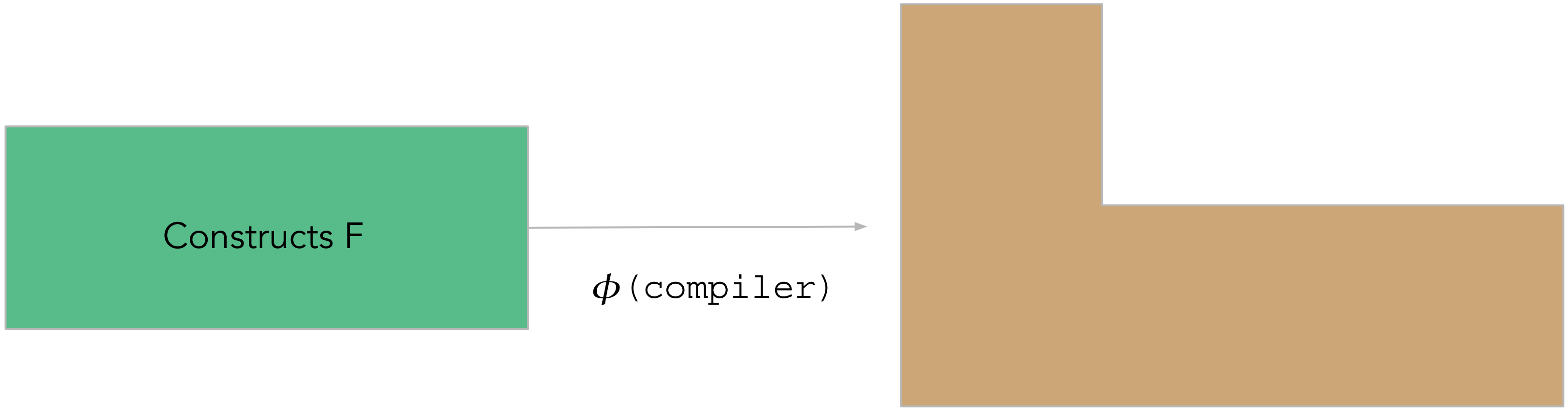
Expressivity



Language L

Language L’

Expressivity



Constructs F

ϕ `(compiler)`

Language L’

Expressivity

```
Let x = init in body
```

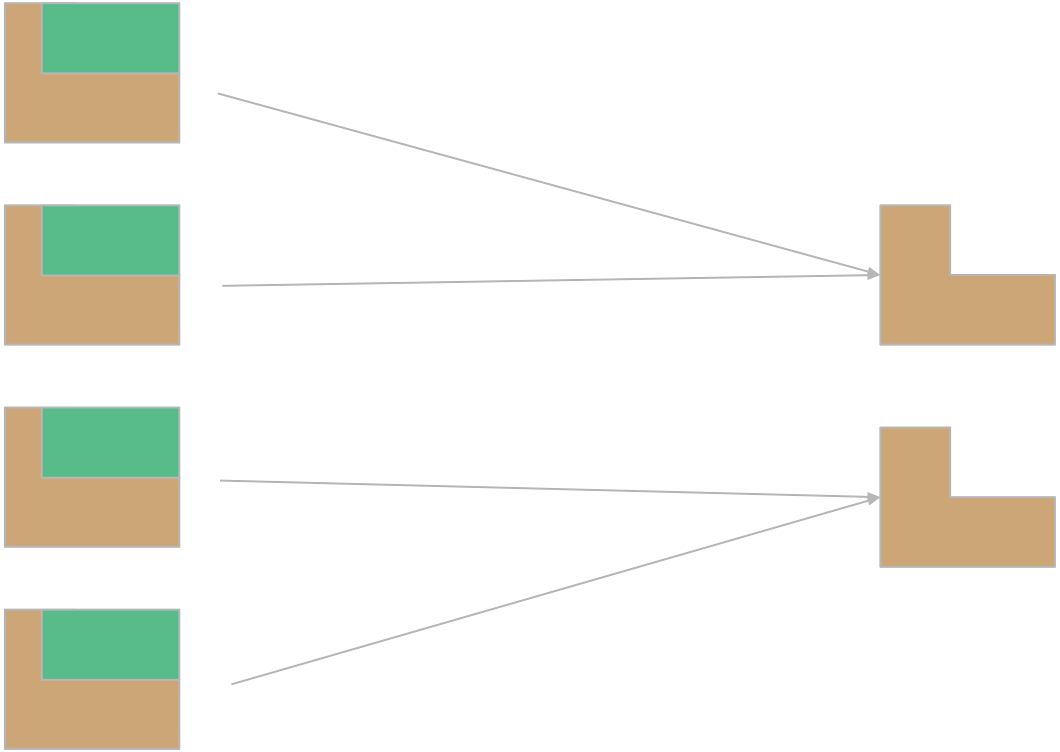
```
(fun x -> body) init
```

```
Let x = ref 0 in x++
```

