CS6840 - Algorithmic Game Theory (2 pages) Spring 2014

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Auctions with More Complex Valuations

So far we studied second price style auctions for the following valuations.

$$v_1 \in \mathbb{R}, \text{Single Item}$$
$$GSP$$
$$\textbf{(a)}\text{Unit Demand} \qquad u_i(A) = \max_j v_{ij}$$
$$\textbf{(b)}\text{Additive} \qquad u_i(A) = \sum_{j \in A} v_{ij}$$

For the additive valuations the optimal solution is $\sum_i \max_i v_{ij}$, where each auction is separate, and no collection between the items.

Today we will consider a General Class of Valuations. – Generalizing ***(a)*** and ***(b)*** – Each i possible ways to use items v_{ij}^k

(i) $v_i(A) = \max_k \sum_{j \in A} v_{ij}^k$

Claim. This class of valuations contains Unit Demand

$$v_{ij}^k = \begin{cases} v_{ij} & \text{if } k = j \\ 0 & \text{otherwise} \end{cases} \qquad (0, \ldots, 0, v_{ij}, 0, \ldots, 0)$$

Theorem. Item Auctions on Second Price each sold separately, bidders conservative,$\sum_{j \in A} b_{ij} \leq v_i(A)$ for all i and all subset of the items, then Social Welfare Nash (or CCE) $\geq \frac{1}{2}$OPT

Assuming Valuations of ***(i)*** form, $b_{ij} = i^{th}$ bid for item i, let the winning bid for item j be $b(j) = \max_i(b_{ij})$.

Proof. Consider OPT location. $O_1, \ldots, O_n$ set items going to bidders $1, \ldots, n$. $V_i(O_i) = \max_k(\sum_{j \in O_i} v_{ij}^k)$, and let k_i be the vector on which the maximum is achieved.

Now define $b_{ij}^* = v_{ij}^{k_i}$, and we claim that this bid satisfies the usual smoothness style inequality.

we have $u_i(b_i^*, b_{-i}) \geq \sum_{j \in O_i}(v_{ij}^{k_i} - b(j))$

(To See why, assume with this bid, person i wins a set A. Now

$$\begin{aligned} u_i(b_i^*, b_{-i}) &= V_i(A) - \sum_{j \in A} b(j) \geq \sum_{j \in A} (v_{ij}^{k_i} - b(j)) \\ &\geq \sum_{j \in (A \cap O_i)} (v_{ij}^{k_i} - b(j)) \\ &\geq \sum_{j \in (A \cap O_)} (v_{ij}^{k_i} - b(j)) \end{aligned}$$

Where the inequality in the top line follows from the definition of V_i, the inequality in teh second line follows as winning additional items $A \setminus O_i$ only make the value higher, and the last inequality follows as the added terms are negative.

Sum Over all players, and using that the bids b form an equilibrium (and hence deviating to b^* doesn't improve player utility), we get:

$$\begin{aligned} \sum u_i(b) &\geq \sum_i \sum_{j \in O_i} v_{ij}^{k_i} - \sum_i \sum_{j \in O_i} b(j) = SW(\text{OPT}) - \sum_j b(j) \\ &\geq SW(\text{OPT}) - \sum_i \sum_{j \in A_i} \geq SW(\text{OPT}) + \sum_i v_i(A_i) = \geq SW(\text{OPT}) + SW(\text{NASH}) \end{aligned}$$

where A_i is the set of items won by player i in Nash, and that last inequality used the assumption of no overbidding.

Now rearranging terms, and using the fact that $\sum u_i(b) \leq SW(\text{NASH})$ we get

$$\sum u_i(b) + \sum v_i(A_i) \geq SW(\text{OPT})$$

□

Next class we will talk about what valuations can be written in the form used in this proof.