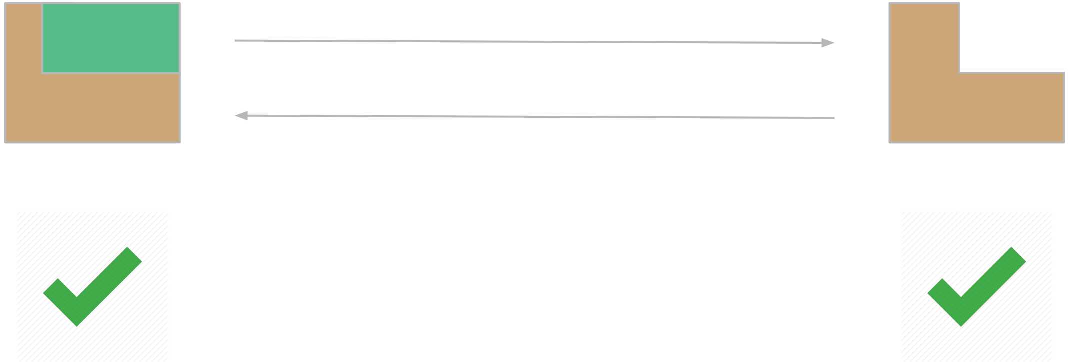
```
Let x = makeBox () in
x.setRef(x.getRef + 1)
```

Eliminable Constructs

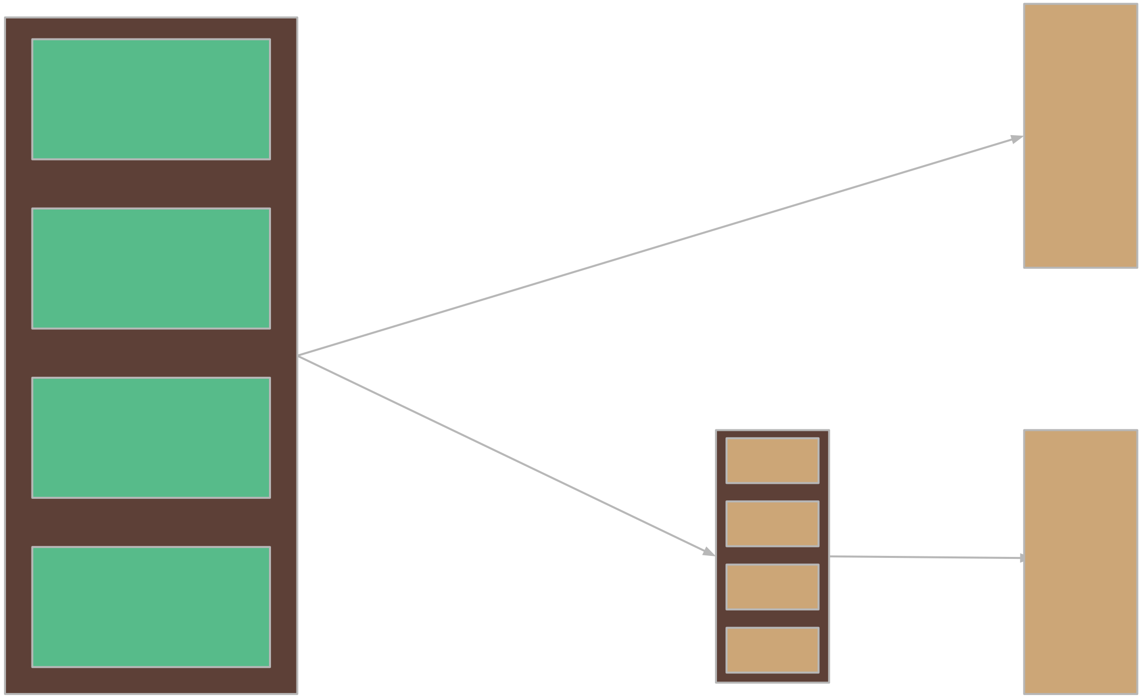
E1 $\varphi(e)$ is an $\mathcal{L}'$-program for all $\mathcal{L}$-programs e;



E3 $eval_{\mathcal{L}}(e)$ holds if and only if $eval_{\mathcal{L}'}(\varphi(e))$ holds for all $\mathcal{L}$-programs e.



E2 $\varphi(\mathbb{F}(e_1, \ldots, e_a)) = \mathbb{F}(\varphi(e_1), \ldots, \varphi(e_a))$ for all facilities $\mathbb{F}$ of $\mathcal{L}'$, i.e., φ is homomorphic in all constructs of $\mathcal{L}'$; and



Eliminable: Example

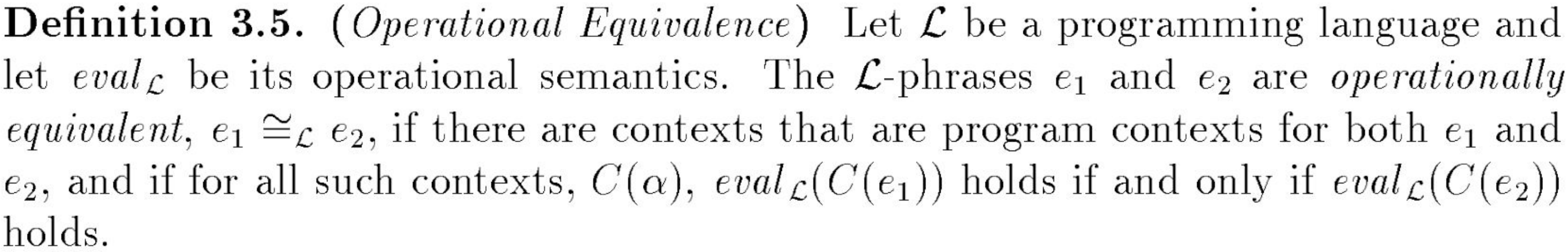
```
Let x = init in body
```

```
φ(Let x = init in body) => (fun x -> φ(body)) φ(init)
```

```
(fun x -> body) init
```

Contextual Equivalence

Or Observational Equivalence



Definition 3.5. (*Operational Equivalence*) Let $\mathcal{L}$ be a programming language and let $eval_{\mathcal{L}}$ be its operational semantics. The $\mathcal{L}$-phrases e_1 and e_2 are *operationally equivalent*, $e_1 \cong_{\mathcal{L}} e_2$, if there are contexts that are program contexts for both e_1 and e_2, and if for all such contexts, $C(\alpha)$, $eval_{\mathcal{L}}(C(e_1))$ holds if and only if $eval_{\mathcal{L}}(C(e_2))$ holds.

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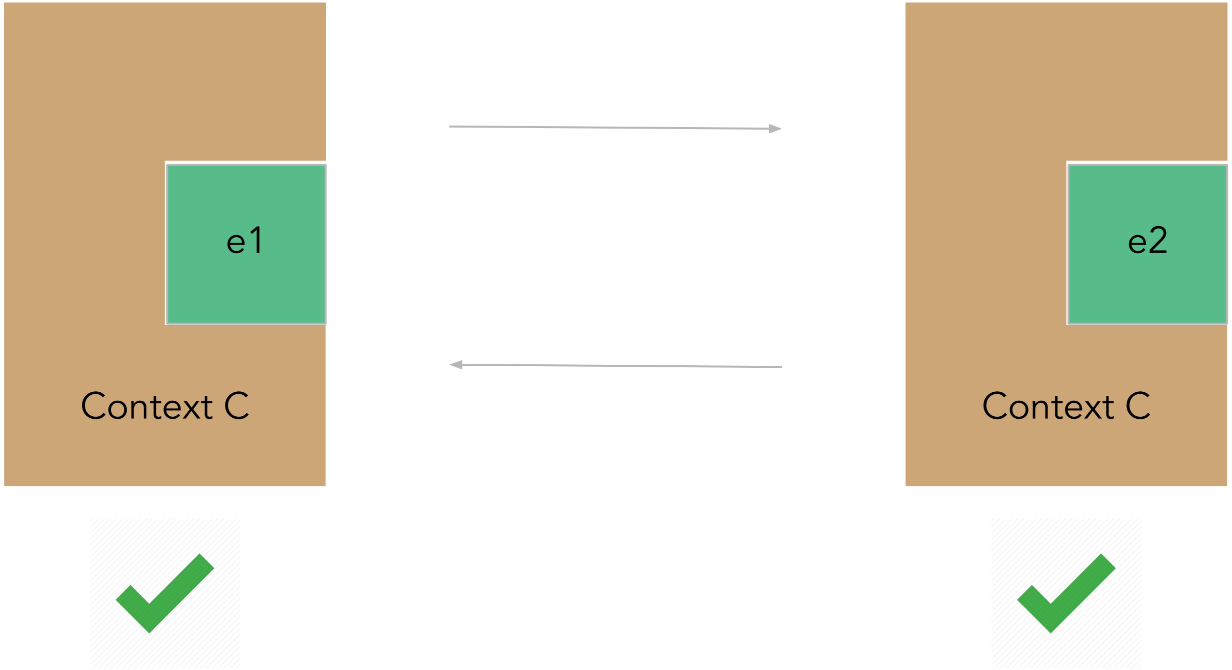

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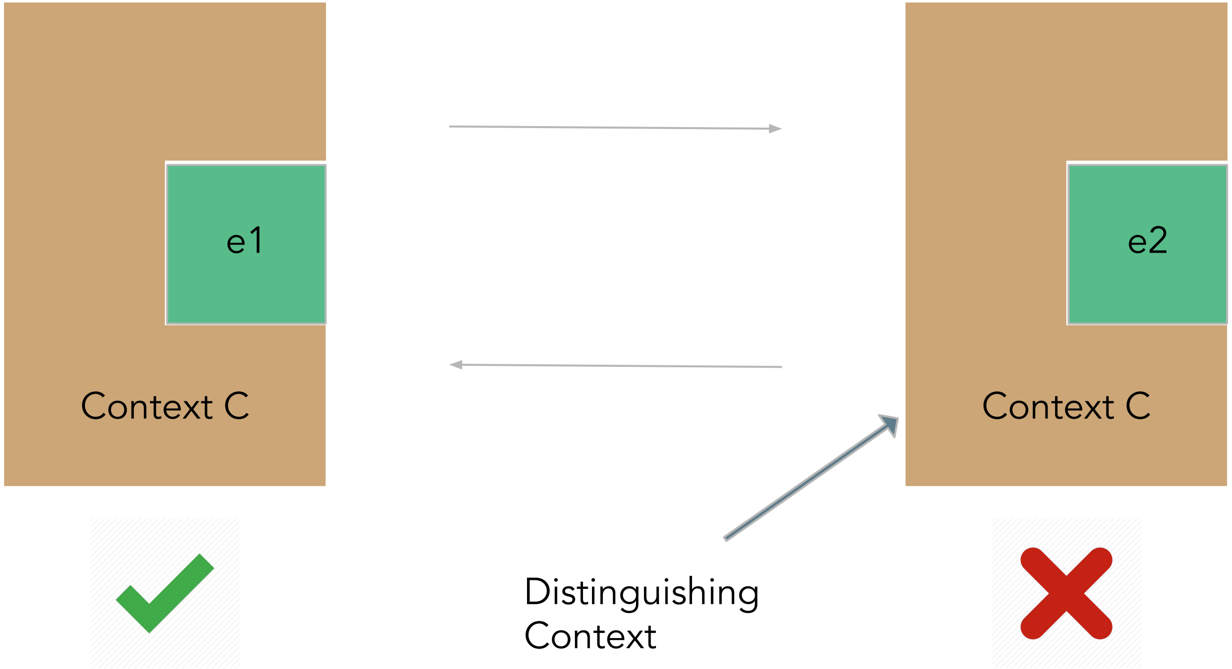
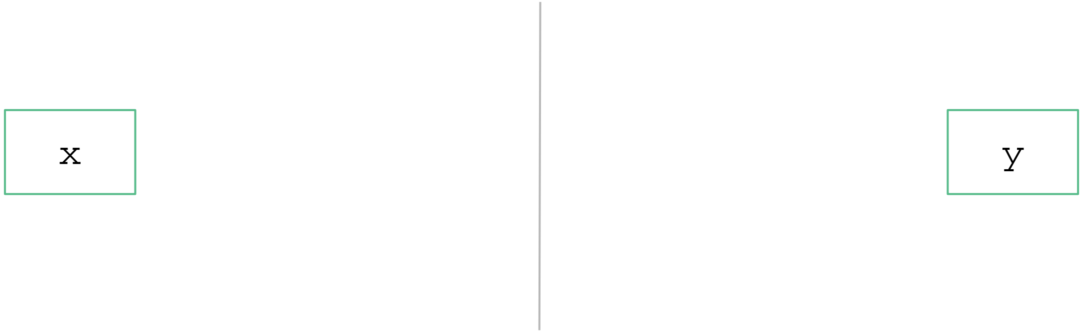


Distinguishing Context



```
C(a) = (fun x, y -> a) 1 (throw 1)
```

Contextual Equivalence: Example

```
(fun x, y -> x) (fun x -> x) (throw 1)
```

```
1
```

```
(fun x, y -> y) (fun x -> x) (throw 1)
```

```
(throw 1)
```

```
⊥
```

Contextual Equivalence: Example

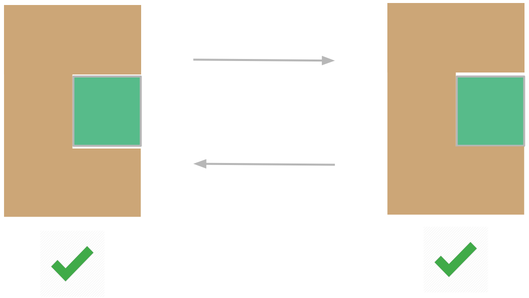
Expressivity

Constructs F is expressible in Language L if

→

ϕ satisfies **E1, E2, E3**

F is eliminable

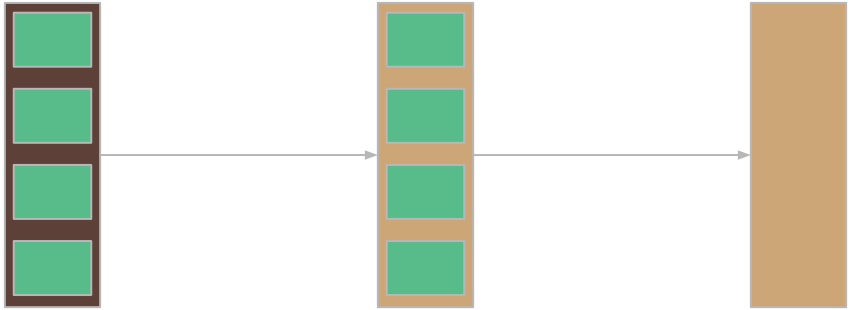


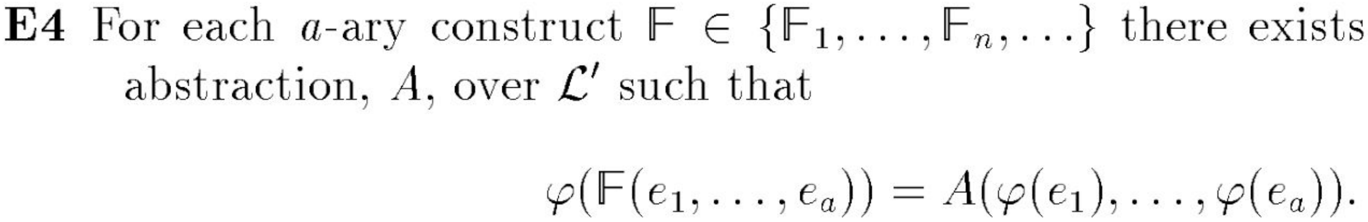
There is no *distinguishing context* for F and ϕ(F).

Macro-expressivity

E4 For each a-ary construct $\mathbb{F} \in \{\mathbb{F}_1, \ldots, \mathbb{F}_n, \ldots\}$ there exists an a-ary syntactic abstraction, A, over $\mathcal{L}'$ such that

$$\varphi(\mathbb{F}(e_1, \ldots, e_a)) = A(\varphi(e_1), \ldots, \varphi(e_a)).$$





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```
For (init, test, update, body)
```

```
While (test, body)
```

Macro expressivity: Example

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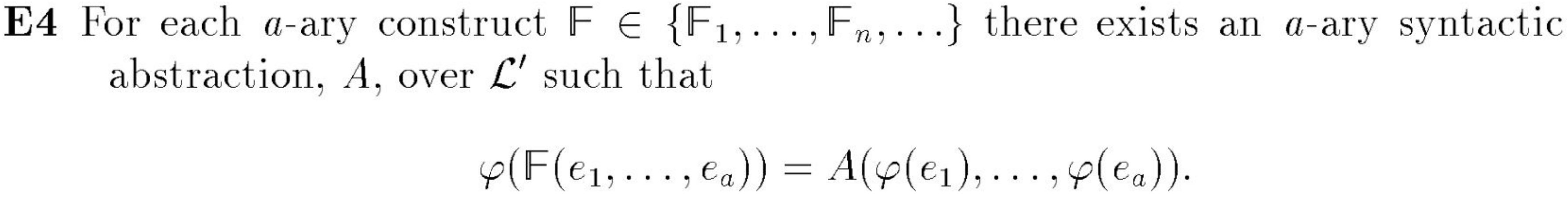

E4 For each a-ary construct $\mathbb{F} \in \{\mathbb{F}_1, \ldots, \mathbb{F}_n, \ldots\}$ there exists an a-ary syntactic abstraction, A, over $\mathcal{L}'$ such that

$$\varphi(\mathbb{F}(e_1, \ldots, e_a)) = A(\varphi(e_1), \ldots, \varphi(e_a)).$$

```
For (init, test, update, body)
```

```
For (init, test, upd, body) => init in While (ϕ(test), ϕ(body) ++ ϕ(update))
```

```
While (test, body)
```

Polymorphic **let**

$$\frac{A \vdash e : \tau; A \vdash b[x/e] : \tau'}{A \vdash \textbf{let}\ x = e\ \textbf{in}\ b : \tau'}$$

Call-by-value STLC

Expressive but Macro-inexpressive

Polymorphic **let**

$$\boxed{\frac{A \vdash e : \tau; A \vdash b[x/e] : \tau'}{A \vdash \mathbf{let}\ x = e\ \mathbf{in}\ b : \tau'}}$$

```
Let (x, e, b) => (fun x -> subst(x, φ(e), φ(b))) φ(e)
```

Call-by-value STLC

Expressive!

Expressive but Macro-inexpressive

AST function, not a *syntactic abstraction!*

```
Let (x, e, b) => (fun x -> subst(x, φ(e), φ(b))) φ(e)
```

Recursive macros are not a problem! Macro-based **subst** implementation will generate scoped macros. **subst** is truly performing a *compile-time computation.*

Expressive but Macro-inexpressive

What do we get?

Eliminability

(Macro) expressivity

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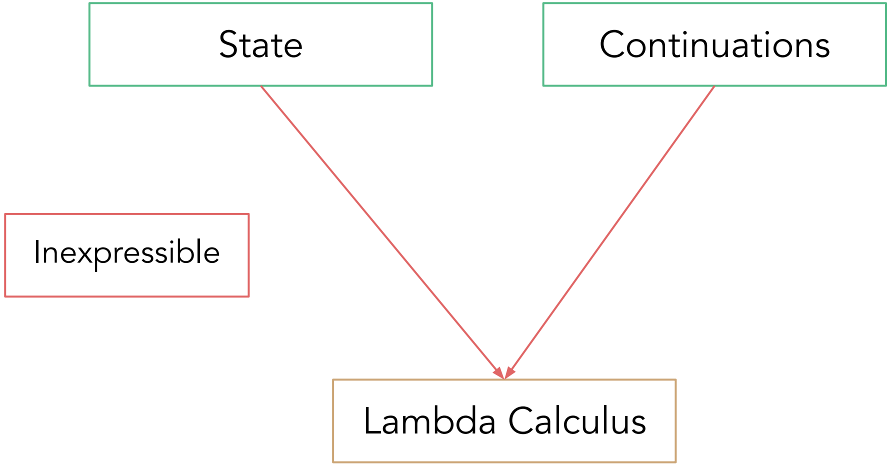
Eliminability

(Macro) expressivity



State

Continuations

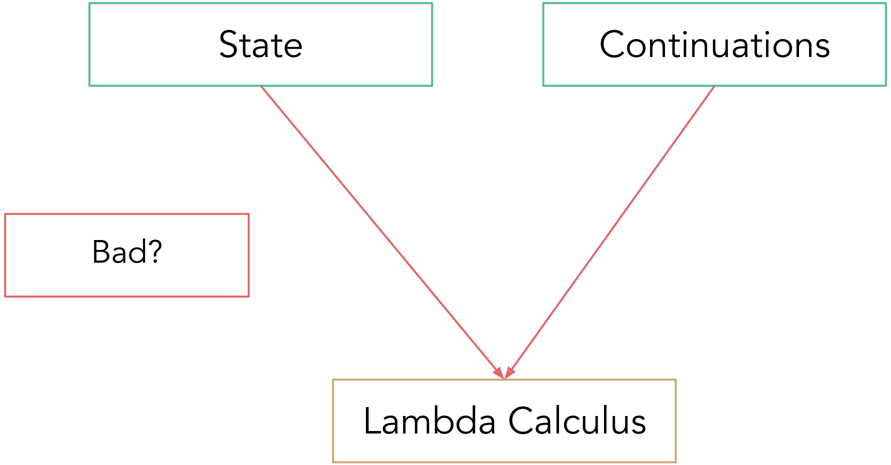
Inexpressible

Lambda Calculus

What do we get?

Eliminability

(Macro) expressivity



State

Continuations

Bad?

Lambda Calculus

Case Study

Essence of JavaScript*

```
let x = {
  a: 10,
  b: 20,
}

> { a: 10, b: 20 }
```

```
with (x) {
  a + b + 10
}

> 30
```

* The Essence of JavaScript (2010); Arjun Guha, Claudiu Saftoiu, and Shriram Krishnamurthi

Essence of JavaScript*

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```

Lambda Calculus + objects

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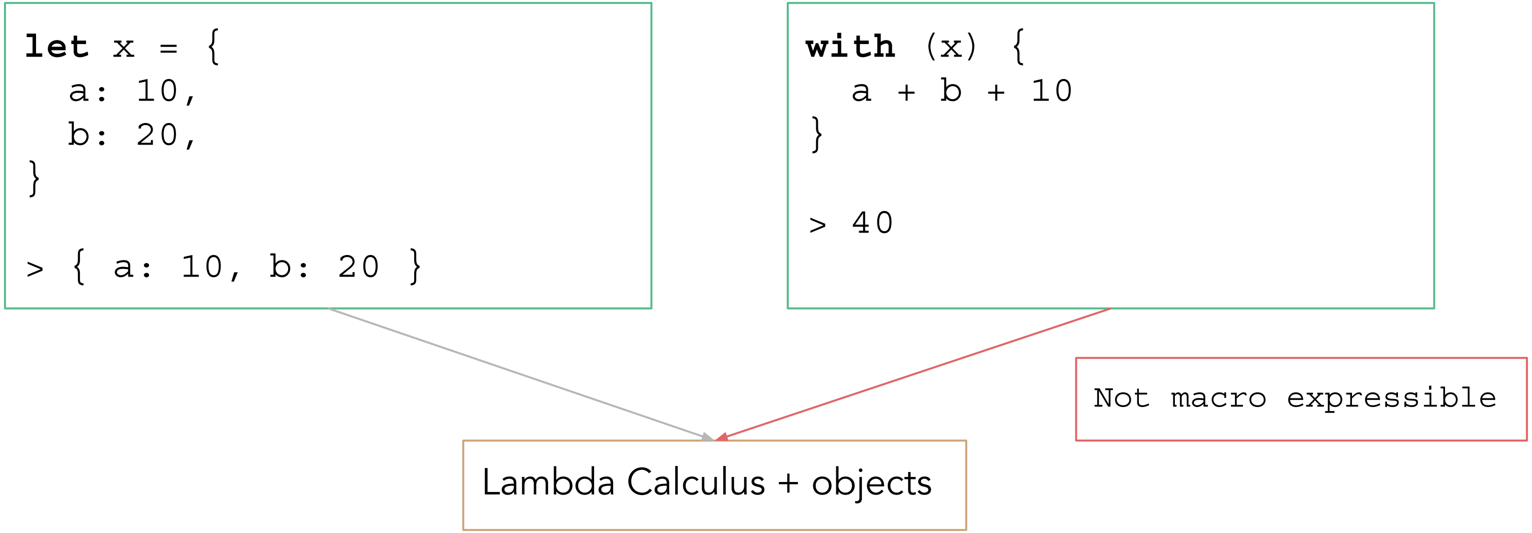
Essence of JavaScript*



```
let x = {
  a: 10,
  b: 20,
}

> { a: 10, b: 20 }
```

```
with (x) {
  a + b + 10
}

> 40
```

Not macro expressible

Lambda Calculus + objects

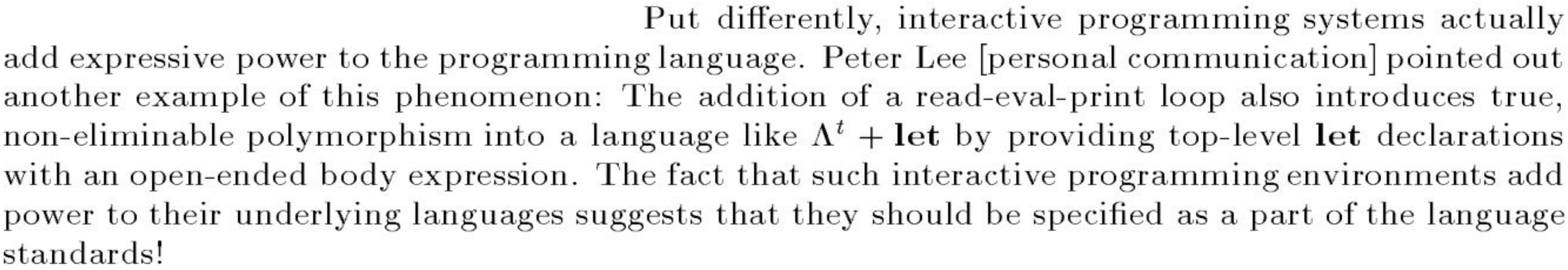
* The Essence of JavaScript (2010); Arjun Guha, Claudiu Saftoiu, and Shriram Krishnamurthi

Thanks!

```
(call/cc
 (lambda (return)
   (while (true)
     (return "Power Overwhelming!"))))
```

Discussion points

- Expressivity as a language design principle vs type directed language design.
- Why is this not the prevailing way of designing languages?
- Programming languages: isolated mathematical formalisms or complete ecosystems?



Put differently, interactive programming systems actually add expressive power to the programming language. Peter Lee [personal communication] pointed out another example of this phenomenon: The addition of a read-eval-print loop also introduces true, non-eliminable polymorphism into a language like $\Lambda^t + \textbf{let}$ by providing top-level **let** declarations with an open-ended body expression. The fact that such interactive programming environments add power to their underlying languages suggests that they should be specified as a part of the language standards!